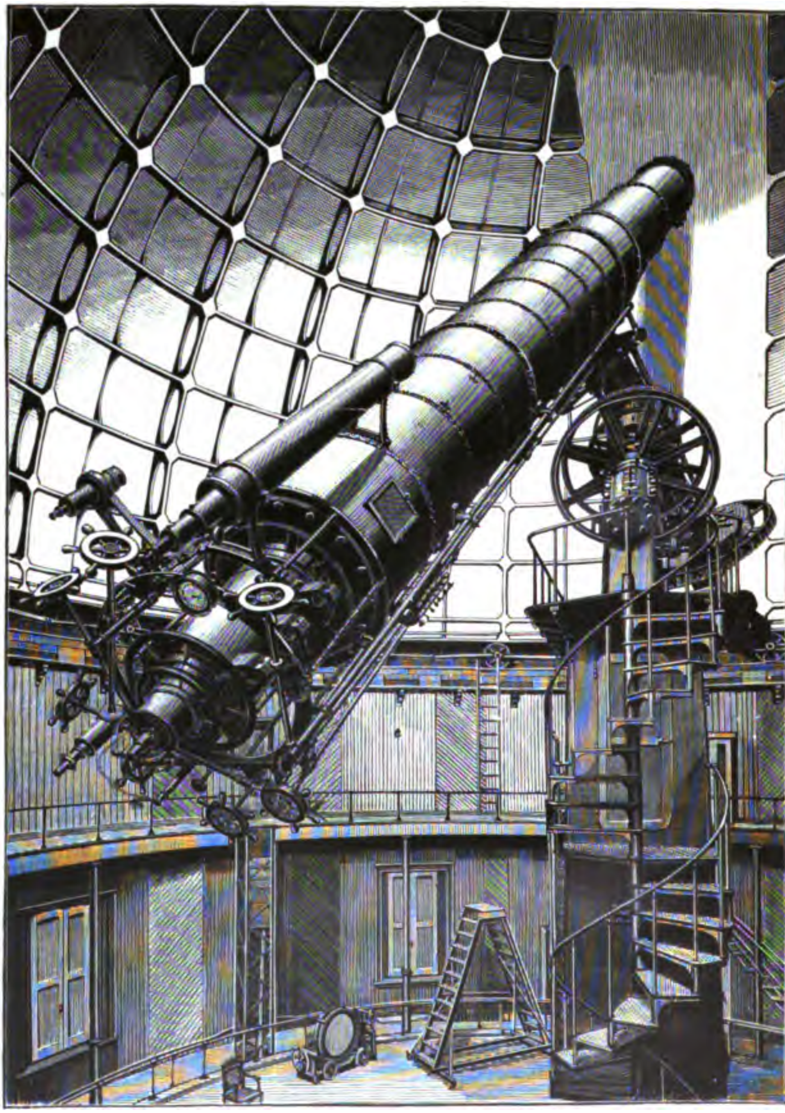

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A TEXT-BOOK

OF

GENERAL ASTRONOMY

FOR

COLLEGES AND SCIENTIFIC SCHOOLS

BY

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REVISED EDITION

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PREFACE TO FIRST EDITION.

The present work is designed as a text-book of Astronomy suited to the *general* course in our colleges and schools of science, and is meant to supply that amount of information upon the subject which may fairly be expected of every "liberally educated" person. While it assumes the previous discipline and mental maturity usually corresponding to the latter years of the college course, it does not demand the peculiar mathematical training and aptitude necessary as the basis of a *special* course in the science — only the most elementary knowledge of Algebra, Geometry, and Trigonometry is required for its reading. Its aim is to give a clear, accurate, and justly proportioned presentation of astronomical facts, principles, and methods in such a form that they can be easily apprehended by the average college student with a reasonable amount of effort.

The limitations of time are such in our college course that probably it will not be possible in most cases for a class to take thoroughly everything in the book. The fine print is to be regarded rather as collateral reading, important to a complete view of the subject, but not essential to the course. Some of the chapters can even be omitted in cases where it is found necessary to abridge the course as much as possible; *e.g.*, the chapters on Instruments and on Perturbations.

While the work is no mere compilation, it makes no claims to special originality: information and help have been drawn from all available sources. The author is under great obligations to the astronomical histories of Grant and Wolf, and especially to Miss Clerke's admirable "History of Astronomy in the Nineteenth Century." Many data also have been drawn from Houzeau's valuable "Vade Mecum de l'Astronome."

It has been intended to bring the book well down to date, and to indicate to the student the sources of information on subjects which are necessarily here treated inadequately on account of the limitations of time and space.

Special acknowledgments are due to Professor Langley and to his publishers, Messrs. Ticknor & Co., for the use of a number of illustrations from his beautiful book, "The New Astronomy"; and also to D. Appleton & Co. for the use of several cuts from the author's little book on the Sun. Professor Trowbridge of Cambridge kindly provided the original negative from which was made the cut illustrating the comparison of the spectrum of iron with that of the sun. Warner & Swasey of Cleveland and Fauth & Co. of Washington have also furnished the engravings of a number of astronomical instruments.

Professors Todd, Emerson, Upton, and McNeill have given most valuable assistance and suggestions in the revision of the proof; as indeed, in hardly a less degree, have several others.

PRINCETON, N. J., August, 1888.

PREFACE TO THE REVISED EDITION.

THE progress of Astronomy has been very rapid since the first publication of this book in 1889, and, although in the meantime the author has attempted as far as possible to keep the successive issues "up to date" by minor changes, notes, and "addenda," it has at last become imperative to give the work a thorough revision, rewriting certain portions and making considerable additions, in order to embody the new and important results which have been obtained during the last ten years.

The Appendix has also been enlarged by several articles giving the demonstration of certain fundamental methods and

formulæ for which, in previous editions, the student was referred to other works not always conveniently accessible. In one or two of these articles the Calculus is necessarily used.

The various tables have been corrected to correspond with the latest and most authoritative data; and a set of illustrative exercises has been added at the end of nearly every chapter.

While the book has thus been necessarily somewhat increased in size, the changes have been so managed that no serious difficulty will be encountered in using the new edition along with the older issues. The original numbering of the *articles* has been retained throughout, with only one or two exceptions, the interpolated matter being designated by numbers with asterisks.

It is believed that the book, so far as its scope extends, may now be taken as fairly representing the present state of the science, although some of the most important recent discoveries are hardly made so prominent as would have been the case if the revision had not been substantially completed and prepared for the press more than two years ago; the actual printing having been much delayed by various causes.

Special acknowledgments are due from the author to the publishers for the liberality with which they have made the extensive and expensive changes in the plates, and to Appleton & Co., and Professors Frost, Hale, Holden, and Pickering for many of the new illustrations.

Princeton University, March, 1898.

PREFACE TO ISSUE OF 1904.

In this issue of the Revised Edition a considerable number of corrections, changes, and additions have been made in the text, and three Addenda have been appended, in order to bring the book up to date as far as possible.

August, 1904.

TABLE OF CONTENTS.

INTRODUCTION.

1. **Astronomy** (ἄστρον νόμος) is the science which treats of the heavenly bodies. As such bodies we reckon the sun and moon, the planets (of which the earth is one) and their satellites, comets and meteors, and finally the stars and nebulæ.

We have to consider in Astronomy: —

(*a*) The motions of these bodies, both real and apparent, and the laws which govern these motions.

(*b*) Their forms, dimensions, and masses.

(*c*) Their nature constitution, and conditions.

(*d*) The effects they produce upon each other by their attractions, radiations, or by any other ascertainable influence.

It was an early, and has been a most persistent, belief that the heavenly bodies have a powerful influence upon human affairs, so that from a knowledge of their positions and "aspects" at critical moments (as for instance at the time of a person's birth) one could draw up a "horoscope" which would indicate the probable future.

The *pseudo-science* which was founded on this belief was named Astrology, — the elder sister of Alchemy, — and for centuries Astronomy was its handmaid; *i.e.*, astronomical observations and calculations were made mainly in order to supply astrological data.

At present the end and object of astronomical study is chiefly knowledge pure and simple; so far as now appears, its development has less direct bearing upon the material interests of mankind than that of any other of the natural sciences. It is not likely that great inventions and new arts will grow out of its laws and principles, such as are continually arising from physical, chemical, and biological discoveries, though of course it would be rash to say that such outgrowths are impossible. But the student of Astronomy must expect his chief profit to be intellectual, in the widening of the range of thought and conception, in the pleasure attending the discovery of simple law working out the most complicated results, in the delight

over the beauty and order revealed by the telescope in systems otherwise invisible, in the recognition of the essential unity of the material universe, and of the kinship between his own mind and the infinite Reason that formed all things and is immanent in them.

At the same time it should be said at once that, even from the lowest point of view, Astronomy is far from a useless science. The *art of navigation* depends for its very possibility upon astronomical prediction. Take away from mankind their almanacs, sextants, and chronometers, and commerce by sea would practically stop. The science also has important applications in the survey of extended regions of country, and the establishment of boundaries, to say nothing of the accurate determination of time and the arrangement of the calendar.

It need hardly be said that Astronomy is not separated from kindred sciences by sharp boundaries. It would be impossible, for instance, to draw a line between Astronomy on one side and Geology and Physical Geography on the other. Many problems relating to the formation and constitution of the earth belong alike to all three.

2. Astronomy is divided into many branches, some of which, as ordinarily recognized, are the following: —

1. **Descriptive Astronomy.** — This, as its name implies, is merely an orderly statement of astronomical facts and principles.

2. **Practical Astronomy.** — This is quite as much an art as a science, and treats of the instruments, the methods of observation, and the processes of calculation by which astronomical facts are ascertained.

3. **Theoretical Astronomy,** which treats of the calculations of orbits and ephemerides, including the effects of so-called "perturbations."

4. **Mechanical Astronomy,** which is simply the application of mechanical principles to explain astronomical facts (chiefly the planetary and lunar motions). It is sometimes called *Gravitational* Astronomy, because, with few exceptions, gravitation is the only force sensibly concerned in the motions of the heavenly bodies. Until within thirty years this branch of the science was generally designated as *Physical Astronomy*, but the term is now objectionable because of late it has been used by many writers to denote a very different and comparatively new branch of the science; viz., —

5. **Astronomical Physics, or Astro-physics.** — This treats of the physical characteristics of the heavenly bodies, their brightness and spectroscopic peculiarities, their temperature and radiation, the nature and condition of their atmospheres and surfaces, and all phenomena which indicate or depend on their physical condition.

6. **Spherical Astronomy.** — This, discarding all consideration of absolute dimensions and distances, treats the heavenly bodies simply as objects moving on the "surface of the celestial sphere": it has to do only with angles and directions, and, strictly regarded, is in fact merely Spherical Trigonometry applied to Astronomy.

3. The above-named branches are not distinct and separate, but they overlap in all directions. Spherical Astronomy, for instance, finds the demonstration of many of its formulæ in Gravitational Astronomy, and their application appears in Theoretical and Practical Astronomy. But valuable works exist bearing all the different titles indicated above, and it is important for the student to know what subjects he may expect to find discussed in each; for this reason it has seemed worth while to name and define the several branches, although they do not distribute the science between them in any strictly logical and mutually exclusive manner.

In the present text-book little regard will be paid to these subdivisions, since the object of the work is not to present a complete and profound discussion of the subject such as would be demanded by a professional astronomer, but only to give so much knowledge of the facts and such an understanding of the principles of the science as may fairly claim to be elements in a liberal education. If this result is gained in the reader's case, it may easily happen that he will wish for more than he can find in these pages, and then he must have recourse to works of a higher order and far more difficult, dealing with the subject more in detail and more thoroughly.

To master the present book no further preparation is necessary than a very elementary knowledge of Algebra, Geometry, and Trigonometry, and a similar acquaintance with Mechanics and Physics, especially Optics. While nothing short of high mathematical attainments will enable one to become eminent in the science, yet a perfect comprehension of all its fundamental methods and principles, and a very satisfactory acquaintance with its main results, is quite within the reach of every person of ordinary intelligence, without any more extensive training than may be had in

our common schools. At the same time the necessary statements and demonstrations are so much facilitated by the use of trigonometrical terms and processes that it would be unwise to dispense with them entirely in a work to be used by pupils who have already become acquainted with them.

In discussing the different subjects which present themselves, the writer will adopt whatever plan appears best fitted to convey to the student clear and definite ideas, and to impress them upon the mind. Usually it will be best to proceed in the Euclidean order, by first stating the fact or principle in question, and then explaining its demonstration. But in some cases the inverse process is preferable, and the conclusion to be reached will appear gradually unfolding itself as the result of the observations upon which it depends, just as its discovery came about.

The occasional references to "Physics" refer to the "Elementary Text-Book of Physics," by Anthony and Brackett; Magie's revised edition, 1897. John Wiley & Sons, N.Y.

CHAPTER I.

THE "DOCTRINE OF THE SPHERE," DEFINITIONS, AND GENERAL CONSIDERATIONS.

Astronomy, like all the other sciences, has a terminology of its own, and uses technical terms in the description of its facts and phenomena. In a popular essay it would of course be proper to avoid such terms as far as possible, even at the expense of circumlocutions and occasional ambiguity; but in a text-book it is desirable that the reader should be introduced to the most important of them at the very outset, and made sufficiently familiar with them to use them intelligently and accurately.

4. The Celestial Sphere. — To an observer looking up to the heavens at night it seems as if the stars were glittering points attached to the inner surface of a dome; since we have no direct perception of their distance there is no reason to imagine some nearer than others, and so we involuntarily think of the surface as *spherical* with ourselves in its centre. Or if we sometimes feel that the stars and other objects in the sky really differ in distance, we still instinctively imagine an immense sphere surrounding and enclosing all. Upon this sphere we imagine lines and circles traced, resembling more or less the meridians and parallels upon the surface of the earth, and by reference to these circles we are able to describe intelligently the apparent positions and motions of the heavenly bodies.

This celestial sphere may be regarded in either of two different ways, both of which are correct and lead to identical results.

(*a*) We may imagine it, in the first place, as transparent, and of merely finite (though undetermined) dimensions, *but in some way so attached to, and connected with, the observer that his eye always remains at its centre wherever he goes.* Each observer, in this way of viewing it, carries his own sky with him, and is the centre of his own heavens.

(*b*) Or, in the second place, — and this is generally the more convenient way of regarding the matter, — we may consider the celestial

sphere as mathematically *infinite* in its dimensions: then, let the observer go where he will, he cannot sensibly get away from its centre. Its radius being "greater than any assignable quantity," the size of continents, the diameter of the earth, the distance of the sun, the orbits of planets and comets, even the spaces between the stars, are all insignificant, and the whole visible universe shrinks *relatively* to a mere point at its centre. In what follows we shall use this conception of the celestial sphere.[1]

The apparent place of any celestial body will then be the point on the celestial sphere where the line drawn from the eye of the observer in the direction in which he sees the object, and produced indefinitely, pierces the sphere. Thus, in Figure 1, *A*, *B*, *C* are the apparent places of *a*, *b*, and *c*, the observer being at *O*. The apparent place of a heavenly body evidently depends solely upon its *direction*, and is wholly independent of its *distance* from the observer.

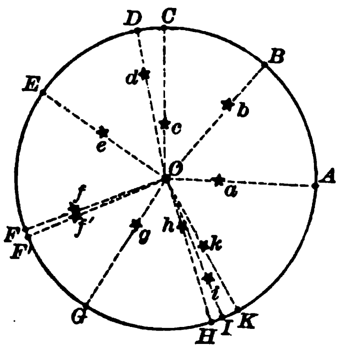

FIG. 1.

5. Linear and Angular Dimensions. —Linear dimensions are such as may be expressed in *linear* units; *i.e.*, in miles, feet, or inches; in metres or millimetres. Angular dimensions are expressed in *angular* units; *i.e.*, in right angles, in radians,[2] or (more commonly in astronomy) in degrees, minutes, and seconds. Thus, for instance, the *linear* semi-

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[1] To most persons the sky appears, not a true hemisphere, but a *flattened* vault, as if the horizon were more remote than the zenith. This is a subjective effect due mainly to the intervening objects between us and the horizon. The sun and moon when rising or setting look much larger than when they are higher up, for the same reason.

[2] *A radian* is the angle which is measured by an arc equal in length to radius. Since a circle whose radius is unity has a circumference of 2π, and contains 360°, or 21,600′, or 1,296,000″, it follows that a *radian* contains $\left(\frac{360}{2\pi}\right)^{\circ}$, or $\left(\frac{21600}{2\pi}\right)'$, or $\left(\frac{1296000}{2\pi}\right)''$; *i.e.* (approximately), a radian $= 57.3° = 3437.7' = 206264.8''$.

Hence, to reduce to seconds of arc an angle expressed in radians, we must multiply it by the number 206264.8; a relation of which we shall have to make frequent use.

diameter of the sun is about 697,000 kilometres (433,000 miles), while its *angular* semidiameter is about 16′, or a little more than a quarter of a degree. Obviously, angular units alone can properly be used in describing apparent distances and dimensions in the sky. For instance, one cannot say correctly that the two stars which are known as "the pointers" are two or five or ten *feet* apart: their distance is about five *degrees*.

It is sometimes convenient to speak of "*angular area*," the unit of which is a "square degree" or a "square minute"; *i.e.*, a small square in the sky of which each side is 1° or 1′. Thus we may compare the angular area of the constellation Orion with that of Taurus, in *square degrees*, just as we might compare Pennsylvania and New Jersey in square miles.

6. Relation between the Distance and Apparent Size of an Object. —Suppose a globe having a radius BC equal to r. As seen from

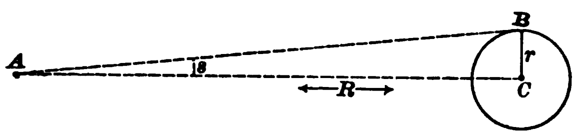

FIG. 2.

the point A (Fig. 2) its apparent (*i.e.*, *angular*) semidiameter will be BAC or s, its distance being AC or R.

We have immediately from Trigonometry, since B is a right angle,

$$\sin s = \frac{r}{R}.$$

If, as is usual in Astronomy, the diameter of the object is small as compared with its distance, we may write $s = \frac{r}{R}$, which gives s in *radians* (not in degrees or seconds). If we wish it in the ordinary angular units,

$$s^\circ = 57.3\frac{r}{R}, \text{ or } s' = 3437.7\frac{r}{R}, \text{ or } s'' = 206264.8\frac{r}{R},$$

where s° means s in *degrees; s', s* in *minutes; s'', s* in *seconds* of arc. In either form of the equation we see that the apparent diameter *varies directly as the linear diameter, and inversely as the distance.*

In the case of the moon, $R =$ about 239,000 miles; and r, 1081 miles. Hence $s = \frac{1081}{239000} = \frac{1}{221}$ of a radian, which is a little more than $\frac{1}{4}$ of a degree, or about 933″.

It may be mentioned here as a rather curious fact that most persons say that the moon appears about *a foot in diameter;* at least, this seems to be the average estimate.[1] This implies that the surface of the sky appears to them only about 110 feet away, since that is the distance at which a disc one foot in diameter would have an angular diameter of $\frac{1}{110}$ of a radian, or $\frac{1}{2}°$.

7. Vanishing Point. — Any system of parallel lines produced in one direction will *appear* to pierce the celestial sphere at a single point. They actually pierce it at different points, separated on the surface of the sphere by linear distances equal to the actual distances between the lines, but on the infinitely distant surface these linear distances, being only finite, become invisible, subtending at the centre angles less than anything assignable. The different points, therefore, coalesce into a *spot* of apparently infinitesimal size — the so-called "vanishing point" of perspective. Thus the axis of the earth and *all lines parallel to this axis* point to the celestial pole.

POINTS AND CIRCLES OF REFERENCE.

8. The Zenith. — The Zenith is the *point vertically overhead, i.e.*, the point where a plumb-line, produced upwards, would pierce the sky: it is determined by the *direction of gravity* where the observer stands.

If the earth were exactly spherical, the zenith might also be defined as the point where a line drawn *from the centre of the earth upward through the observer* meets the sky. But since, as we shall see hereafter, the earth is not an exact globe, this second definition indicates a point known as the *Geocentric Zenith*, which is not identical with the *True* or *Astronomical Zenith*, determined by the direction of gravity.

9. The Nadir. — The Nadir is the point opposite the zenith — under foot, of course.

Both zenith and nadir are derived from the Arabic, which language has also given us many other astronomical terms.

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[1] See note on p. 20, at the end of the chapter.

10. Horizon. — The Horizon[1] is a great circle of the celestial sphere, having the zenith and nadir as its poles: it is therefore half-way between them, and 90° from each.

A *horizontal plane*, or the *plane of the horizon*, is a plane perpendicular to the direction of gravity, and the horizon may also be correctly defined as the intersection of the celestial sphere by this plane.

Many writers make a distinction between the *sensible* and *rational* horizons. The plane of the sensible horizon passes through the observer; the plane of the rational horizon passes through the centre of the earth, parallel to the plane of the sensible horizon: these two planes, parallel to each other, and everywhere about 4000 miles apart, trace out on the sky the two horizons, the sensible and the rational. It is evident, however, that on the infinitely distant surface of the celestial sphere, the two traces sensibly coalesce into one single great circle, which is the horizon as first defined. We get, therefore, but one *horizon circle* in the sky.

11. The Visible Horizon is the line where sky and earth meet. On land it is an irregular line, broken by hills and trees, and of no astronomical value; but at sea it is a true circle, and of great importance in observation. It is not, however, a *great* circle, but, technically speaking, only a *small* circle; depressed below the true horizon by an amount depending upon the observer's elevation above the water. This depression is called the *Dip of the Horizon*, and will be discussed further on.

12. Vertical Circles. — These are great circles passing through the zenith and nadir, and therefore necessarily perpendicular to the horizon — *secondaries* to it, to use the technical term.

Parallels of Altitude, or Almucantars. — These are small circles parallel to the horizon: the term Almucantar is seldom used.

The points and circles thus far defined are determined entirely by the *direction of gravity* at the station occupied by the observer.

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13. The Diurnal Rotation of the Heavens. — If one watches the sky for a few hours some night, he will find that, while certain stars rise in the east, others set in the west, and nearly all the constellations change their places. Watching longer and more closely, it will

[1] Beware of the common, but vulgar, pronunciation, *Hórizon.*

appear that the stars move in circles, uniformly, in such a way as not to disturb their relative configurations, but as if they were attached to the inner surface of a revolving sphere, turning on its axis once a day. The path thus daily described by a star is called its "*diurnal circle.*"

It is soon evident that in our latitude the visible "pole" of this sphere — the point about which it turns — is in the north, not quite half-way up from the horizon to the zenith, for in that region the stars hardly move at all, but keep their places all night long.

14. The Poles. — The Poles may be defined as the two points in the sky, one in the northern hemisphere and one in the southern,

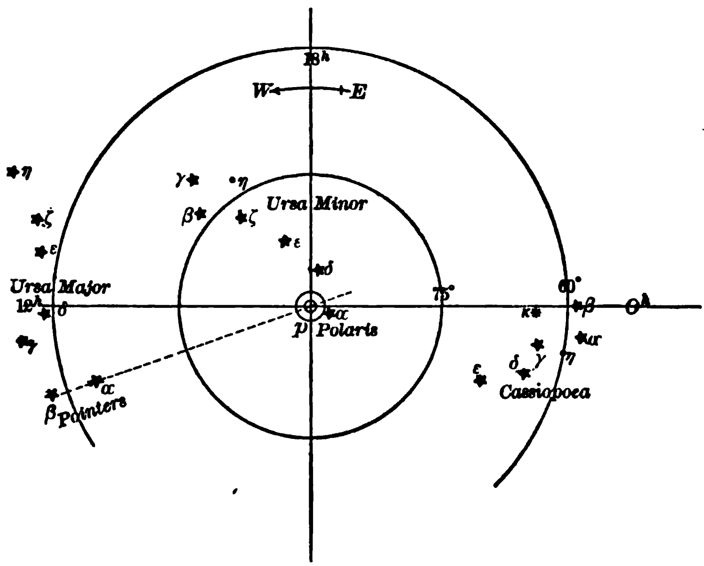

FIG. 3. — The Pole Star and the Pointers.

where a *star's diurnal circle reduces to zero ; i.e.*, points where, if a star were placed, it would suffer no apparent change of place during the whole twenty-four hours. The line joining these poles is, of course, the *axis* of the celestial sphere, about which it seems to rotate daily.

The exact place of the pole may be found by observing some star very near the pole at two times 12 hours apart, and taking the middle point between the two observed places of the star.

The definition of the pole just given is independent of any theory as to the cause of the apparent rotation of the heavens. If, how-

ever, we admit that it is due to the earth's rotation on its axis, then we may define the poles as the *points where the earth's axis produced pierces the celestial sphere.*

15. The Pole-star (Polaris).—The place of the northern pole is very conveniently marked by the *Pole-star*, a star of the second magnitude, which is now only about $1\frac{1}{4}°$ from the pole: we say *now*, because on account of a slow change in the direction of the earth's axis, called "precession" (to be discussed later), the distance between the pole-star and the pole is constantly changing, and has been for several centuries gradually decreasing.

The pole-star stands comparatively solitary in the sky, and may easily be recognized by means of the so-called "pointers,"—two stars in the "dipper" (in the constellation of Ursa Major)—which point very nearly to it, as shown in Fig. 3. The pole is very nearly on the line joining Polaris with the star Mizar (ζ Urs. Maj., at the bend in the handle of the dipper), and at a distance just about one-quarter of the distance between the pointers, which are nearly 5° apart.

The southern pole, unfortunately, is not so marked by any conspicuous star.

16. The Celestial Equator, or Equinoctial Circle.—This is a great circle midway between the two poles, and of course 90° from each. It may also be defined as the intersection of the plane of the earth's equator with the celestial sphere. It derives its name from the fact that, at the two dates in the year when the sun crosses this circle—about March 20 and Sept. 22—the day and night are equal in length.

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17. The Vernal Equinox, or First of Aries.—The Equinox, strictly speaking, is the *time when* the sun crosses the equator, but the term has come by accommodation to denote also the *point where* it crosses. This crossing occurs twice a year, about March 20th and September 22d, and the *Vernal Equinox is the point on the equator where the sun crosses it in the spring.* It is sometimes called the *Greenwich of the Celestial Sphere,* because it is used as a reference point in the sky, much as Greenwich is on the earth. Its position is not marked by any conspicuous star.

Why this point is also called the "First of Aries" will appear later, when we come to speak of the zodiac and its "signs."

18. Hour-Circles. — Hour-circles are great circles of the celestial sphere passing through its poles, and consequently perpendicular to the celestial equator. They correspond exactly to the meridians of the earth, and some writers call them "Celestial Meridians"; but the term is objectionable, as likely to lead to confusion with *the* Meridian, to be noted immediately.

19. The Meridian and Prime Vertical. — *The Meridian is the great circle passing through the pole and the zenith.* Since it is a great circle, it must necessarily pass through *both* poles, and through the nadir as well as the zenith, and must be perpendicular both to the equator and to the horizon.

It may also be correctly defined as the *Vertical Circle* which passes through the *pole;* or, again, as the *Hour-Circle* which passes through the *zenith*, since all vertical circles must pass through the zenith, and all hour-circles through the pole.

The Prime Vertical is the Vertical Circle (passing through the zenith) at right angles to the meridian; hence lying *east and west* on the celestial sphere.

20. The Cardinal Points. — The North and South Points are the points on the horizon where it is intersected by the meridian; the East and West Points are where it is cut by the prime vertical, and also by the equator. The North *Point*, which is on the horizon, must not be confounded with the North *Pole*, which is not on the horizon, but at an elevation equal (see Art. 30) to the latitude of the observer.

With these circles and points of reference we have now the means to describe intelligibly the position of a heavenly body, in several different ways.

We may give its *altitude* and *azimuth*, or its *declination* and *hour-angle;* or, if we know the time, its *declination* and *right ascension*. Either of these pairs of co-ordinates, as they are called, will define its place in the sky.

21. Altitude and Zenith Distance (Fig. 4). — The Altitude of a heavenly body is *its angular elevation above the horizon*, and is measured by the arc of the vertical circle passing through the body, and intercepted between it and the horizon.

The Zenith Distance of a body is simply its angular distance from the zenith, and is the complement of the altitude. Altitude + Zenith Distance = 90°.

22. Azimuth and Amplitude (Fig. 4).—The Azimuth of a body is the *angle at the zenith, between the meridian and the vertical circle, which passes through the body.* It is measured also by the arc of the horizon intercepted between the north or south point, and the foot of this vertical. The word is of Arabic origin, and has the same meaning as the *true bearing* in surveying and navigation.

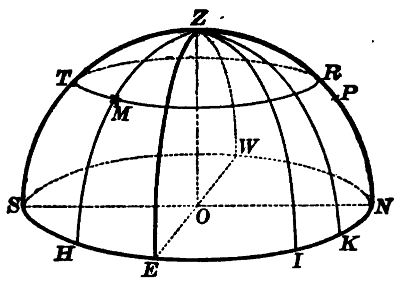

FIG. 4.—The Horizon and Vertical Circles.

O, the place of the Observer.	*M*, some Star.
OZ, the Observer's Vertical.	*ZMH*, arc of the Star's Vertical Circle.
Z, the Zenith; *P*, the Pole.	*TMR*, the Star's Almucantar.
SENW, the Horizon.	Angle *TZM*, or arc *SWNEH*, Star's *Azimuth*.
SZPN, the Meridian.	Arc *HM*, Star's *Altitude*.
EZW, the Prime Vertical.	Arc *ZM*, Star's *Zenith Distance*.

There are various ways of reckoning azimuth. Many writers express it in the same manner as *the bearing* is expressed in surveying; *i.e.*, so many degrees east or west of north or south; N. 20° E., etc. The more usual way at present is, however, to reckon it in degrees from the south point clear round through the west to the point of beginning: thus an object in the SW. would have an azimuth of 45°; in the NW., 135°; in the N., 180°; in the NE., 225°; and in the SE., 315°. For example, to find a star whose azimuth is 260°, and altitude 60°, we must face N. 80° E., and then look up two-thirds of the way to the zenith. The object in this case has an *amplitude* of 10° N. of E., and a zenith distance of 30°. Evidently both the azimuth and altitude of a heavenly body are continually changing.

The *Amplitude* of a body is the angle intercepted between the Prime vertical and the Vertical circle which passes through the body.

In Fig. 4, *SENW* represents the horizon, *S* being the south point, and *Z* the zenith. The angle *SZM*, which numerically equals the arc *SH*, is the *Azimuth* of the star *M*; while *EZM*, or *EH*, is its *Amplitude*. *MH* is its *Altitude*, and *ZM* its *Zenith Distance*.

23. Declination and Polar Distance (Fig. 5). — The Declination of a heavenly body is its *angular distance north or south of the celestial equator*, and is measured by the arc of the hour-circle passing through the object, intercepted between it and the equator. It is reckoned positive (+) north of the celestial equator, and negative (—) south of it. Evidently it is precisely analogous to the latitude of a place on the earth. The *north-polar distance* of a star is its angular distance from the North Pole, and is simply the complement of the declination. Declination + North-Polar Distance = 90°.

The declination of a star remains always the same; at least, the slow changes that it undergoes need not be considered for our present purpose. "*Parallels of Declination*" are small circles parallel to the celestial equator.

24. The Hour-Angle (Fig. 5). — The Hour-Angle of a star is the *angle at the pole between the meridian and the hour-circle passing through the star.* It may be reckoned in degrees; but it also may be, and most commonly is, reckoned in *hours, minutes, and seconds of time;* the hour being equivalent to fifteen degrees, and the minute and second of time being equal to fifteen minutes and seconds of arc respectively.

Of course the hour-angle of an object is continually changing, being zero when the object is on the meridian, one hour, or fifteen degrees, when it has moved that amount westward, and so on.

25. Right Ascension (Fig. 5). — The Right Ascension of a star is *the angle at the pole between the star's hour-circle and the hour-circle (called the Equinoctial Colure), which passes through the vernal equinox.*

It may be defined also as the arc of the equator, intercepted between the vernal equinox and the foot of the star's hour-circle.

It is always reckoned from the equinox *toward the east;* sometimes in degrees, but usually in *hours, minutes, and seconds of time. The right ascension, like the declination, remains unchanged by the diurnal motion.*

26. Sidereal Time (Fig. 5). — For many astronomical purposes it is convenient to reckon time, not by the sun's position in the sky, but by that of the vernal equinox.

The Sidereal Time at any moment may be defined as *the hour-angle of the vernal equinox.* It is sidereal *noon*, when the equinoctial point is on the meridian; 1 o'clock (sidereal) when its hour-angle is 15°; and 23 o'clock when its hour-angle is 345°, *i.e.*, when the vernal equinox is an hour *east* of the meridian; the time being reckoned round through the whole 24 hours. On account of the annual motion of the sun among the stars, the *Solar Day*, by which

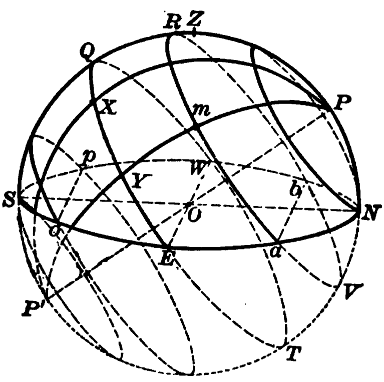

Fig. 5. — Hour-Circles, etc.

O, place of the Observer; *Z*, his Zenith.
SENW, the Horizon.
POP′, the Axis of the Celestial Sphere.
P and *P′*, the two Poles of the Heavens.
EQWT, the Celestial Equator, or Equinoctial.
X, the Vernal Equinox, or "First of Aries."
PXP′, the Equinoctial Colure, or Zero Hour-Circle.

m, some Star.
Ym, the Star's *Declination*; *Pm*, its *North-polar Distance*.
Angle mPR = arc QY, the Star's (eastern) *Hour-Angle*; $= 24^h$ *minus* Star's (western) Hour-Angle.
Angle XPm = arc XY, Star's *Right Ascension*. Sidereal time at the moment $= 24^h$ *minus* angle XPQ.

time is reckoned for ordinary purposes, is about 4 minutes longer than the sidereal day. The exact difference is $3^m\,56^s.556$ (sidereal), or just one day in a year; there being 366¼ *sidereal* days in the year, as against 365¼ *solar* days. See also Arts. 110 and 1000.

27. Observatory Definition of Right Ascension. — It is evident from the above definition of sidereal time that we may also define the Right Ascension of a star as *the sidereal time when the star crosses the meridian.* The Star and the Vernal Equinox are (practically)

fixed points in the sky, and do not change their relative position during the sky's apparent daily revolution; a given star, therefore, always comes to the meridian of any observer the same number of hours after the vernal equinox has passed; and this number of hours is the sidereal time at the moment of the star's transit, and measures its right ascension. In the observatory, this definition of right ascension is the most natural and convenient.

It is obvious that the right ascension of a star corresponds in the sky exactly with the *longitude* of a place on the earth; terrestrial longitude being reckoned from Greenwich, just as right ascension is reckoned from the vernal equinox.

N.B. *We shall find hereafter that the heavenly bodies have latitudes and longitudes of their own; but unfortunately these celestial latitudes and longitudes do not correspond to the terrestrial, and great care is necessary to prevent confusion.* (See Art. 179.)

28. An *armillary sphere*, or some equivalent apparatus, is almost essential to enable a beginner to get correct ideas of the points, circles, and co-ordinates defined above, but the figures will perhaps be of assistance.

The first of them (Fig. 4) represents the horizon, meridian, and prime vertical, and shows how the position of a star is indicated by its altitude and azimuth. This framework of circles, depending upon the direction of gravity, to an observer at any given station always remains *apparently* unchanged in position, while the sky apparently turns around outside it.

The other figure (Fig. 5) represents the system of points and circles which depend upon the earth's rotation, and are independent of the direction of gravity. The vernal equinox and the hour-circles apparently revolve with the stars while the pole remains fixed upon the meridian, and the equator and parallels of declination, revolving truly in their own planes, also appear to be at rest in the sky. But the whole system of lines and points represented in the figure (horizon and meridian alone excepted) may be considered as attached to, or marked out upon, the inner surface of the celestial vault and whirling with it.

It need hardly be said that the "appearances are deceitful" — that which is really carried around by the earth's rotation is the observer, with his plumb-line and zenith, his horizon and meridian; while the stars stand still — at least, their motions in a day are insensible as seen from the earth.

At the poles of the earth, which are, mathematically speaking, "singular" points, the definitions of the Meridian, of North and South, etc., break down.

There the pole (celestial) and zenith coincide, and any number of circles may be drawn through the two points, which have now become one. The horizon and equator coalesce, and the only direction on the earth's surface is due south (or north) — east and west have vanished.

A single step of the observer will, however, remedy the confusion: zenith and pole will separate, and his meridian will again become determinate.

29. To recapitulate: The *direction of gravity* at the point where the observer stands determines the Zenith and Nadir, the Horizon, and the Almucantars (parallel to the Horizon), and all the vertical circles. One of the verticals, the *Meridian*, is singled out from the rest by the circumstance that it passes through the *pole* of the sky, marking the North and South Points where it cuts the horizon.

Altitude and Azimuth (or their complements, Zenith Distance and Amplitude) are the co-ordinates which designate the position of a body by reference to the Zenith and the Meridian.

Similarly, the *direction of the earth's axis* (which is independent of the observer's place on the earth) determines the Poles, the Equator, the Parallels of Declination, and the Hour-Circles. *Two* of these Hour-Circles are singled out as reference lines; one of them, the Meridian, which passes through the Zenith, and is a purely *local* reference line; the other, the Equinoctial Colure, which passes through the Vernal Equinox, a point chosen from its relation to the sun's annual motion. Declination and *Hour-Angle* are the co-ordinates which refer the place of a star to the Pole and the Meridian; while Declination and *Right Ascension* refer it to the Pole and Equinoctial Colure. The latter are the co-ordinates usually employed in star-catalogues and ephemerides to define the positions of stars and planets, and correspond to Latitude and Longitude on the earth.

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30. **Relation of the Apparent Diurnal Motion of the Sky to the Observer's Latitude.** — Evidently the apparent motions of the stars will be considerably influenced by the station of the observer, since the place of the pole in the sky will depend upon it. The *Altitude* of the pole, or its *height in degrees above* the horizon, is always equal to the *Latitude* of the observer. Indeed, the German word for latitude (astronomical) is *Polhöhe; i.e.*, simply "Pole-height."

This will be clear from Fig. 6. The latitude of a place is the *angle between its plumb-line and the plane of the equator;* the angle ONQ in the figure. [If the earth were truly spherical, N would coincide with C, the centre of the earth. The ordinary definition of latitude given in the geographies disregards the slight difference.]

Now the angle $H'OP''$ is equal to ONQ, because their sides are mutually perpendicular; and it is also the *altitude of the pole*, because the line HH' is horizontal at O, and OP'', being directed towards the celestial pole, is therefore parallel to $pCPP'$, the axis of the earth.

This fundamental relation, *that the altitude of the celestial pole is the Latitude of the observer*, cannot be too strongly impressed on the student's mind. The usual symbol for the latitude of a place is ϕ.

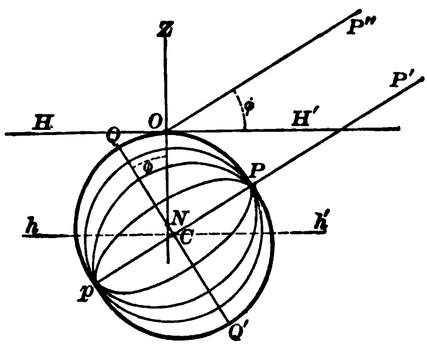

FIG. 6. — Relation of Latitude to the Elevation of the Pole.

31. The Right Sphere. — If the observer is situated at the earth's equator, *i.e.*, in latitude zero ($\phi = 0$), the pole will be in the horizon, and the equator will pass vertically overhead through the zenith.

The stars will rise and set vertically, and their diurnal circles will all be bisected by the horizon, so that they will be 12 hours above it and 12 below. This aspect of the heavens is called the *Right Sphere*.

32. The Parallel Sphere. — If the observer is at the pole of the earth ($\phi = 90°$), then the celestial pole will be in the zenith, and the equator will coincide with the horizon. If he is at the *North* Pole, all stars north of the celestial equator will remain permanently

above the horizon, never rising or falling at all, but sailing around on circles of altitude (or *Almucantars*) parallel to the horizon. Stars in the Southern Hemisphere, on the other hand, would never rise to view. As the sun and the moon move in such a way that during half the time they are alternately north and south of the equator, they will be half the time above the horizon and half the time below it. The moon would be visible for about a fortnight at a time, and the sun for six months.

33. The Oblique Sphere (Fig. 7). — At any station between the pole and equator the stars will move in circles oblique to the horizon, *SENW* in the figure. Those whose distance from the elevated pole is less than the latitude of the place will, of course, never sink below the horizon, — the radius of the "*Circle of Perpetual Apparition*,"

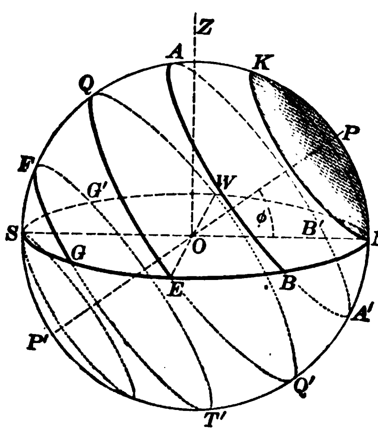

FIG. 7. — The Oblique Sphere and Diurnal Circles.

as it is called (the shaded cap around P in the figure), being just equal to the height of the pole, and becoming larger as the latitude increases. On the other hand, stars within the same distance of the depressed pole will lie within the "*Circle of Perpetual Occultation*," and will never rise above the horizon.

A star exactly on the celestial equator will have its diurnal circle $EQWQ'$ bisected by the horizon, and will be above the horizon just as long as below it. A star north of the equator (if the North Pole is the elevated one) will have more than half of its diurnal circle above the horizon, and will be visible more than half the time; as, for instance, a star at A: and of course the reverse will be true of stars

on the other side of the equator.[1] Whenever the sun is north of the equator, the day will therefore be longer than the night for all stations in northern latitude: how much longer will depend both on the latitude of the place and the sun's distance from the celestial equator.

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[1] A Celestial Globe will be of great assistance in studying these diurnal circles. The north pole of the globe must be elevated to an angle equal to the latitude of the observer, which can be done by means of the degrees marked on the brass meridian. It will then at once be easily seen what stars never set, which ones never rise, and during what part of the 24 hours any heavenly body at a known distance from the equator is above or below the horizon.

Note to Art. 6.

The ordinary estimate of the apparent size of the sun and moon as "about a foot in diameter" probably rests upon a physiological fact, — *viz.*, that in judging moderate distances, where we are not helped by intervening objects, we have to depend upon the muscular sensation of strain in converging our eyes towards the object looked at. For distances not exceeding fifty or sixty feet this is fairly accurate, but for distances above a hundred feet it entirely fails. When, therefore, we look at the moon in mid-heaven, our eyes directly inform us that it is at least a hundred feet away; on the other hand, from the absence of intervening objects we instinctively estimate the distance as the least possible consistent with the non-convergence of our eyes, and accordingly imagine the size of the disc to be about that of a ball which at a distance of a hundred feet or so would subtend the same angle of half a degree; *i.e.*, about a foot.

CHAPTER II.

ASTRONOMICAL INSTRUMENTS.

34. ASTRONOMICAL observations are of various kinds: sometimes we desire to ascertain the apparent distance between two bodies at a given time; sometimes the position which a body occupies at a given time, or the moment it arrives at a given circle of the sky, usually the meridian. Sometimes we wish merely to examine its surface, to measure its light, or to investigate its spectrum; and for all these purposes special instruments have been devised.

We propose in this chapter to describe very briefly a few of the most important

35. Telescopes in General.—Telescopes are of two kinds, refracting and reflecting. The former were first invented, and are much more used, but the largest instruments ever made are reflectors. In both the fundamental principle is the same. The large lens, or mirror, of the instrument forms at its focus a *real image* of the object looked at, and this image is then examined and magnified by the eye-piece, which in principle is only a magnifying-glass.

In the form of telescope, however, introduced by Galileo,[1] and still used as the "opera-glass," the rays from the object-glass are intercepted by a concave lens which performs the office of an eye-piece *before* they meet at the focus to form the "real image." But on account of the smallness of the field of view, and other objections, this form of telescope is never used when any considerable power is needed.

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[1] In strictness, Galileo did not invent the telescope. Its *first* invention seems to have been in 1608, by Lipperhey, a spectacle-maker of Middleburg, in Holland; though the honor has also been claimed for two or three other Dutch opticians. Galileo, in his "Nuncius Sydereus," published in March, 1610, himself says that he had heard of the Dutch instruments in 1609, and by so hearing was led to construct his own, which, however, far excelled in power any that had been made previously; and he was the first to apply the telescope to Astronomy. See Grant's "History of Astronomy," pp. 514 and seqq.

36. Simple Refracting Telescope. — This consists essentially as shown in the figure (Fig. 8), of a tube containing two lenses: a single convex lens, *A*, called the object-glass; and another, of smaller size and short focus, *B*, called the eye-piece. Recalling the principles of lenses the student will see that if the instrument be directed at a distant object, the moon, for instance, all the rays, $a_0a_1a_2$, which fall upon the object-glass from a point at the *top* of the moon, will be collected at a in the focal plane, at the *bottom* of the image. Similarly rays from the *bottom* of the moon will go to b at the *top* of the image; moreover, since the rays that pass through the optical centre of the lens, o, are undeviated,[1] the angle a_0ob_0 will equal boa; or, in other words, if the focal length of the lens be five feet, for instance, then the image of the moon, seen from a distance of five feet, will appear just as large as the moon itself does in the sky, — it will subtend the same angle. If we look at it from a smaller distance,

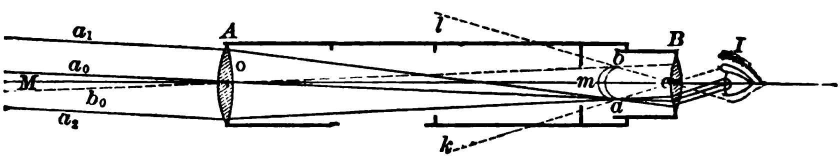

Fig. 8. — Path of the Rays in the Astronomical Telescope.

say from a distance of one foot, the image will look larger than the moon; and in fact, without using an eye-piece at all, a person with normal eyes can obtain considerable magnifying power from the object-glass of a large telescope. With a lens of ten feet focal length, such as is ordinarily used in an 8-inch telescope, one can easily see the mountains on the moon and the satellites of Jupiter, by taking out the eye-piece, and putting the eye in the line of vision some eight or ten inches back of the eye-piece hole.

The image is a *real* one; *i.e.*, the rays that come from different points in the object *actually meet* at corresponding points in the image, so that if a photographic plate were inserted at ab, and properly exposed, a picture would be obtained.

If we look at the image with the naked eye, we cannot come nearer

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[1] In this explanation, we use the approximate theory of lenses (in which their thickness is neglected), as given in the elementary text-books. The more exact theory of Gauss and later writers would require some slight modifications in our statements, but none of any material importance. For a thorough discussion, see Jamin, "*Traité de Physique*," or Encyc. Britannica, — Optics.

to the image (unless near-sighted) than eight or ten inches, and so cannot get any great magnifying power; but if we use a magnifying-glass, we can approach much closer.

37. Magnifying Power. — If the eye-piece B is set at a distance from the image equal to its principal focal distance, then any pencil of rays from any point of the image will, after passing the lens, be converted into a parallel beam, and will appear to the eye to come from a point at an infinite distance, as if from an object in the sky. The rays which came from the top of the moon, for instance, and are collected at a in the image, will reach the eye as a beam *parallel to the line ac, which connects a with the optical centre of the eye-piece.* Similarly with the rays which meet at b. The observer, therefore, will see the *top* of the moon's disc in the direction ck, and the *bottom* in the direction cl. It will appear to him *inverted*, and greatly magnified; its apparent diameter, as seen by the naked eye and measured by the angle aob (or its equal b_0oa_0), having been increased to acb. Since both these angles are subtended by the same line ab, and are *small* (the figure, of course, is much out of proportion), they must be inversely proportional to the distance ob and cb; *i.e.*, $boa : bca = cb : ob$; or, putting this into words: The ratio between the natural apparent diameter of the object, and its diameter as seen through the telescope, *is equal to the ratio between the focal lengths of the eye-lens and object-glass.* This ratio is called the *magnifying power* of the telescope, and is therefore given by the simple formula $M = \frac{F}{f}$, where F is the focal length of the object-glass and f that of eye-piece,[1] while M is the magnifying power.

If, for example, the object-glass have a focal length of thirty feet, and the eye-piece of one inch, the magnifying power will be 360; the power may be changed at pleasure by substituting different eye-pieces, of which every large telescope has an extensive stock.

38. Brightness of Image. — Since all the rays from a star which fall upon the large object-glass are transmitted to the observer's eye (neglecting the losses by absorption and reflection), he obviously re-

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[1] A magnifying power of 1 is no magnifying power at all. Object and image subtend equal angles. A magnifying power denoted by a fraction, say $\frac{1}{2}$, would be a *minifying* power, making the object look *smaller*, as when we look at an object through the wrong end of a spy-glass.

ceives a quantity of light much greater than he would naturally get: as many times greater as the area of the object-lens is greater than that of the pupil of the eye. If we estimate this latter as having a diameter of one-fifth of an inch, then a 1-inch telescope would increase the light twenty-five times, a 10-inch instrument 2500 times, and the great Lick telescope, of thirty-six inches' aperture, 32,400 times, the amount being proportional to the *square* of the diameter of the lens.

It must not be supposed, however, that the apparent brightness of an object like the moon, or a planet which shows a disc, is increased in any such ratio, since the eye-piece spreads out the light to cover a vastly more extensive angular area, according to its magnifying power; in fact, it can be shown that no optical arrangement can show an *extended surface* brighter than it appears to the naked eye. But the *total quantity* of light utilized is greatly increased by the telescope, and in consequence, multitudes of stars, far too faint to be visible to the unassisted eye, are revealed; and, what is practically very important, *the brighter stars are easily seen by day* with the telescope.

39. Distinctness of Image. — This depends upon the exactness with which the lens gathers to a single *point* in the focal image all the rays which emanate from the corresponding point in the object. A single lens, with spherical surfaces, cannot do this very perfectly, the "aberrations" being of two kinds, the *spherical* aberration and the *chromatic*. The former could be corrected, if it were worth while, by slightly modifying the form of the lens-surfaces; but the latter, which is far more troublesome, cannot be cured in any such way. The violet rays are more refrangible than the red, and come to a focus nearer the lens; so that the image of a star formed by such a lens can never be a luminous point, but is a round patch of light of different color at centre and edge.

40. Long Telescopes. — By making the diameter of the lens very small as compared with its focal length, the aberration becomes less conspicuous; and refractors were used, about 1680, having a length of more than 100 feet and a diameter of five or six inches. The object-glass was mounted at the top of a high pole and the eye-piece was on a separate stand below. Huyghens and Cassini both used such "aerial telescopes," and one of Huyghens' object-glasses, of six inches aperture and 123 feet focus, is still preserved in the Museum of the Royal Society in London.

41. The Achromatic Telescope. — The chromatic aberration of a lens, as has been said, cannot be cured by any modification of the lens itself; but it was discovered in England about 1760 that it can be nearly corrected by making the object-glass of *two* (or more) lenses, of *different kinds of glass*, one of the lenses being convex and the other concave. The convex lens is usually made of *crown* glass, the concave of *flint* glass. At the same time, by properly choosing the curves, the *spherical* aberration can also be destroyed, so that such a compound object-glass comes reasonably near to fulfilling the condition, that it should gather to a mathematical point in the image all the rays that reach the object-glass from a single point in the object.

These object-glasses admit of a considerable variety of forms. Formerly they were generally made, as in Fig. 9, No. 3, having the two lenses close together, and the adjacent surfaces of the same, or nearly the same, curvature. In small object-glasses the lenses are often cemented together with Canada balsam or some other transparent medium. At present some of the best makers separate the two lenses by a considerable distance, so as to admit a free circulation of air between them; in the Pulkowa and Princeton object-glasses, constructed by Clark, the lenses are seven inches apart, and in the Lick telescope six and a half inches; as in No. 1. In a form devised by Gauss (No. 2), which has some advantages, but is difficult of construction, the curves are very deep, and both the lenses are of watch-glass form — concave on one side and convex on the other. In all these forms the crown glass is outside; Steinheil, Hastings, and others have constructed lenses with the *flint-glass* lens outside. Object-glasses are sometimes made with *three* lenses instead of two; a slightly better correction of aberrations can be obtained in this way, but the gain is too small to pay for the extra expense and loss of light.

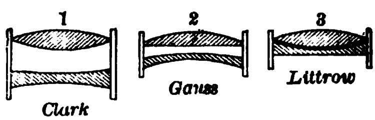

FIG. 9. — Different Forms of the Achromatic Object-glass.

42. Secondary Spectrum. — It is not, however, possible with the kinds of glass ordinarily employed to secure a perfect correction of the color. Our best achromatic lenses bring the yellowish green rays to a focus *nearer the lens* than they do the red and violet. In consequence, the image of a bright star is surrounded by a purple halo, which is not very noticeable in a good telescope of small size, but is very conspicuous and troublesome in a large instrument.

This imperfection of achromatism makes it unsatisfactory to use an ordinary lens (*visually* corrected) for astronomical photography. To fit it to make good photographs, it must either be specially corrected for the rays

that are most effective in photography, the blue and violet (in which case it will be almost worthless visually), or else a subsidiary lens, known as a "photographic corrector," may be provided, which can be put on in front of the object-glass when needed. A new form of object-glass, devised independently by Pickering in this country and Stokes in England, avoids the necessity of a third lens by making the crown-glass lens of such a form that when put close to the flint lens, with the *flatter side out*, it makes a perfect object-glass for visual purposes; but by simply reversing the crown lens, with the more convex side outward, and separating the lenses an inch or two, it becomes a photographic object-glass.

42*. Photo-visual Objectives. — Much is hoped from the new kinds of glass now made at Jena, but there has been great difficulty in producing discs satisfactorily homogeneous, of such chemical composition that the surfaces will not "rust," and large enough for telescopes of any size. Since 1894, however, the English opticians, Cooke & Sons, have been advertising "perfectly achromatic" triple object-glasses, which are asserted to be equally perfect for visual and photographic use. They offer to make lenses twenty inches in diameter, but up to 1896 had produced only a few as large as six or eight inches, which have been examined and very favorably reported on by eminent astronomers. Possibly the new century will open a new era in telescope-making.

43. Diffraction and Spurious Disc. — Even if a lens were perfect as regards the correction of aberrations, the "wave" nature of light prevents the image of a luminous point from being also a point; the image must *necessarily* consist of a central *disc*, brightest in the centre and fading to darkness at the edge, and this is surrounded by a series of bright rings, of which, however, only the smallest one is generally easily seen. The size of this disc-and-ring system can be calculated from the known wave-lengths of light and the dimensions of the lens, and the results agree very precisely with observation. The diameter of the "spurious disc" *varies inversely* with the aperture of the telescope. According to Dawes, it is about $4''.5$ for a 1-inch telescope; and consequently $1''$ for a $4\frac{1}{2}$-inch instrument, $0''.5$ for a 9-inch, and so on.

This circumstance has much to do with the superiority of large instruments in showing minute details. No increase of magnifying power on a small telescope can exhibit things as sharply as the same power on the larger one; provided, of course, that the larger object-glass is equally perfect in workmanship, and the *air* in good optical condition.

If the telescope is a good one, and if the air is perfectly steady, — which unfortunately is seldom the case, — the apparent disc of a star should be

Resolving Power = $\frac{4''.5}{a}$

Minimum useful mag. power = 3a

perfectly round and well defined, without wings or tails of any kind, having around it from one to three bright rings, separated by distances somewhat greater than the diameter of the disc. If, however, the magnifying power is more than about 50 to the inch of aperture, the edge of the disc will begin to appear hazy. There is seldom any advantage in the use of a magnifying power exceeding 75 to the inch, and for most purposes powers ranging from 20 to 40 to the inch are most satisfactory.

44. Eye-Pieces. — For many purposes, as for instance the examination of close double stars, there is no better eye-piece than the simple convex lens; but it performs well only when the object is exactly in the centre of the field. Usually it is best to employ for the *eye-piece* a combination of two or more lenses.

Eye-pieces belong to two classes, the *positive* and the *negative*. The former, which are much more generally useful, act as simple magnifying-glasses, and can be used as hand magnifiers if desired. The focal image formed by the object-glass lies *outside* of the eye-piece.

In the *negative* eye-pieces, on the other hand, the rays from the object-glass are intercepted before they come to the focus, and the image is formed between the lenses of the eye-piece. Such an eye-piece cannot be used as a hand magnifier.

45. The simplest and most common forms of these eye-pieces are the Ramsden (positive) and Huyghenian (negative). Each is composed of two plano-convex lenses, but the arrangement and curves differ, as shown in Fig. 10. The former gives a very flat field of view, but is not achromatic; the latter is more nearly achromatic, and possibly defines a little better just at the centre of the field; but the fact that it is a *negative* eye-piece greatly restricts its usefulness. In the Ramsden eye-piece the focal lengths of the two component lenses, both of which have their flat sides out, are about equal to each other, and their distance is about one-third of the sum of the focal lengths. In the Huyghenian the curved sides of the lenses are both turned towards the object-glass; the focal distance of the field lens should be exactly *three* times that of the lens next the eye, and the distance between the lenses one-half the sum of the focal lengths.

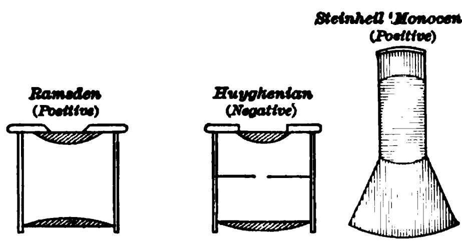

Fig. 10. — Various Forms of Telescope Eye-piece.

There are numerous other forms of eye-piece, each with its own advantages and disadvantages. The *erecting* eye-piece, used in spy-glasses, is

essentially a compound microscope, and gives erect vision by again inverting the already inverted image formed by the object-glass.

It is obvious that in a telescope of any size the object-glass is the most important and expensive part of the instrument. Its cost varies from a few hundred dollars to many thousands, while the eye-pieces generally cost only from $5 to $20 apiece.

46. Reticle. — When a telescope is used for *pointing*, as in most astronomical instruments, it must be provided with a *reticle* of some sort. This is usually a metallic frame with *spider lines* stretched across it, placed, not near the object-glass itself (as is often supposed), but at the *focus* of the object-glass, where the image is formed, as at *a b* in Fig. 8.

It is usually so arranged that it can be moved in or out a little to get it exactly into the focal plane, and then, when the eye-piece (positive) is adjusted for the observer's eye to give distinct vision of the object, the "wires," as they are called, will also be equally distinct. As spider-threads are very fragile, and likely to get broken and displaced, it is often better to substitute filaments of *quartz*, or a thin plate of glass with lines ruled upon it and blackened. The field of view, or the threads themselves, must be illuminated in order to make them visible in darkness.

47. The Reflecting Telescope. — When the chromatic aberration of lenses came to be understood through the optical discovery of the dispersion of light by Newton, the reflecting telescope was invented, and held its place as the instrument for star-gazing until well into the present century, when large achromatics began to be made. There are several varieties of reflecting telescope, all agreeing in the substitution of a large concave mirror in place of the object-glass of the refractor, but differing in the way in which they get at the image formed by this mirror at its focus in order to examine it with the eye-piece.

48. In the Herschelian form, which is the simplest, but only suited to very large instruments, the mirror is *tipped* a little, so as to throw the image to the side of the tube, and the observer stands with his back to the object and looks down into the tube. If the telescope is as much as two or three feet in diameter, his head will not intercept enough light to do much harm, — not nearly so much as would be lost by the second reflection necessary in the other forms of the instrument. But the inclination of the mirror, and the heat from the observer's person, are fatal to any very accurate definition, and unfit this form of instrument for anything but the observation of nebulæ and objects which mainly require light-gathering power.

In the Newtonian telescope, a small plane reflector standing at an angle of 45° is placed in the centre of the tube, so as to intercept the rays reflected by the large mirror a little before they come to their focus, and throw them to the side of the tube, where the eye-piece is placed.

In the Gregorian form (which was the first invented), the large mirror is pierced through its centre, and the rays from it are reflected through the hole by a small *concave* mirror, placed a little outside of the principal focus at the mouth of the tube. With this instrument one looks directly at the stars as with a refractor, and the image is erect.

The Cassegrainian form is very similar, except that the small concave mirror of the Gregorian is replaced by a *convex* mirror, placed a little inside the focus of the large mirror, which makes the instrument a little shorter, and gives a flatter field of view.

Formerly the great mirror was always made of a composition of copper and tin (two parts of copper to one of tin) known as "speculum metal." At present it is usually made of glass *silvered* on the front surface, by a chemical process which deposits the metal in a thin, brilliant film. These silver-on-glass reflectors, when new, reflect much more light than the old specula, but the film does not retain its polish so long. It is, however, a comparatively simple matter to renew the film when necessary.

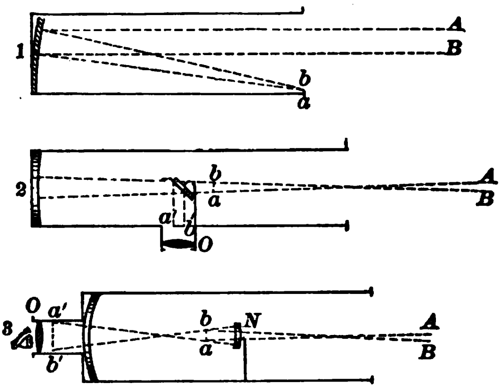

FIG. 11. — Different Forms of Reflecting Telescope.
1. The Herschelian; 2. The Newtonian; 3. The Gregorian.

The largest telescopes ever made have been reflectors. At the head of the list stands the enormous instrument of Lord Rosse, constructed in 1842, with a mirror six feet in diameter and sixty feet focal length. Next in order comes the five-foot silver-on-glass reflector of Mr. Common[1] (1889), and after it a number of instruments of four feet aperture, first among which is the great telescope of the elder Herschel, built in 1789, followed by the telescope erected by Lassell at Malta in 1860, the Melbourne reflector by Grubb in 1870, and the still more recent silver-on-glass reflector of the Paris observatory.

49. Relative Advantages of Refractors and Reflectors. — There has been a good deal of discussion on this point, and each construction has its partisans.

In favor of the reflectors we may mention, —

First. *Ease of construction and consequent cheapness.* The concave mirror

[1] Acquired and mounted by Harvard College Observatory in 1905.

has but one surface to figure and polish, while an object-glass has four. Moreover, as the light goes *through* an object-glass, it is evident that the glass employed must be perfectly clear and of uniform density through and through; while in the case of the mirror, the light does not penetrate the material at all. This makes it vastly easier to get the material for a large mirror than for a large lens.

Second (and immediately connected with the preceding). *The possibility of making reflectors much larger than refractors.* Lord Rosse's great reflector is six feet in diameter, while the Yerkes telescope, the largest of all refractors, is only forty inches. (Its focal length, however, is sixty-five feet.)

Third. *Perfect achromatism.* This is unquestionably a very great advantage, especially in photographic and spectroscopic work.

But, on the whole, the advantages are generally considered to lie with the refractors.

In their favor we mention:—

First. *Great superiority in light.* No mirror (unless, perhaps, a *freshly polished* silver-on-glass film) reflects much more than three-quarters of the incident light; while a good (single) lens transmits over 90 per cent. In a good refractor about 80 per cent of the light reaches the eye, after passing through the four lenses of the object-glass and eye-piece. In a Newtonian reflector, in average condition, the percentage seldom exceeds 50 per cent, and more frequently is lower than higher.

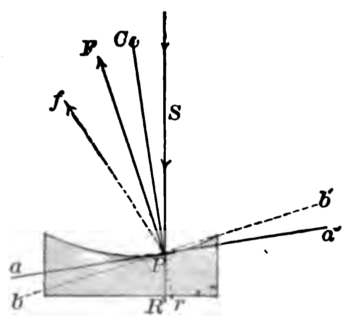

FIG. 12.—Effect of Surface Errors in a Mirror and in a Lens.

Second. *Better definition.*—Any slight error at a point in the surface of a glass lens, whether caused by faulty workmanship or by distortion, affects the direction of the ray passing through it only *one-third* as much as the same error on the surface of a mirror would do.

If, for instance, in Fig. 12, an element of the surface at P is turned out of its proper direction, aa', by a small angle, so as to take the direction bb', then the *reflected* ray will be sent to f, and its deviation will be *twice* the angle aPb. But since the index of refraction of glass is about 1.5 the change in the direction of the *refracted* ray from R to r will only be about *two-thirds* of aPb.

Moreover, so far as distortions are concerned, when a lens bends a little by its own weight, *both sides are affected in a nearly compensatory manner*, while in a mirror there is no such compensation. As a consequence, mirrors very seldom indeed give any such definition as lenses do. The least fault of workmanship, the least distortion by their own weight, the slightest difference of temperature, between front and back, will absolutely ruin the image, while a lens would be but slightly affected in its performance by the same circumstances.

Third. *Permanence.* The lens, once made, and fairly taken care of, suffers no deterioration from age; but the metallic speculum or the silver film soon tarnishes, and must be repolished every few years. This alone is decisive in most cases, and relegates the reflector mainly to the use of those who are themselves able to construct their own instruments.

To these considerations we may add that a refractor, though more expensive than a reflector of similar power, is not only more permanent, and less likely to have its performance affected by accidental circumstances, but is lighter and more convenient to use.

50. Time-Keepers and Time-Recorders. — *The Clock, Chronometer, and Chronograph.* — Modern practical astronomy owes its development as much to the clock and chronometer as to the telescope. The ancients possessed no accurate instruments for the measurement of time, and until within 200 years, the only reasonably precise method of fixing the time of an important observation, as, for instance, of an eclipse, was by noting the *altitude* of the sun, or of some known star at or very near the moment.

It is true that the Arabian astronomer Ibn Jounis had made some use of the pendulum about the year 1000 A.D., more than 500 years before Galileo introduced it to Europeans. But it was not until nearly a century after Galileo's discovery that Huyghens applied it to the construction of clocks (in 1657).

So far as the principles of construction are concerned, there is no difference between an astronomical clock and any other. As a matter of convenience, however, the astronomical clock is almost invariably made to beat seconds (rarely half-seconds), and has a conspicuous second-hand, while the hour-hand makes but *one* revolution a day, instead of two, as usual, and the face is marked for twenty-four hours instead of twelve. Of course it is constructed with extreme care in all respects.

The Escapement, or *"Scapement,"* is often of the form known as the "Graham Dead-beat"; but it is also frequently one of the numerous "gravity" escapements which have been invented by ingenious mechanicians. The office of the escapement is to be "unlocked" by the pendulum at each vibration, so as to permit the wheel-work to advance one step, marking a second (or sometimes two seconds), upon the clock-face; while, at the same time, the escapement gives the pendulum a slight impulse, just equal to the resistance it has suffered in performing the unlocking. The work done by the pendulum in "unlocking" the train, *and the corresponding impulse, ought to be perfectly constant,* in spite of all changes in the condition of the train of wheels; and it is *desirable,* though not *essential,* that this work should be as *small* as possible.

51. The pendulum itself is usually suspended by a flat spring, and great pains should be taken to have the support extremely firm: this is often neglected, and the clock then cannot perform well.

Compensation for Temperature. — In order to keep perfect time, the pendulum must be a "compensation pendulum"; *i.e.*, constructed in such a way that changes of temperature will not change its length.

An uncompensated pendulum, with steel rod, changes its daily rate about one-third of a second for each degree of temperature (centigrade). A wooden pendulum rod is much less affected by temperature, but is very apt to be disturbed by changes of *moisture.*

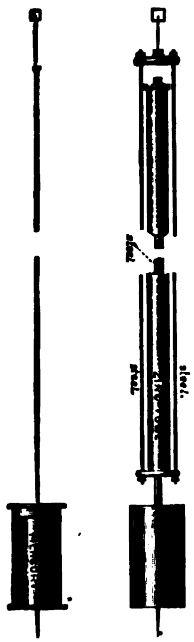

Fig. 13.
Compensation Pendulums.
1. Graham's Pendulum.
2. Zinc-Steel Pendulum.

Graham's mercurial pendulum (Fig. 13) is the one most commonly used. It consists simply of a jar (usually steel), three or four inches in diameter, and about eight inches high, containing forty or fifty pounds of mercury, and suspended at the end of a steel rod. When the temperature rises, the rod lengthens (which would make the clock go slower); but, at the same time, the mercury expands, from the bottom upwards, just enough to compensate. This pendulum will perform well only when not exposed to *rapid changes* of temperature. Under rapid changes the compensation *lags.* If, for instance, it grows warm quickly, the rod will expand before the mercury does; so that, *while the mercury is growing warmer*, the clock will run slow, though after it has become warm the rate may be all right.

A compensation pendulum, constructed on the principle of the old gridiron pendulum of Harrison, but of zinc and steel instead of brass and steel, is now much used. The compensation is not so easily adjusted as in the mercurial pendulum, but when properly made the mechanism acts well, and bears rapid alterations of temperature much better than the mercurial pendulum. The heavy pendulum-bob, a lead cylinder, is hung at the end of a steel rod, which is suspended from the top of a zinc tube, and hangs through the centre of it. This tube is itself supported *at the bottom* by three or four steel rods which hang from a piece attached to the pendulum spring. The standard clock at Greenwich has a pendulum of this kind.

52. Effect of Atmospheric Pressure. — In consequence of the buoyancy of the air, and its resistance to motion, a pendulum swings

a little more slowly than it would *in vacuo*, and every change in the density of the air affects its rate more or less. With mercurial pendulums, of ordinary construction, the "*barometric coefficient*," as it is called, is about one-third of a second for an inch of the barometer; *i.e.*, an increase of atmospheric density which would raise the barometer one inch would make the clock *lose* about one-third of a second daily. It varies considerably, however, with different pendulums.

It is not very usual to take any notice of this slight disturbance; but when the extremest accuracy of time-keeping is aimed at, the clock is either sealed in an air-tight case from which the air is partially exhausted (as at Berlin), or else some special mechanism, controlled by a barometer, is devised to compensate for the barometric changes, as at Greenwich. In the Greenwich clock a magnet is raised or lowered by the rise or fall of the mercury in a barometer attached to the clock-case. When the magnet rises, it approaches a bit of iron two or three inches above it, fixed to the bottom of the pendulum, and the increase of attraction accelerates the rate just enough to balance the retardation due to the air's increased density and viscosity. There are several other contrivances for the same purpose.

53. Error and Rate. — The "*error*," or "*correction*" of a clock is the amount that *must be added* to the indication of the clock-face at any moment in order to give the *true time;* it is, therefore, *plus* (+) when the clock is *slow*, and *minus* (−) when it is *fast*. The *rate* of a clock is the amount of its *daily gain or loss; plus* (+) when the clock is *losing*. Sometimes the *hourly rate* is used, but "*hourly*" is then always specified.

A *perfect* clock is one that has a *constant rate*, whether that rate be large or small. It is desirable, for convenience' sake, that both error and rate should be small; but this is a mere matter of adjustment by the user of the clock, who adjusts the error by setting the hands, and the rate by raising or lowering the pendulum-bob.

The final adjustment of rate is often obtained by first setting the pendulum-bob so that the clock will run slow a second or two daily, and then putting on the top of the bob little weights of a gramme or two, which will accelerate the motion. They can be dropped into place or knocked off without stopping the clock or perceptibly disturbing it.

The very best clocks will run three or four years without being stopped for cleaning, and will retain their rate without a change of more than one-fifth of a second, one way or the other, during the whole time. But this is

exceptional performance. In a run as long as that, most clocks would be liable to change their rate as much as half a second or more, and to do it somewhat irregularly.

54. The Chronometer. — The pendulum-clock not being portable, it is necessary to provide time-keepers that are. The chronometer is merely a carefully made watch, with a balance wheel compensated to run, as nearly as possible, at the same rate in different temperatures, and with a peculiar escapement, which, though unsuited to watches exposed to ordinary rough usage, gives better results than any other when treated carefully.

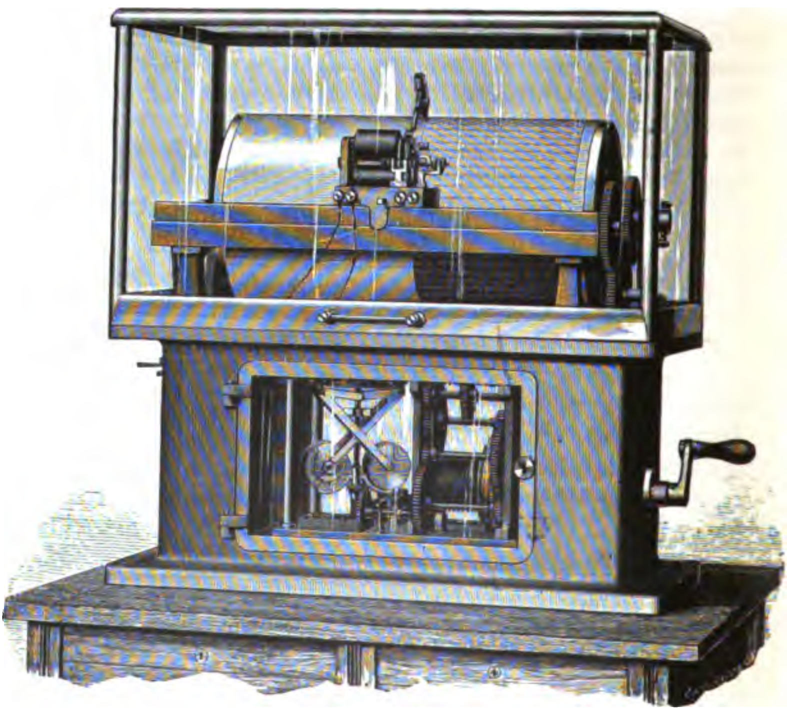

FIG. 14. — A Chronograph by Warner and Swasey.

The *box-chronometer* used on ship-board is usually about twice the diameter of a common pocket watch, and is mounted on gimbals, so as to keep horizontal at all times, notwithstanding the motion of the vessel. It usually beats half-seconds. It is not possible to secure in the chronometer-balance as perfect a temperature correction as in the pendulum. For this and other

reasons the best chronometers cannot quite compete with the best clocks in precision of time-keeping; but they are sufficiently accurate for most purposes, and of course are vastly more convenient for field operations. They are simply indispensable at sea. *Never turn the hands of a chronometer backward.*

55. Before the invention of the telegraph it was customary to note time merely "by eye and ear." The observer, keeping his time-piece near him, listened to the clock-beats, and estimated as closely as he could, in seconds and *tenths* of seconds, the moment when the phenomenon he was watching occurred — the moment, for instance, when a star passed across a wire in the reticle of his telescope. At present the record is usually made by simply pressing a "key" in the hand of the observer, and this, by a telegraphic connection, makes a mark upon a strip or sheet of paper, which is moved at a uniform rate by clock-work, and graduated by seconds-signals from the clock or chronometer.

56. The Chronograph. — This is the instrument which carries the marking-pen and moves the paper on which the time-record is made.

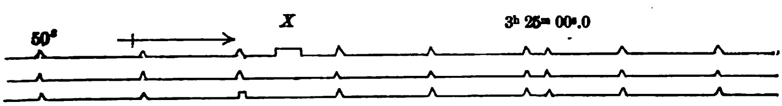

FIG. 15. — Part of a Chronograph Record.

The paper is wrapped upon a cylinder, six or seven inches in diameter, and fifteen or sixteen inches long. This cylinder is made to revolve once a minute, by clock-work, while the pen rests lightly upon the paper and is slowly drawn along by a screw-motion, so that it marks a continuous spiral. The pen is carried on the armature of an electro-magnet, which every other second (or sometimes every second) receives a momentary current from the clock, causing it to make a mark like those which break the lines in the figure annexed.

The beginning of a new minute (the 60th sec.) is indicated either by a double mark as shown, or by the omission of a mark. When the observer touches his key he also sends a current through the magnet, and thus interpolates a mark of his own on the record, as at X in the figure: the *beginning* of the mark is the instant noted — in this case 54.9^s. Of course the minutes when the chronograph was started and stopped are noted by the observer on the sheet, and so enable him to identify the minutes and seconds all through the record.

Many European observatories use chronographs in which the record is made upon a long fillet of paper, instead of a sheet on a cylinder. The instrument is lighter and cheaper than the American form, but much less convenient.

The regulator of the clock-work must be a "continuous" regulator, working continuously, and not by beats like a clock-escapement. There are various forms, most of which are centrifugal governors, acting either by friction (like the one in the figure) or by the resistance of the air; or else "spring-governors," in which the motion of a train, with a pretty heavy fly-wheel, is slightly checked at regular intervals by a pendulum.

57. Clock-Breaks. — The arrangements by which the clock is made to send regular electric signals are also various. One of the earliest and simplest is a fine platinum wire attached to the pendulum, which swings through a drop of mercury at each vibration. All of the arrangements, however, in which the pendulum itself has to make the electric contact are objectionable, and for clocks using the Graham dead-beat scapement no absolutely satisfactory means of giving the signals has yet been devised. Clocks with the gravity escapements have a decided advantage in this respect. Their wheel-work has no direct action in driving the pendulum, and so may be made to do any reasonable amount of outside work in the way of "key-manipulation" without affecting the clock-rate in the least. Usually a wheel on the axis of the scape-wheel is made to give the electric signals by touching a light spring with one of its teeth every other second.

Chronometers are now also fitted up in the same way, to be used with the chronograph.

The signals sent are sometimes "breaks" in a continuous current, and sometimes "makes" in an open circuit. Usage varies in this respect, and each method has its advantages. The break-circuit system is a little simpler in its connections, and possibly the signals are a little more sharp, but it involves a much greater consumption of battery material, as the current is always circulating, except during the momentary breaks.

58. Meridian Observations. — A large proportion of all astronomical observations are made at the time when the heavenly body observed is crossing the meridian, or very near it. At that moment the effects of refraction and parallax (to be discussed hereafter) are a minimum, and as they act only in a vertical plane, they do not have any influence on the *time* at which the body crosses the meridian.

59. The Transit Instrument is the instrument used, in connection with a clock or chronometer, and often with a chronograph also, to observe the time of a star's "transit" across the meridian.

If the error of the (sidereal) clock is known at the moment, this observation will determine the right ascension of the body, which, it

will be remembered, is simply *the sidereal time at which it crosses the meridian; i.e.*, the number of hours, minutes, and seconds by which it follows the vernal equinox.

Vice versa, if the right ascension is known, the error or correction of the clock will be determined.

The instrument (Fig. 16) consists essentially of a telescope mounted upon a stiff axis perpendicular to the telescope tube. This axis is placed horizontal, east and west, and turns on *pivots* at its extremities, in Y-bearings upon the top of two fixed piers or pillars. A

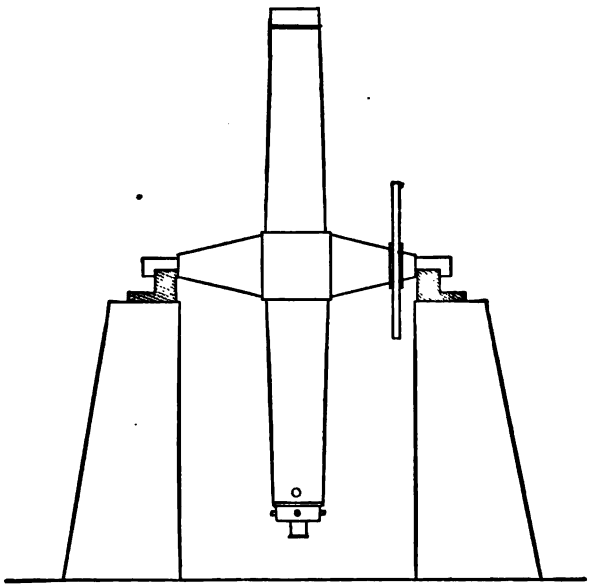

FIG. 16. — The Transit Instrument (Schematic).

small graduated circle is attached, to facilitate "setting" the telescope at any designated altitude or declination.

The telescope carries at the eye-end, in the focal plane of the object-glass, a reticle of some odd number of vertical wires, — five or more, — one of which is always in the centre, and the others are usually placed at equal distances on each side of it. One or two wires also cross the field horizontally.

If the pivots are true, and the instrument accurately adjusted, it is evident that the *central vertical wire will always follow the meridian as the instrument is turned;* and the instant when a star crosses this wire will be the true moment of the star's meridian transit. The

object in having a number of wires is, of course, simply to gain accuracy by taking the mean of a number of observations instead of depending upon a single one.

In order to "*level*" the axis properly, a delicate spirit-level is an essential adjunct; it is usual, also, (and important) to provide a convenient "*reversing apparatus*," by which the instrument can be turned half round, making the eastern and western pivots change places.

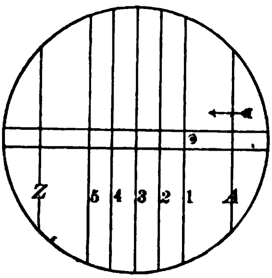

FIG. 17.—Reticle of the Transit Instrument.

The instrument must be thoroughly stiff and rigid, without loose joints or shaky screws; and the two pivots must be *accurately round*, *precisely in line with each other*, *free from taper*, and precisely *of the same size;* all of which conditions may be summed up by saying that they *must be portions of one and the same geometrical cylinder.*

The proper construction and grinding of these pivots, which are usually of hard bell metal (sometimes of steel), taxes the art of the most skilful mechanician. The level, also, is a delicate instrument, and difficult to construct.

Provision is made, of course, for illuminating the field of view at night so as to make the reticle wires visible. Usually one (or both) of the pivots is pierced, and a lamp throws light through the opening upon a small mirror in the centre of the tube, which reflects it down upon the reticle.

The Y's are used instead of round bearings, in order to prevent any *rolling* or *shake* of the pivots as the instrument turns.

Fig. 18 shows a modern transit instrument (portable) as actually constructed by Fauth & Co.

Another form of the instrument is much used, which is often designated as the "Broken Transit." A reflector in the central cube throws the rays coming from the object-glass, out at right angles through one end of the axis, where the eye-piece is placed; so that the observer does not have to change his position at all for different stars, but simply looks straight forward horizontally. It is very convenient and rapid in actual work, but the observations require a considerable correction for *flexure of the axis.*

60. *Adjustments.*—(1) Focus and verticality of wires. (2) Collimation. (3) Level. (4) Azimuth.

First. The first thing to do after the instrument is set on its supports and the axis roughly levelled, is *to adjust the reticle.* The eye-piece is drawn out

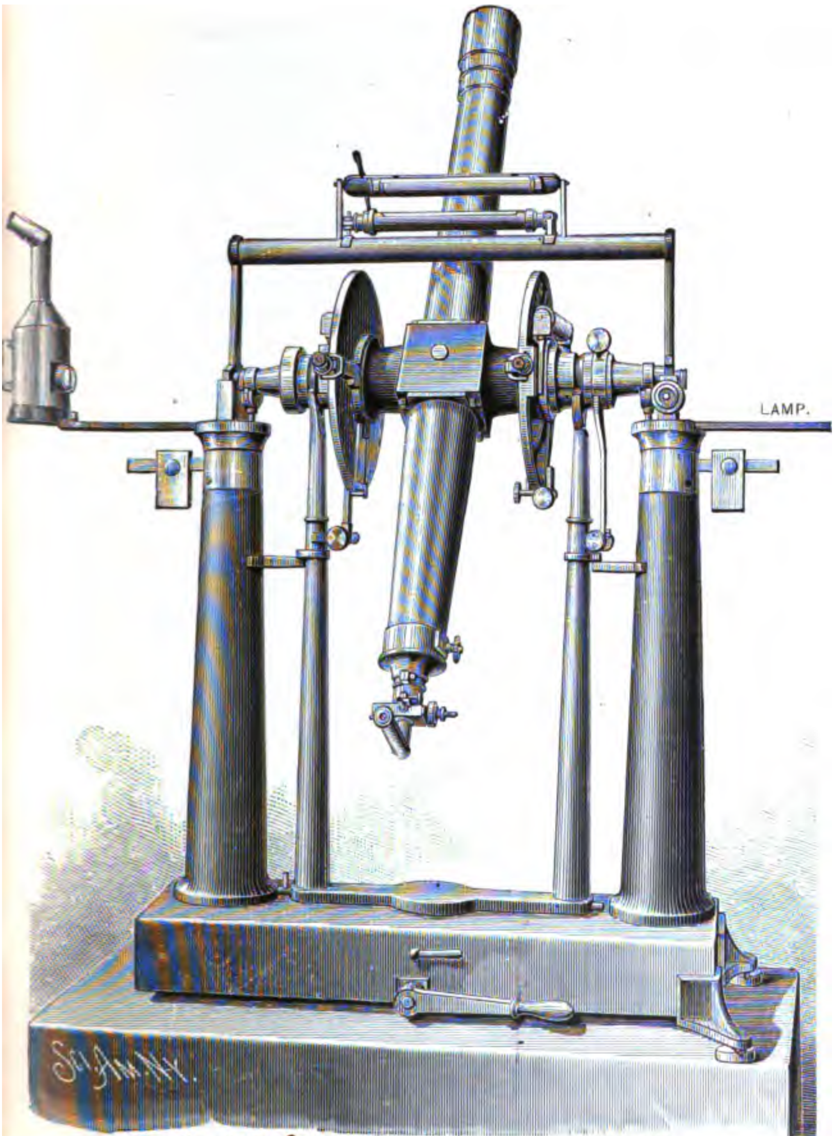

FIG. 18. — A 3-inch Transit, with reversing apparatus. Fauth & Co.

or pushed in until the wires appear perfectly sharp, and then the instrument is directed to a star or to some *distant* object (not less than a mile away), and without disturbing the eye-piece, the sliding-tube, which carries the reticle, is drawn out or pushed in until the object is also distinct at the

same time with the wires. If this adjustment is correctly made, motion of the eye in front of the eye-piece will not produce any apparent displacement of the object in the field, with reference to the wires. To test the verticality of the wires, the telescope is moved up and down a little, while looking at the object; if the axis is level and the wires vertical, the wire will not move off from the object sideways. There are screws provided to turn the reticle a little, so as to effect this adjustment.

When the wires have been thus adjusted for focus and verticality, the reticle-slide should be tightly clamped and never disturbed again. The *eye-piece* can be moved in and out at pleasure, to secure distinct vision for different eyes, but it is *essential that the distance between the object-glass and the reticle remain constant.*

Second. *Collimation.* The line joining the optical centre of the object-glass with the middle wire of the reticle is called the "*line of collimation,*" and this line must be made exactly perpendicular to the axis of rotation by moving the reticle slightly to one side or the other by means of the adjusting screws provided for the purpose. The simplest way of effecting the adjustment is to point the instrument on some well-defined distant object, like a nail-head or a joint in brickwork, and then carefully to "reverse" the instrument without disturbing the stand. If the middle wire, after reversal, points just as it did before, the "collimation" is correct; if not, the middle wire must be moved *half way* towards the object by the screws.

Collimator. — It is not always easy to find a distant object on which to make this adjustment, and a "*collimator*" may be substituted with advantage. This is simply a telescope mounted horizontally on a pier in front of the transit instrument, so that when the transit telescope is horizontal, it can look straight into the collimator, which ought to be of about the same size as the transit itself.

In the focus of the collimator object-glass are placed two wires forming an X, and thus placed they can be seen by a telescope looking into the collimator just as distinctly as if they were at an infinite distance and really celestial objects. The instrument furnishes us a mark *optically* celestial, but *mechanically* within reach of our finger-ends for illumination, adjustment, etc. If the pier on which it is mounted is firm, the collimator cross is in all respects as good as a star, and much more convenient.

Third. *Level.* The adjustment for *level* is made by setting a striding level on the pivots of the axis, reading the level, then reversing the level (not the transit) and reading it again. If the pivots are round and of the same size, the difference between the level-readings direct and reversed will indicate the amount by which one pivot is higher than the other. One of the Y's is made so that it can be raised and lowered slightly by means of a screw, and this gives the means of making the axis horizontal. If the pivots are not of the same size (and they never are *absolutely*), the astronomer must determine and allow for the difference.

Fourth. *Azimuth.* In order that the instrument may indicate the meridian truly, its axis must lie exactly east and west; *i.e.*, its *azimuth* must be 90°. This adjustment must be made by means of observations upon the stars, and is an excellent example of the method of successive approximations, which is so characteristic of astronomical investigation. (*a*) After adjusting carefully the focus and collimation of the instrument, we set it north and south *by guess*, and level it as precisely as possible. By looking at the pole star, and remembering how the pole itself lies with reference to it, one can easily set the instrument *pretty nearly; i.e.*, within half a degree or so. The middle wire will now describe in the sky a vertical circle, which crosses the meridian at the zenith, and lies very near the meridian for a considerable distance each side of the zenith.

(*b*) We must next get an "approximate" time; *i.e.*, set our clock or chronometer *nearly* right. To do this, we select from the list of several hundred stars in the Nautical Almanac (which is to be regarded in about the same light with the clock and the spirit level, as an indispensable accessory to the transit) a star which is about to cross the meridian *near the zenith.* The difference between the right ascension of the star as given in the Almanac, and the time shown by the clock-face, will be *very nearly* the error of the clock at the time of the observation: not *exactly*, unless the declination of the star is such that it passes *exactly* through the zenith, but *very nearly*, since the star crosses the meridian near the zenith. We now have the time within a second or two.

(*c*) Next turn down the telescope upon some Almanac star, which is soon to cross the meridian within 10° of the pole. It will appear to move very slowly. A little before the time it should reach the meridian, move the whole frame of the instrument until the middle wire points upon it, and then, by means of the "Azimuth Screw," which gives a slight horizontal motion to one of the Y's, *follow the star* until the *indicated moment of its transit; i.e.*, until the clock (corrected for clock error) shows on its face the star's right ascension. If the clock correction had been known with absolute exactness, the instrument would now be *truly* in the meridian: as the clock error, however, is only approximate, the instrument will only be approximately in the meridian; but — and this is the essential point — it will be *very much more nearly so* than at the beginning of the operation. The supposed incorrectness, amounting perhaps to one or two seconds, in the time at which the instrument was set on the circumpolar star will, on account of the slcw motion of the star, make almost no perceptible difference in the direction given to the axis.

A repetition of the operation may possibly be needed to secure all the desired precision. The accuracy of this azimuth adjustment can then be verified by three successive "culminations" or transits of the pole star, or any other circumpolar. The interval occupied in passing from the upper to the lower culmination on the west side of the meridian ought, of course, to be exactly equal to the time on the eastern side; *i.e.*, twelve sidereal hours.

61. The final test of *all* the adjustments, and of the accurate going of the clock, is obtained by observing a number of Almanac stars of widely different declination. If they all indicate *identically* the same clock correction, the instrument is in adjustment; if not, and if the differences are not very great, it is possible to deduce from the observations themselves the true clock error, and the adjustment errors of the instrument.

It is to be added, in this connection, that the astronomer can never assume that *adjustments are perfect:* even if once perfect, they would not stay so, on account of changes of temperature and other causes. Nor are observations ever absolutely accurate. The problem is, from observations more or less *inaccurate* but *honest*, with instruments more or less *maladjusted* but *firm*, to find the result that would have been obtained by a perfect observation with a perfect and perfectly adjusted instrument. It can be more nearly done than one might suppose. But the discussion of the subject belongs to Practical Astronomy, and cannot be entered into here.

62. Prime Vertical Instrument.—For certain purposes, a Transit Instrument, provided with an apparatus for rapid reversal, is turned quarter-way round and mounted with the axis *north* and *south*, so that the plane of rotation lies *east* and *west*, instead of in the meridian. It is then called a Prime Vertical Transit.

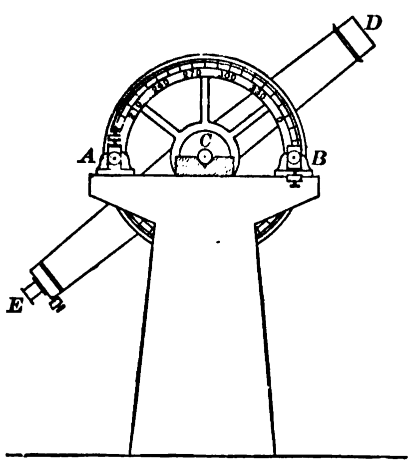

FIG. 19.—The Meridian Circle (Schematic).

63. The Meridian Circle.—In order to determine the *Declination* or *Polar Distance* of an object, it is necessary to have some instrument for measuring angles; mere time-observations will not suffice. The instrument most used for this purpose is the *Meridian Circle*, or *Transit Circle*, which is simply a transit instrument, with a graduated circle attached to its axis, and revolving with the telescope. Sometimes there are two circles, one at each end of the axis.

Fig. 19 represents the instrument "schematically," showing merely the essential parts. Fig. 20 is a meridian circle, with a 4-inch telescope, constructed by Fauth & Co.

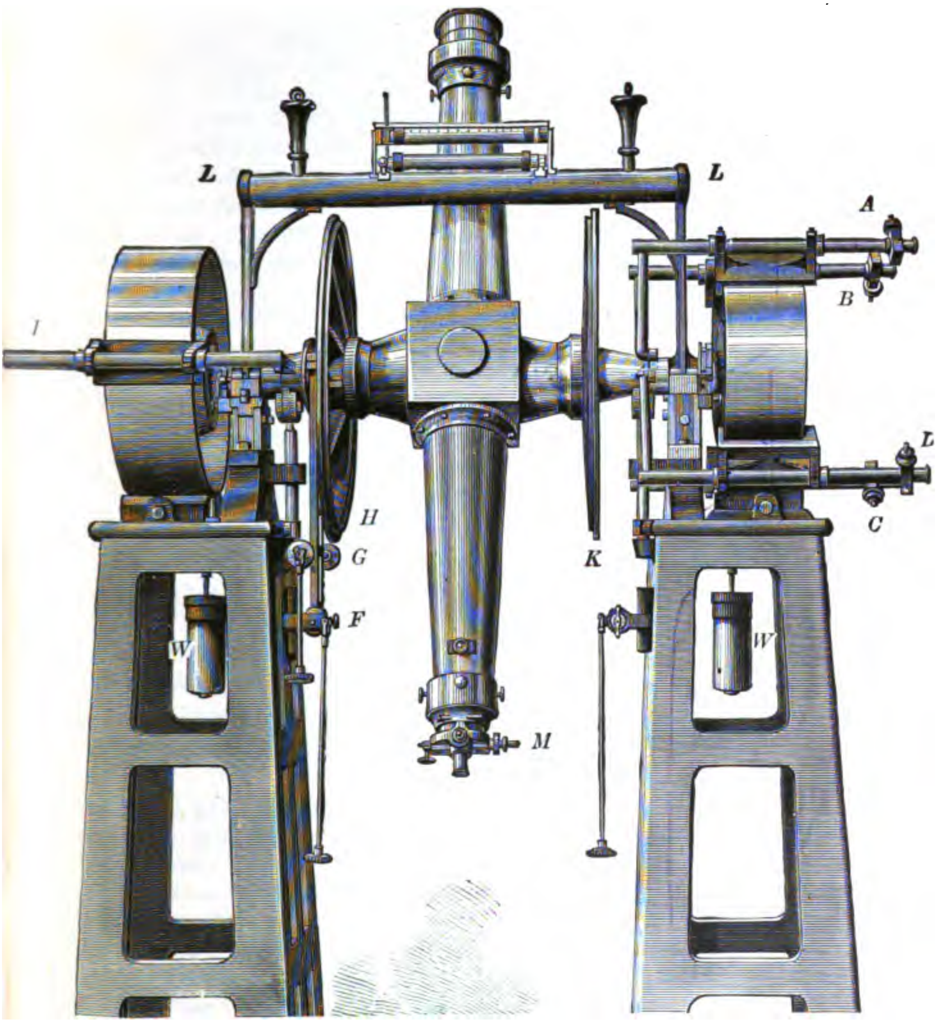

FIG. 20. — Meridian Circle.

A, *B*, *C*, *D*, the Reading Microscopes.
K, the Graduated Circle.
H, the Roughly Graduated Setting Circle.
I, the Index Microscope. This is usually, however, placed halfway between *A* and *D*.

F, the Clamp. *G*, the Tangent Screw.
LL, the Level, placed in position only occasionally.
M, the Right Ascension Micrometer.
W W, Counterpoises, which take part of the weight of the instrument off from the Y's.

In observatory instruments the circle is usually from two to four feet in diameter; larger circles were once used, but it is found that their weight, and the consequent strains and flexures, render them actually less accurate than the smaller ones. The utmost resources of mechanical art are exhausted in making the graduation as precise as possible and in providing for its accurate reading, as well as in securing the maximum firmness and stability of every part of the instrument. The actual divisions are usually 5′ apart (in very large instruments sometimes only 2′), but the circle is "read" to seconds and tenths of seconds of arc by means of *reading microscopes*, from two to six in number, fixed to the pier of the instrument. In a circle of forty inches diameter, 1″ is a little less than $\frac{1}{10000}$ of an inch, ($\frac{20}{206265}$ inch), so that the necessity of fine workmanship is obvious.

64. The Reading Microscope (Fig. 21). — This consists essentially of a compound microscope, which forms a magnified image of the graduation at the focus of its object-glass, where this image is viewed by a positive eye-piece. At the place where the image is formed a pair of parallel spider-lines or a cross is placed, movable in the plane of the image by a "micrometer screw"; *i.e.*, a fine screw with a graduated head, usually divided into sixty parts. One revolution of the screw carries the wire 1′ of arc, which makes one division of the screw-head 1″, the tenths of seconds being estimated.

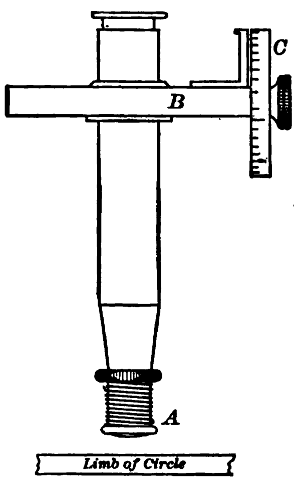

Fig. 21. — The Reading Microscope.

The adjustment of the microscope for "runs," as it is called (that is, to make one revolution of the micrometer screw exactly equal to 1′), is effected as follows. By setting the wires first on one of the graduation marks visible in the field of view, and then on the next mark, it is immediately evident whether five revolutions of the screw "run" over or fall short of 5′ of the graduation. If they *overrun*, it shows that the image of the graduation formed by the microscope objective is too small to fit the screw, and *vice-versa*. Now, by simply increasing or decreasing the distance AB between the objective and the micrometer box, the size of the image can be altered at will, and the objective is therefore so mounted that this can be done. Of course, every change in the length of the microscope tube will also require a readjustment of the distance between the "limb," or graduated surface, of the circle and the microscope, in order to secure distinct vision; but by a few trials the adjustment is easily made sufficiently precise.

The reading of the circle is as follows: An extra index-microscope, with low power and large field of view, shows by inspection the degrees and minutes. The reading-microscopes are only used to give the odd seconds, which is done by turning the screw until the parallel spider-lines are made to include one of the graduation lines half-way between themselves; the head of the screw then shows directly the seconds and tenths, to be added to the degrees and minutes shown by the index. Thus in Fig. 22, the reading of the microscope is 3′ 22″.1, the 3′ being given by the *scale* in the field, the 22″.1 by the screw-head.

65. Method of observing a Star.—A minute or two before the star reaches the meridian the instrument is approximately pointed, so that the star will come into the field of view. As soon as it makes its appearance, the instrument is moved by the slow-motion tangent-screw until the star is "bisected" by the fixed horizontal wire of the reticle, and the star is kept bisected until it reaches the middle vertical wire which marks the meridian. The microscopes are then read, and their mean result is the star's "circle-reading."

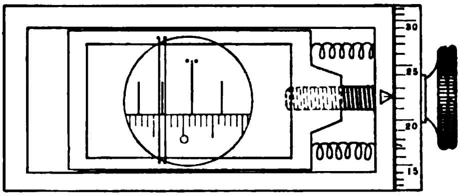

FIG. 22.—Field of View of Reading Microscope.

Frequently the star is bisected, not by moving the whole instrument, but by means of a "micrometer wire," which moves up and down in the field of view. The micrometer reading then has to be combined with the reading of the microscope, to get the true circle-reading.

66. Zero Points.—In determining the declination or meridian altitude of a star by means of its circle-reading, it is necessary to know the "*zero point*" of the circle. For declinations, the "zero point" is either the polar or the equatorial reading of the circle; *i.e.*, the reading of the circle when the telescope is pointed at the pole or at the equator.

The "polar point" may be found by observing some circumpolar star above the pole, and again, twelve hours later, below it. When the two circle-readings have been *duly corrected for refraction and instrumental errors*, their mean will be the polar point.

Suppose, for instance, that δ Ursæ Minoris, at the "upper culmination," gives a corrected reading of 52° 18′ 25″.3, while at the lower culmination the reading is 45° 31′ 35″.7, then the mean of these, 48° 55′ 00″.5, is the polar point, and of course the equatorial reading is 138° 55′ 00″.5,—just 90° greater. The *polar distance* of the star would be the *half-difference* of the two readings, or 3° 23′ 24″.8.

67. Nadir Point.—The determination of the polar point requires two observations of the same star at an interval of twelve hours. It is often difficult to obtain such a pair; moreover, the *refraction* complicates the matter, and renders the result less trustworthy. Accordingly it is now usual to use the nadir or the horizontal reading as the zero, rather than the polar point.

The *nadir point* is determined by pointing the telescope downwards to a basin of mercury, moving the telescope until the image of the horizontal wire of the reticle, as seen by reflection, coincides with the wire itself. Since the reticle is exactly in the principal focus of the object-glass, rays of light emitted by any point in the reticle will become a parallel beam after passing the lens, and if this beam strikes a plane mirror perpendicularly and is returned, the rays will come just as if from a real object in the sky, and will form an image at the focal plane. When, therefore, the image of the central wire of the reticle, seen in the mercury basin by reflection, coincides with the wire itself, we know that the line of collimation must be exactly perpendicular to the surface of the mercury; *i.e.*, vertical.

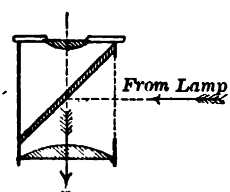

FIG. 23.
The Collimating Eye-Piece.

To make the image visible it is necessary to illuminate the reticle by light thrown *towards* the object-glass from behind the wires, instead of light coming from the object-glass towards the eye as usual. This peculiar illumination is commonly effected by means of Bohnenberger's "collimating eye-piece," shown in Fig. 23. In the simplest form it is merely a common Ramsden eye-piece, with a hole in one side, and a thin glass plate inserted at an angle of 45°. A light from one side, entering through the hole, will be (partially) reflected towards the wires, and will illuminate them sufficiently.

The *horizontal point* of course differs just 90° from the nadir point. It may also be found independently by noting the circle-readings of some star observed one night directly, and the next night by reflection in mercury; or, if the star is a close circumpolar, both observations may be made the same evening, one a few minutes before its meridian passage, the other just as long after. But the method of the collimating eye-piece is fully as accurate and vastly more convenient.

68. Differential Use of the Instrument.—We now know the places of several hundred stars with so much precision that in many cases it is quite sufficient to observe one or two of these "*standard stars*" in connection with the bodies whose places we wish to determine. The difference between the declination of the known star and that of any star whose place is to be determined, will, of course, be simply the difference of their circle-readings, corrected for refraction, etc. The meridian circle is said to be used "*differentially*" when thus treated.

69. Errors of Graduation, etc.—If the circle is from a reputable maker, and has four or six microscopes, and if the observations are carefully made and all the microscopes read each time, results of sufficient precision for most purposes may be obtained by merely correcting the observations for "runs" and refraction. The outstanding errors ought not to exceed a second or two. But when the *tenths* of a second are in question, the case is different. It will not then do for the astronomer to assume the accuracy of the graduation of his circle, but he must investigate the *errors of its divisions*, the errors of the *micrometer screws* in the microscopes, the *flexure* of the telescope, and the effect of differences of temperature in shifting the zero points of the circle, by slightly disturbing the position or direction of the microscopes. Of course this is not the place to enter into such details, but it is an opportunity to impress again upon the student the fact that truth and accuracy are only attainable by immense painstaking and labor.

70. Mural Circle.—This instrument is in principle the same as the meridian circle, which has superseded it. It consists of a circle, carrying a telescope mounted on the face of a *wall* of masonry (as its name implies) and free to revolve in the plane of the meridian. The wall furnishes a convenient support for the microscopes.

71. Altitude and Azimuth, or Universal, Instrument.—Since the transit instrument and meridian circle are confined to the plane of the meridian, their usefulness is obviously limited. Meridian observations are better and more easily used than any others, but are not always attainable. We must therefore have instruments which will follow an object to any part of the heavens.

The *altitude and azimuth instrument* is simply a surveyor's theodolite on a large scale. It has a horizontal circle turning upon a *vertical axis*, and read by verniers or microscopes. Upon this circle, and turning with it, are supports which carry the *horizontal axis* of the telescope with its vertical circle, also read by microscopes. Obvi-

ously the readings of these two circles, when the instrument is properly adjusted and the zero points determined, will give the altitude

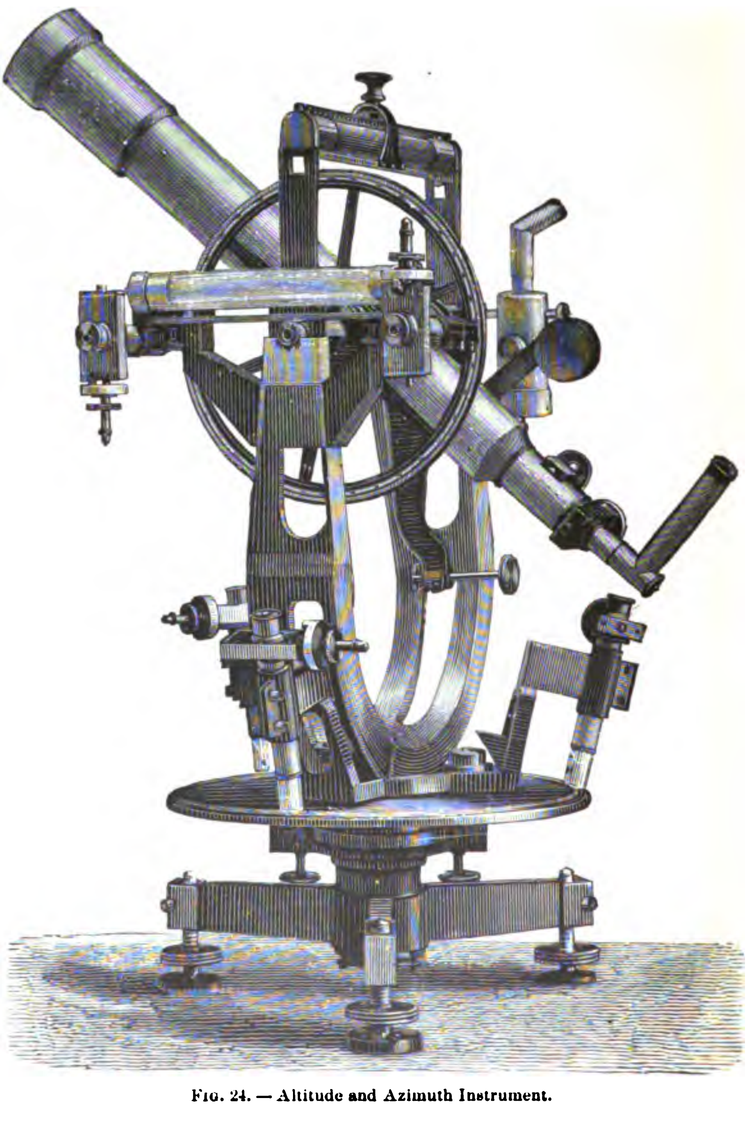

FIG. 24. — Altitude and Azimuth Instrument.

and azimuth of the body pointed on. Fig. 24 represents a small instrument of this kind.

72. The Equatorial.—The essential characteristic of this instrument is that its principal axis, *i.e.*, the axis which rests in *fixed* bearings, instead of being either horizontal or vertical, is inclined at an angle equal to the latitude of the place, and directed towards the pole, thus placing it parallel to the earth's axis of rotation. This axis of the instrument is called its *polar axis;* and the graduated circle which it carries, and which is parallel to the celestial equator, is called the *hour-circle*, because its reading gives the *hour-angle* of the body upon which the telescope happens to be pointed. Sometimes, also, it is called the Right Ascension Circle. Upon this polar-axis are secured the bearings of the *declination axis*, which is perpendicular to the polar axis, and carries the telescope itself and the declination circle.

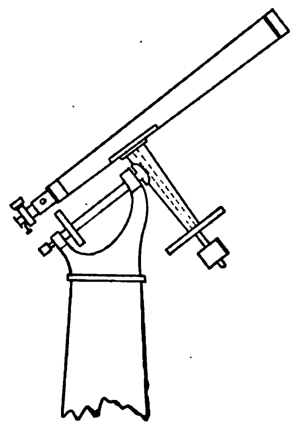
FIG. 25.
The Equatorial (Schematic).

In the instruments before described, the telescope is a mere *pointer*, and wholly subsidiary to the circles; in the equatorial the telescope is usually the main thing, and the circles are subordinate, serving only to aid the observer in finding or identifying the body upon which the telescope is directed.

Fig. 25 exhibits schematically the ordinary form of equatorial mounting, of which there are numerous modifications. Fig 26 is the 23-inch Clark telescope at Princeton, and Fig. 27 is the 4-foot Melbourne reflector. The frontispiece is the great Lick telescope of thirty-six inches diameter.

The advantages of the equatorial mounting for a large telescope are very great as regards convenience. In the first place, when the telescope is once pointed upon a star or planet, it is only necessary to turn the polar axis with a uniform motion in order to "follow" the star, which otherwise would be carried out of the field of view in a few moments by the diurnal motion. This motion, since it is uniform, can be, and in all large instruments usually is, given by clock-work, with a continuous regulator of some kind, similar to that used in the chronograph. The instrument once directed and clamped, and the clock-work started, the object will continue apparently immovable in the field of view as long as may be desired.

In the next place, it is very easy to find an object, even if invisible to the naked eye, like a faint comet or nebula, or a star in the day-

time, provided we know its declination and right ascension, and have the sidereal time; for which reason a sidereal clock or chronometer is an indispensable adjunct of the equatorial.

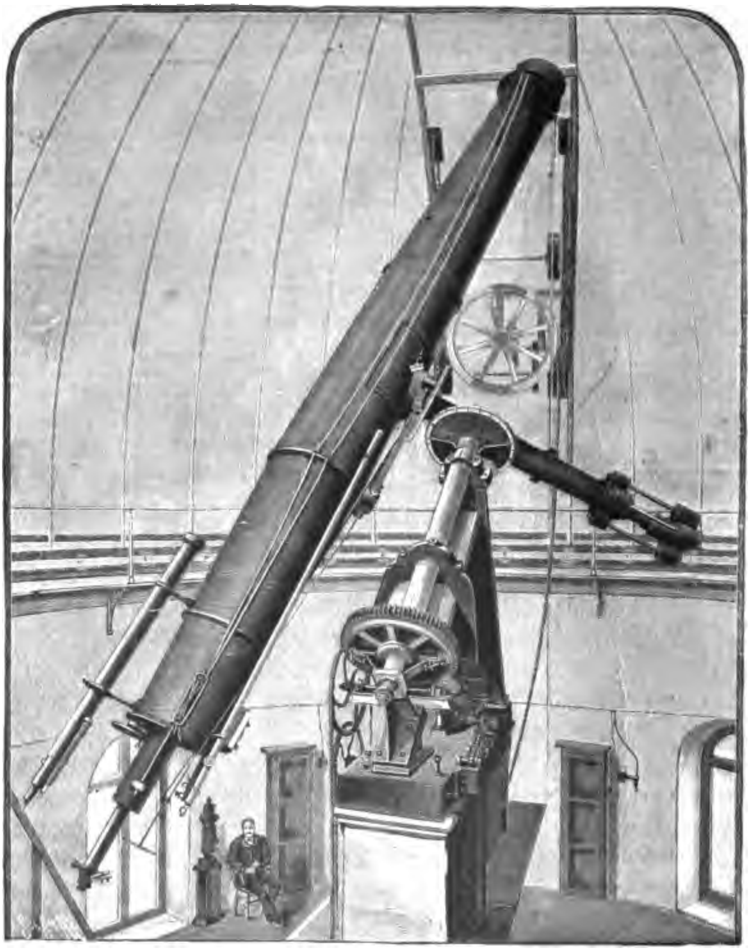

FIG. 26. — The 23-inch Princeton Telescope.

To find an object, the telescope is turned in declination until the reading of the declination circle corresponds to the declination of the object, and then the polar axis is turned until the hour-circle of the instrument (not to be confounded with an hour-circle in the sky) reads the *hour-angle* of the object. This hour-angle, it will be remembered, is simply the difference be-

tween the sidereal time and the right ascension of the object. The hour-angle is east if the right ascension exceeds the time; west, if it is less. When the telescope is thus set, the object will be found (with a low magnifying power) in the field of view, unless it is near the horizon, in which case refraction must be taken into account.

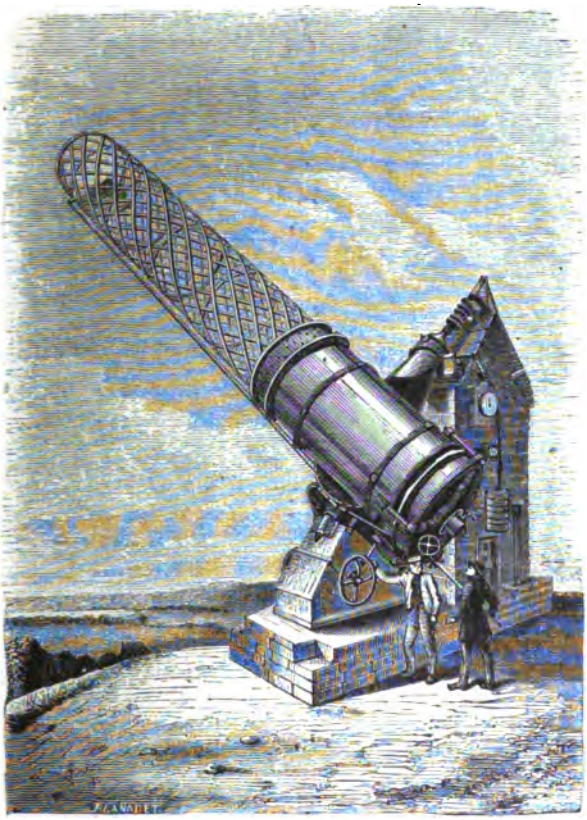

FIG. 27.—The Melbourne Reflector.

While the instrument cannot give very accurate determinations of the positions of bodies by the direct readings of its circles, on account of the irregular flexures of its axes, it may do so indirectly; that is, it may be used to determine very accurately the *difference* between the right ascension and declination of a comet or planet, for instance, and that of some neighboring star, whose place has been already determined by the meridian circle; and this is one of the most important uses of the instrument.

73. The Micrometer. — Micrometers of various sorts are employed for the purpose. The most common and most generally useful is the so-called "*filar position-micrometer*," Fig. 28, which is an indispensable auxiliary of every good telescope.

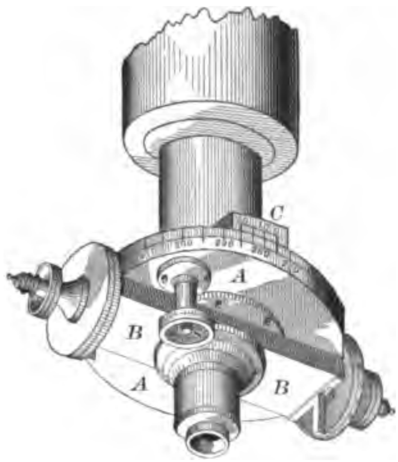

FIG. 28. — The Filar Position-Micrometer.

It is a small instrument, much like the upper part of the reading microscope, but more complicated. It usually contains a reticle of fixed wires, two or three parallel to each other, and crossed at right angles by a second set. Then there are two or three wires parallel to the first set, and movable by an accurately made screw with a graduated head and a counter, or scale, for indicating the number of entire revolutions made by the screw. The box containing these wires, and carrying the eye-piece and screw, can itself be turned around in a plane perpendicular to the optical axis of the telescope, and set in any desired position; for example, so that the movable wires shall be parallel to the celestial equator, while the

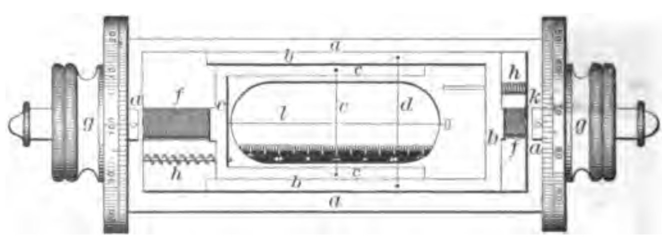

FIG. 29. — Construction of the Micrometer.

other set run north and south. This "position angle" is read on a graduated circle, which forms part of the instrument. Means of illumination are provided, giving at pleasure either dark wires in a bright field, or *vice versa*.

With this instrument one can measure the distance (in seconds of arc), and the direction between any two stars which are near enough to be seen at once in the same field of view. This range in small

telescopes may reach 30′ of arc; while in the larger instruments, which, with the same eye-pieces have much higher magnifying powers, it is necessarily less, — not more than from 5′ to 10′.

74. A new form of equatorial, known as the *Equatorial Coudé*, or *Elbowed* Equatorial, has been recently introduced at the Paris Observatory. With large instruments of the ordinary form a great deal of inconvenience is encountered by the observer, in moving about to follow the eye-piece into the various positions into which it is forced by the inconsiderateness of the

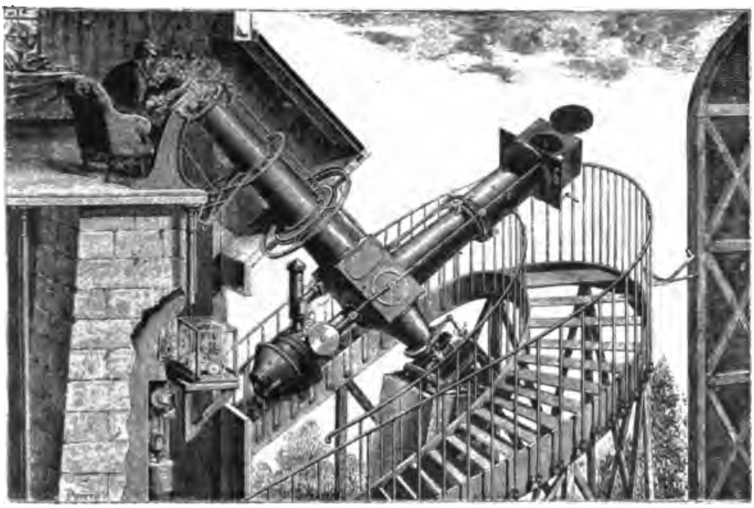

FIG. 30. — The Equatorial Coudé.

heavenly bodies. Moreover, the revolving dome, which is usually erected to shelter a great telescope, is an exceedingly cumbrous and expensive affair.

In the Equatorial Coudé, Fig. 30, these difficulties are overcome by the use of mirrors. The observer sits always in one fixed position, looking obliquely down through the polar axis, which is also the telescope tube.[1]

The instrument (figured above) had an aperture of about ten inches, and proved so satisfactory that in 1891 a much larger one with a twenty-four-inch lens was also mounted, and both are now in constant use.

75. All instruments so far described, except the chronometer, are *fixed* instruments; of use only when they can be set up firmly and carefully adjusted to established positions. Not one of them would be of the slightest use on shipboard.

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[1] For description of the siderostat and cœlostat see Addendum A.

We have now to describe the instrument which, with the help of the chronometer, is the main dependence of the mariner. It is an instrument with which the observer measures the angular distance between two objects; as, for instance, the sun and the visible horizon, not by pointing first on one and then afterwards on the other, but by *sighting them both, simultaneously and in apparent coincidence;* which can be done even when he has no fixed position or stable footing.

76. The Sextant. — The graduated limb of the sextant is carried by a light framework, usually of metal, provided with a suitable handle *X*. The arc is about one-sixth of a circle, as the name implies, and

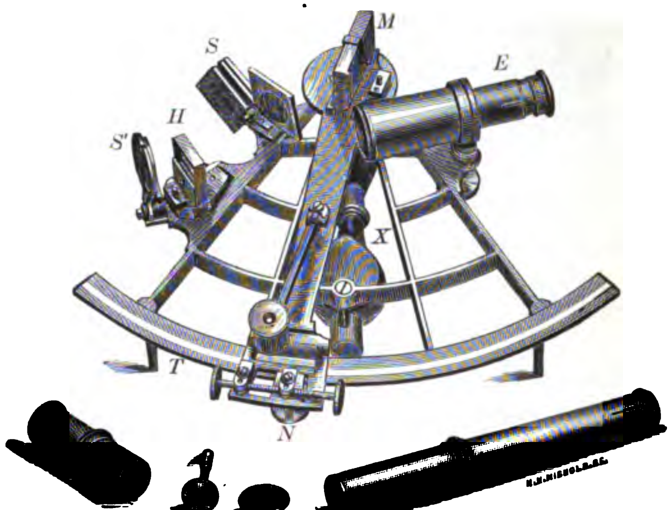

FIG. 31. — The Sextant.

is usually from five to eight inches radius. It bears a graduation of half-degrees, *numbered as whole degrees*, so that it can measure any angle less than 120°.

An "*index-arm*," *MN* in the figure, is pivoted at the centre of the arc, and carries a vernier which slides along the limb, and can be fixed at any point by a clamp and delicately moved by the attached tangent screw, *T*. The reading of this vernier gives the angle measured by the instrument. The best instruments read to 10″.

Just over the centre of motion, the "*index-mirror*" *M*, about two inches by one and one-half in size, is fastened securely to the index-arm, so as to be perpendicular to the plane of the limb. At

H, the "horizon-glass," about an inch wide and of the same height as the index-glass, is secured firmly to the frame of the instrument, in such position that, when the vernier of the index-arm reads *zero*, the index-mirror and horizon-glass will be parallel to each other. Only *half* of the horizon-glass is silvered, the upper half being left transparent. *E* is a small telescope.

If the vernier stands *near*, but not *at* zero, the observer looking into the telescope will see together in the field of view *two* separate images of the object; and if, while still looking, he slides the vernier a little, he will see that one of the images remains fixed, while the other moves. The fixed image is due to the rays which reach the object-glass of the telescope directly, coming through the unsilvered half of the horizon-glass: the movable image, on the other hand, is produced by rays which have suffered two reflections, — first, from the index-mirror to the horizon-glass; and second, at the lower half of the horizon-glass. When the two mirrors are parallel, and the vernier reads zero, the two images coincide, provided the object is at a considerable distance.

If now the vernier does not stand at or near zero, the observer, looking at any object directly through the horizon-glass, will see, not only that object, but also whatever other object is so situated as to send its rays to the telescope by reflection upon the mirrors; *and the reading of the vernier will give the angle at the instrument between the two objects whose images thus coincide;* the angle between the planes of the two mirrors being just half that between the objects, and the half-degrees on the limb being numbered as whole ones.

77. The principal use of the instrument is in measuring the altitude of the sun. At sea the observer, holding the instrument with his right hand and keeping the plane of the arc vertical, looks *directly* towards the visible horizon at the point under the sun, through the horizon-glass (whence its name); then by moving the vernier with his left hand, he inclines the index-glass upwards until one edge of the reflected image of the sun is brought just to touch the horizon-line, noting the exact time by the chronometer, if necessary. The reading of the vernier, after correcting for the semi-diameter of the sun, the dip of the horizon, the refraction, and the parallax (and for the "index-error" of the sextant, if the vernier does not read strictly zero when the mirrors are parallel) gives the sun's true altitude at the moment.

78. On land the *visible* horizon is of no use, and we have recourse to an "*artificial horizon,*" as it is called. This is merely a shallow basin of mercury, covered, when necessary to protect it from the wind, with a roof made of glass plates having their sides *plane* and *parallel.*

In this case we measure the angle between the sun's image reflected in the mercury and the sun itself. The reading of the instrument, corrected for index-error, gives *twice the sun's apparent altitude;* which apparent altitude, corrected as before for refraction and parallax, but not for dip of the horizon, gives the true altitude. The skilful use of the sextant requires steadiness of hand and considerable dexterity, and from the small size of the telescope the angles measured are of course less precise than if determined by large fixed instruments. But its portability and applicability at sea render it absolutely invaluable.

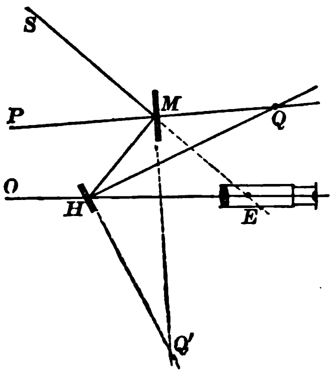

FIG. 32.—Principle of the Sextant.

79. The principle that the true angle between the objects whose images coincide is twice the angle between the mirrors (or between their normals) is easily demonstrated as follows (Fig. 32):—

The ray SM coming from an object, after reflection first at M (the index-mirror), and then at H (the horizon-glass), is made to coincide with the ray OH coming from the horizon. We must prove that the angle SEO, between the object and the horizon, as seen from the point E in the instrument, is double the angle Q, between MQ and HQ, which are normals to the mirrors, and therefore double Q', which is the angle between the planes of the mirrors.

First, from the law of reflection, we have,

$$SMP = HMP, \quad \text{or} \quad SMH = 2 \times PMH.$$

Similarly, $$MHE = 2 \times MHQ.$$

From the geometric principle that the exterior angle SMH of the triangle HME is equal to the sum of the opposite interior angles at H and E, we get

$$HEM = SMH - MHE = 2\,PMH - 2\,MHQ = 2\,(PMH - MHQ).$$

Similarly, from the triangle HMQ, we have

$$HQM = PMH - MHQ,$$

which is half the value just found for HEM, and proves the proposition.

Of course with the sextant, as with all other instruments, it is necessary for the observer who aims at the utmost precision to investigate, and take into account its errors of graduation, construction and adjustment; but their discussion lies beyond our scope.

80. Besides the instruments we have described, there are many others designed for special work, some of which, as the zenith telescope and heliometer, will be mentioned hereafter as it becomes necessary. There is also a whole class of physical instruments, photometers, spectroscopes, heat-measuring appliances, and photographic apparatus, which will have to be considered in due time.

But with clock, meridian circle, and equatorial and their usual accessories, all the fundamental observations of theoretical and spherical astronomy can be supplied. The chronometer and sextant are practically the only astronomical instruments of any use at sea.

CHAPTER III.

CORRECTIONS TO ASTRONOMICAL OBSERVATIONS, DIP OF THE HORIZON, PARALLAX, SEMI-DIAMETER, REFRACTION, AND TWILIGHT.

81. Dip of the Horizon. — In observations of the altitude of a heavenly body at sea, where the measurement is made from the *sea-line*, a correction is needed on account of the fact that this visible horizon does not coincide with the true astronomical horizon (which is 90° from the zenith), but falls sensibly below it by an amount known as the *Dip of the Horizon*. The amount of this dip depends upon the size of the earth and the height of the observer's eye above the sea-level.

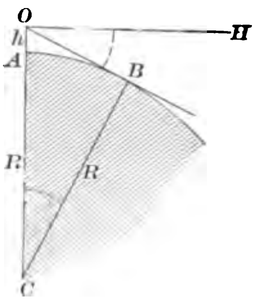

FIG. 33. — Dip of the Horizon.

In Fig. 33, C is the centre of the earth, AB a portion of its level surface, and O the observer, at an elevation h above A. The line OH is truly horizontal, while the tangent line, OB, corresponds to the line drawn from the eye to the visible horizon. The angle HOB is the *dip*. This is obviously equal to the angle OCB at the centre of the earth, if we regard the earth as spherical, as we may do with quite sufficient accuracy for the purpose in hand.

From the right-angled triangle OBC we have directly

$$\cos OCB = \frac{BC}{CO}.$$

Putting R for the radius of the earth, and Δ for the dip, this becomes

$$\cos \Delta = \frac{R}{R+h}.$$

This formula is exact, but inconvenient, because it gives the small angle Δ by means of its cosine. Since, however, $1 - \cos \Delta = 2 \sin^2 \frac{1}{2} \Delta$, we easily obtain the following: —

$$\sin \tfrac{1}{2} \Delta = \sqrt{\frac{h}{2(R+h)}}.$$

This gives the true depression of the sea horizon, as it would be if the line of sight, drawn from the eye to the horizon line, were *straight*. On account of refraction it is not straight, however, and the amount of this "terrestrial refraction" is very variable and uncertain. It is usual to diminish the dip computed from the formula by one-eighth its whole amount.

An approximate formula[1] for the dip is

$$\Delta \text{ (in minutes of arc)} = \sqrt{h \text{ (feet)}};$$

or, in words, *the square root of the elevation of the eye (in feet) gives the dip in minutes.* This gives a value about $\frac{1}{20}$ part too large.

Since the dip is applicable only to sextant observations made at sea, where, from the nature of the instrument, and the rising and falling of the observer with the vessel's motion, it is not possible to measure altitudes more closely than within about 15″, there is no need of any extreme precision in its calculation.

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[1] This approximate formula may be obtained thus: —

$$2 \sin^2(\tfrac{1}{2}\Delta) = \frac{h}{R+h} = \left(\frac{h}{R}\right) \div \left(1 + \frac{h}{R}\right).$$

But since $\frac{h}{R}$ is a very small fraction, it may be neglected in the divisor $\left(1+\frac{h}{R}\right)$, and the expression becomes simply,

$$2 \sin^2 \tfrac{1}{2}\Delta = \frac{h}{R}; \text{ whence } \sin \tfrac{1}{2}\Delta = \sqrt{\frac{h}{2R}}.$$

Since Δ is a very small angle,

$$\Delta = \sin \Delta = 2 \sin \tfrac{1}{2}\Delta, \quad \text{so that}$$

$$\Delta \text{ (in } radians\text{)} = 2\sqrt{\frac{h}{2R}} = \sqrt{\frac{h}{\frac{1}{2}R}}.$$

To reduce radians to minutes, we must multiply by 3438, the number of minutes in a radian. (Art. 6, page 7.) Accordingly,

$$\Delta' \text{ (in minutes of arc)} = 3438\sqrt{\frac{h}{\frac{1}{2}R}}.$$

If we express h in feet, we must also use the same units for R. The mean radius of the earth is about 20,884,000 feet, one-half of which is 10,442,000, and the square root of this is 3231; so that the formula becomes

$$\Delta' = \frac{3438}{3231}\sqrt{h \text{ (feet)}},$$

which is near enough to that given in the text.

In fact, the refraction makes so much difference that even after taking the numerical factor, $\frac{3438}{3231}$, as unity, the formula still gives Δ' about $\frac{1}{20}$ part too large.

The formula $\Delta' = \sqrt{3\,h \text{ (metres)}}$ is yet more nearly correct.

82. Parallax. — In the most general sense, "parallax" is the change of a body's direction resulting from the observer's displacement. In the restricted and technical sense in which we are to employ it now, it may be defined as the difference *between the direction of a body as actually observed and the direction it would have if seen from the earth's centre.* Thus in the figure, Fig. 34, where the observer is supposed to be at O, the position of P in the sky (as seen from O) would be marked by the point where OP produced would pierce the celestial sphere. Its position as seen from C would be determined in the same way by producing CP to which OX is drawn parallel. The angle POX, therefore, or its equal, OPC, is *the parallax of P for an observer at O.*

Obviously, from the figure, we may also give the following definition of the parallax. *It is the angular distance* (number of seconds of arc) *between the observer's station and the centre of the earth's disc, as seen from the body observed.* The moon's parallax at any moment for me is my angular distance from the earth's centre, as seen by "the man in the moon."

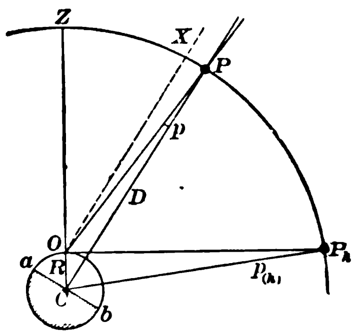

Fig. 34. — Diurnal Parallax.

When a body is in the zenith its parallax is zero, and it is a maximum at the horizon. In all cases it *depresses* a body, diminishing the altitude *without changing the azimuth.*

The "law" of the parallax is, that *it varies as the sine of the zenith distance directly, and inversely as the linear distance* (in miles) *of the body.*

This follows easily from the triangle COP, where we have $PC : OC = \sin COP : \sin CPO$.

Put D for PC, the distance of the body from the earth; R for the earth's radius, CO; p for CPO, the parallax; ζ for ZOP, the apparent zenith distance, and remember that the sine of ζ is equal to the sine of its supplement, COP: we then have as the translation of the above proportion,

$$D : R = \sin \zeta : \sin p.$$

This gives us
$$\sin p = \frac{R}{D} \cdot \sin \zeta;$$

or, from Art. 6, since p is always a small angle,

$$p'' = 206265'' \frac{R}{D} \cdot \sin \zeta.$$

83. **Horizontal Parallax.** — When a body is at the horizon (P_h in the figure), then ζ becomes 90°, and $\sin \zeta = 1$. In this case the parallax reaches its maximum value, which is called *the horizontal parallax* of the body. Taking p_h as the symbol for this, we have

$$\sin p_h = \frac{R}{D}; \text{ or, nearly enough, } p_h = 206265'' \frac{R}{D}.$$

Comparing this with the formula above, we see that the parallax of a body at any zenith distance equals the *horizontal parallax multiplied by the sine of the zenith distance;* i.e., $p = p_h \sin \zeta$.

N.B. A glance at the figure will show that we may define the *horizontal parallax*, OP_hC, of any body, as *the angular semi-diameter of the earth seen from that body.* To say, for instance, that the sun's horizontal parallax is 8″.8, amounts to saying that, *seen from the sun*, the earth's apparent diameter is twice 8″.8, or 17″.6.

84. **Relation between Horizontal Parallax and Distance.** — Since we have

$$\sin p_h = \frac{R}{D},$$

it follows of course that $D = R \div \sin p_h$;

or, (nearly) $$D = \frac{206265''}{p_h''} \times R.$$

If the sun's parallax equals 8″·8,

$$\text{its distance} = \frac{206265}{8.8} \times R = 23439\, R.$$

85. **Equatorial Parallax.** — Owing to the "ellipticity" or "oblateness" of the earth the horizontal parallax of a body varies slightly at different places, being a maximum at the equator, where the distance of an observer from the earth's centre is greatest. It is agreed to take as the standard the *equatorial* horizontal parallax; *i.e.*, the earth's *equatorial* semi-diameter as seen from the body.

86. **Diurnal Parallax.** — The parallax we have been discussing is sometimes called the *diurnal parallax*, because it runs through all its possible changes in one day.

When the sun, for instance, is rising, its parallax is a maximum, and by throwing it down towards the east, increases its apparent right ascension. At noon, when the sun is on the meridian, its parallax is a minimum, and

affects only the declination. At sunset it is again a maximum, but now throws the sun's apparent place down towards the west. Although the sun is invisible while below the horizon, yet the parallax, *geometrically considered*, again becomes a minimum at midnight, regaining its original value at the next sunrise.

The qualifier, "diurnal," is seldom used except when it is necessary to distinguish between this kind of parallax and the *annual* parallax of the fixed stars, which is due to the earth's orbital motion. The stars are so far away that they have no sensible *diurnal* parallax (the earth is an infinitesimal point as seen from them); but some of them do have a slight and measurable *annual* parallax, by means of which we can roughly determine their distances. (Chap. XIX.)

87. Smallness of Parallax. — The horizontal *parallax* of even the nearest of the heavenly bodies is always small. In the case of the moon the average value is about 57′, varying with her continually changing distance. Excepting now and then a stray comet, no other heavenly body ever comes within a distance a hundred times as great as hers. Venus and Mars approach nearest, but the parallax of neither of them ever reaches 40″.

88. Semi-Diameter. — In order to obtain the true altitude of an object it is necessary, if the edge, or "*limb*," as it is called, has been observed, to add or deduct the apparent semi-diameter of the object. In most cases this will be sensibly the same in all parts of the sky, but the moon is so near that there is quite a perceptible difference between her diameter when in the zenith and in the horizon.

A glance at Fig. 34 shows that in the zenith the moon's distance is less than at the horizon, by almost exactly the earth's radius — the difference between the lines OZ and OP_1. Now this is very nearly one-sixtieth part of the moon's distance, and consequently the moon, on a night when its apparent diameter at rising is 30′, will be 30″ *larger* when near the zenith. Since the semi-diameter given in the almanac is what would be seen from the *centre of the earth*, every measure of the moon's distance from stars or from the horizon will require us to take into account this "augmentation of the semi-diameter," as it is technically called.

The formula, easily deduced from the figure by remembering that the angle $PCO = \zeta - p$ (zenith distance − parallax), and that the apparent and "almanac" diameters will be inversely proportional to the two distances OP and CP, is

$$\text{apparent semi-diameter} = \text{almanac s. d.} \times \frac{\sin \zeta}{\sin (\zeta - p)}.$$

This measurable increase of the moon's angular diameter at high altitudes has nothing to do with the purely subjective illusion which makes the disc *look* larger to us when *near the horizon.* That it *is* a mere illusion may be made evident by simply looking through a dark glass just dense enough to hide the horizon and intervening landscape. The moon or sun then seems to shrink at once to normal dimensions.

89. Refraction. — Rays of light have their direction changed by refraction in passing through the air, and as the *direction in which we see a body is that in which its light reaches the eye*, it follows that this refraction apparently displaces the stars and all bodies seen through the atmosphere. So far as the action is regular, the effect is to bend the rays directly *downwards*, and thus to make the objects appear *higher* in the sky. Refraction *increases the altitude* of a celestial object *without altering the azimuth.* Like parallax, it is zero at the zenith and a maximum at the horizon; but it follows a different law. It is entirely independent of the distance of the object, and its amount varies (nearly) as the *tangent* of the zenith distance — not as *the sine*, as in the case of parallax.

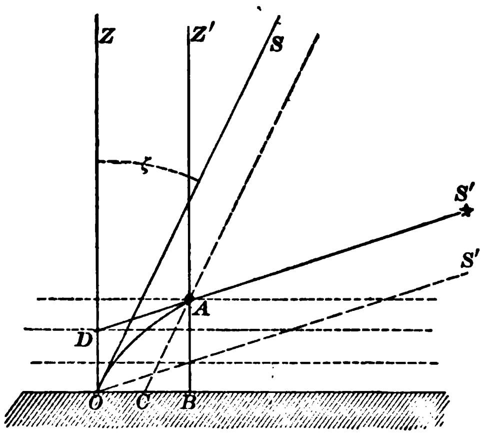

FIG. 35. — Atmospheric Refraction.

90. This approximate law of the refraction is easily proved.

Suppose in Fig. 35 that the observer at O sees a star in the direction OS, at the zenith distance ZOS or ζ. The light has reached him from S' by a path which was straight until the ray met the upper surface of the air at A, but afterwards curved continually downwards as it passed from rarer to denser regions.

We know that the atmosphere is very shallow as compared with the size of the earth, and it is exceedingly rare in the upper portions, so that, as far as concerns refraction, we may assume that the point A, where the first perceptible bending of the ray occurs, is not more than fifty miles high, and that the *vertical* AZ' *is sensibly parallel* to OZ; consequently, also

that *all the successive "strata of equal density" are parallel to each other and to the upper surface of the air.*

[This amounts to neglecting the earth's curvature between O and B.]

The *true* zenith distance (as it would be if there were no refraction) is ZDS', which equals $Z'AS'$; and since the refraction, r, may be defined as the difference between the true and apparent zenith distances, this true zenith distance will $= \zeta + r$.

Now from optical principles, when a ray of light passes through a medium composed of parallel strata, the final direction of the ray is the same as if the medium had throughout the density of the last stratum, and therefore the final direction, SO, will be the same as if all the air, from A down, had the same density as at O, with the same index of refraction, n. We may therefore apply the law of refraction directly at A, and write $\sin Z'AS' = n \sin BAC$ $(= ZOS)$, or $\sin(\zeta + r) = n \sin \zeta$; AC being drawn parallel to OS.

Developing the first member, we have

$$\sin \zeta \cos r + \cos \zeta \sin r = n \sin \zeta.$$

But r is always a small angle, never exceeding $40'$; we may therefore take $\cos r = 1$. Doing this and transposing the first term, we get

$$\cos \zeta \sin r = n \sin \zeta - \sin \zeta = (n-1) \sin \zeta.$$

Whence, $\quad \sin r = (n-1) \tan \zeta;$

or, $\quad r'' = (n-1)\, 206265 \tan \zeta$ (nearly).

The index of refraction for air, *at zero centigrade* and *a barometric pressure of* 760^{mm}, is 1.000294; whence,

$$r'' = .000294 \times 206265 \times \tan \zeta = 60''.6 \tan \zeta.$$

This equation holds very nearly indeed down to a zenith distance of 70°, but fails as we approach the horizon. For rays coming nearly horizontal, the points A and B are so far from O that the normal AZ' is no longer practically parallel to OZ; and many of the other fundamental assumptions on which the formula is based also break down.

At the horizon, where $\zeta = 90°$ and $\tan \zeta =$ *infinity*, the formula would give $\sin r =$ *infinity* also; an absurdity, since no sine can exceed unity. The refraction there is really about $37'$, under the circumstances of temperature and pressure above indicated.

91. Effect of Temperature and Barometric Pressure. — The index of refraction of air depends of course upon its temperature and pressure. As the air grows *warmer*, its refractive power *decreases;* as it grows *denser*, the refraction *increases*. Hence, in all precise observations of the altitude (or zenith distance), it is necessary to note both the thermometer and the barometer, in order to compute the refraction with accuracy. For rough work, like ordinary sextant observations, it will answer to use the "mean refraction," corresponding to an average state of things. See Appendix, Table VIII.

Tables of Refraction. — The exact computation of the refraction is best effected by special tables for the purpose; of these, Bessel's tables are the most convenient, best known, and probably even yet the most accurate. It must be always borne in mind, however, that from the action of wind and other causes the condition of the air along the path of the ray is seldom perfectly normal; in consequence, the actual refraction in any given case is liable to differ from the computed by as much as one or even two per cent. No amount of care in observation can evade this difficulty; the only remedy is a sufficient repetition of observations under varying atmospheric conditions. Observations at an altitude below 10° or 15° are never much to be trusted.

Lateral Refraction. — When the air is much disturbed, sometimes objects are displaced horizontally as well as vertically. Indeed, as a general rule, when one looks at a star with a large telescope and high power, it will seem to "dance" more or less — the effect of the varying refraction which continually displaces the image.

92. Effect on the Time of Sunrise and Sunset. — The horizontal refraction, ranging as it does from 32′ to 40′, according to temperature, is always somewhat greater than the diameter of either the sun or the moon. At the moment, therefore, when the sun's lower limb appears to be just rising, the whole disc is really below the plane of the horizon; and the *time* of sunrise in our latitudes is thus accelerated from two to four minutes, according to the inclination of the sun's diurnal circle to the horizon, which inclination varies with the time of the year. Of course, sunset is delayed by the same amount, and thus the day is lengthened by refraction from four to eight minutes, at the expense of the night.

93. Effect on the Form and Size of the Discs of the Sun and Moon. — Near the horizon the refraction changes very rapidly. While under ordinary summer temperature it is about 35′ *at* the horizon, it is

only 29′ at an elevation of half a degree; so that, as the sun or moon rises, the bottom of the disc is lifted 6′ more than the top, and the vertical diameter is thus made apparently about one-fifth part shorter than the horizontal. This distorts the disc into the form of an oval, flattened on the under side. In cold weather the effect is much more marked. As the horizontal diameter is not at all increased by the refraction, the apparent *area* of the disc is notably diminished by it; so that it is evident that refraction cannot be held in any way responsible for the apparent enlargement of the rising luminary.

94. Determination of the Refraction. — 1. *Physical Method.* Theory furnishes the *law* of astronomical refraction, though the mathematical expression becomes rather complicated when we attempt to make it exact. In order, therefore, to determine the astronomical refraction under all possible circumstances, it is only necessary to determine the index of refraction of air, and its variations with temperature and pressure, by laboratory experiments, and to introduce the constants thus obtained into the formulæ. It is difficult, however, to make these determinations with the necessary precision. In fact, at present our knowledge of the constants of air rests mainly on astronomical work.

2. *By Observations of Circumpolar Stars.* At an observatory whose latitude exceeds 45° select some star which passes *through the zenith* at the upper culmination. (Its declination must equal the latitude of the observatory.) It will not be affected by refraction at the zenith, while at the lower culmination, twelve hours later, it will. With the meridian circle observe its *polar distance* in both positions, determining the "polar point" of the circle as described on pp. 46–47. If the polar point were not itself affected by refraction, the simple difference between the two results for the star's polar distance, obtained from the upper and lower observations, would be the refraction at the lower point.

As a *first approximation*, however, we may neglect the refraction at the pole, and thus obtain a *first* approximate lower refraction. By means of this we may compute an *approximate* polar refraction, and so get a first "corrected polar point." With this compute a *second* approximate lower refraction, which will be much more nearly right than the first; this will give a *second* "corrected polar point"; this will in turn give us a *third* approximation to the refraction; and so on. But it would never be necessary to go beyond the third, as the approximation is very rapid. If the star does not go exactly

through the zenith, it is only necessary to compute each time approximate refractions for its upper observation, as well as for the polar point.

At present, however, the refraction is so well known that the method actually used is to form "equations of condition" from the observations of the altitude of known stars under varying circumstances, and from these to deduce such corrections to the star places and refraction constants as will best harmonize the whole mass of material.

95. 3. *By Observations of the Altitudes of Equatorial Stars made at an Observatory near the Equator.* For an observer so situated, stars that are on the celestial equator ($\delta = 0$) will come to the meridian at the zenith, and will rise and fall *vertically, with a motion strictly proportional to the time;* the *true* zenith distance of the star at any moment being just equal to its *hour-angle* in degrees. We have only, then, to observe the *apparent* zenith distance of a star with the corresponding time, and the refraction comes out directly.

If the station is not exactly on the equator, and if the star's declination is not exactly zero, it is only necessary to know the latitude and declination *approximately* in order to get the refraction very accurately: a considerable error in either latitude or declination will affect the result but slightly.

4. The French astronomer Loewy devised a new method which has proved excellent. He puts a pair of reflectors, inclined to each other at a convenient angle of from 45° to 50° (a glass wedge with silvered sides), in front of the object-glass of an equatorial. This will bring to the eye two rays which make a strictly constant angle with each other, and there is no difficulty in finding pairs of stars so situated that their images will come into the field of view together. Now, were it not for refraction, these images would always keep their relative position unchanged, notwithstanding the diurnal motion; but on account of the changes in the refraction, as one star rises and the other falls, they will shift in the field, and micrometric measures will determine the shifting, and so the refraction, with great precision.

96. Twilight. — (Although this subject is outside the main purpose of this chapter, which deals with corrections to be applied to astronomical observations, we treat it here because, like refraction, it is a purely atmospheric phenomenon, and finds no other more convenient place.)

Twilight, the illumination of the sky which begins before sunrise, and continues after sunset, is caused by the reflection of light to the observer from the upper regions of the earth's atmosphere. It is not yet certain whether this is due to reflection from foreign matter in the air, such as minute crystals of ice and salt, particles of dust of various kinds, and infinitesimal drops of water, or whether the pure gases themselves have some power of reflecting light. There is no

doubt, however, that air, under the ordinary conditions, possesses considerable power of reflection; so that, as long as any air upon which the sun is shining is visible to the observer, it will send him more or less light, and appear illuminated.

Suppose the atmosphere to have the depth indicated in the figure. Then, if the sun is at S, Fig. 36, it will just have set to an observer at 1, but all the air within his range of vision will still be illuminated. When, by the earth's rotation, he has been transported to 2, he will see the "twilight bow" rising in the east, a faintly reddish arc separating the illuminated part of the sky from the darkened part below, which lies in the shadow of the earth. When he reaches 3, the western half of the sky alone remains bright, but the arc of separation between the light and darkness has become vague and indefinite; when he reaches 4, only a glow remains in the west; and when he comes to 5, night closes in on him. Nothing remains in sight on which the sun is shining.

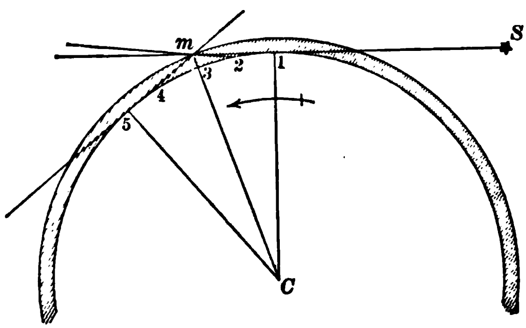

FIG. 36. — Twilight.

97. *Duration of Twilight.* — This depends upon the height of the atmosphere, and the angle at which the sun's diurnal circle cuts the horizon. It is found as a matter of observation, not admitting, however, of much precision, that twilight lasts until the sun has sunk about 18° below the horizon; that is to say, the angle 1 C 5 in the figure is about 18°.

The time required to reach this point in latitude 40° varies from two hours at the longest days in summer, to one hour thirty minutes about Oct. 12 and March 1, when it is least. At the winter solstice it is about one hour and thirty-five minutes.

In higher latitudes the twilight lasts longer, and the variation is more considerable: the date of the minimum also shifts.

Near the equator the duration is shorter, hardly exceeding an hour at the

sea-level; while at high elevations (where the amount of air above the observer's level is less) it becomes very brief. At Quito and Lima it is said not to last more than twenty minutes. Probably, also, in mountain regions the clearness of the air, and its purity contribute to the effect.

98. Height of the Atmosphere.—It is evident from the figure that at the moment twilight ceases, the last visible portion of illuminated air is at the top of the atmosphere, and just half-way between the observer and the nearest point where the sun is setting. If the whole arc 1, 5 is 18°, 1, 3 is 9°: then calling the height of the atmosphere H and the earth's radius R, and neglecting refraction (*i.e.*, supposing the lines $1m$ and $5m$ to be straight), we have from the right-angled triangle $1\,Cm$, $Cm = 1\,C \times \sec 9°$, or $R + H = R \times \sec 9°$; whence $H = R\,(\sec 9° - 1) = 0.0125\,R$, or almost exactly fifty miles. This must be diminished about one-fifth part on account of the curvature of the lines $1m$ and $5m$ by refraction, making the height of the atmosphere about forty miles.

The result must not, however, be accepted too confidently. It only proves that we get no sensible *twilight illumination* from air at a greater height: above that elevation the air is either too rare, or too *pure* from foreign particles, to send us any perceptible reflection. There is abundant evidence from the phenomena of meteors that the atmosphere extends to a height of 100 miles at least, and it cannot be asserted positively that it has *any* definite upper limit.

99. Aberration. — There is yet one more correction which has to be applied in order to get the true direction of the line which at the instant of observation joins the eye of the observer to the star he is pointing at. The *aberration of light* is an apparent displacement of the object observed, due to the combination of the earth's orbital motion with the progressive motion of light. It can be better discussed, however, in a different connection (see Chap. VI.), and we content ourselves with merely mentioning it here.

Exercises on Chapter I.

1. What point in the celestial sphere has both its right ascension and declination zero?

2. What are the celestial latitude and longitude of this point?

3. What are the hour-angle and azimuth of the zenith?

4. What angle does the celestial equator make with the horizon at a place in latitude 40°?

5. Name the fourteen principal points on the celestial sphere, zenith, poles, equinoxes, etc.

6. What important circles on the celestial sphere have no correlatives on the surface of the earth?

7. What are the approximate right ascension and declination of the sun on September 22?

8. If a certain star culminates (comes to the meridian) at eight o'clock to-night, at what time will it culminate ten days hence?

9. What is the altitude of the sun on March 21 at noon for an observer in latitude 40° 30′?

10. On March 21, one hour after sunset, whereabouts in the sky would be the position of a star having a right ascension of 7 hours and a declination of + 40°, the observer being in latitude 45°?

Exercises on Chapter II.

1. If a firefly were to light on the object-glass of a telescope, what would be the appearance to an observer looking through the instrument?

2. If a triangular piece of paper were pasted on the object-glass of a telescope pointed at the moon, how would it affect the appearance of the moon as seen by the observer?

3. If a certain eye-piece gives a magnifying power of 60 when used with a telescope of 5 feet focal length, what power will it give when used on a telescope of 30 feet focus?

4. If the wires of a micrometer (Fig. 29, Art. 73) are so set that, used with a telescope of 10 feet focal length, a star moving along the right ascension wire will occupy 15 seconds in passing from *d* to *e*, how long will it take when the micrometer is transferred to a telescope of 50 feet focus?

5. What is theoretically the angular distance between the centres of two star discs which are just barely separated by the Yerkes telescope of 40 inches aperture?

6. What is magnitude of 1″ on a graduated circle of 2 feet diameter?

7. Why is it important that the two pivots of a transit instrument should be of exactly the same diameter?

EXERCISES ON CHAPTER III.

1. What is the approximate dip of the horizon from a hill 900 feet high?

2. How high must a mountain be in order that the dip of the horizon from its summit may be 2°?

3. Does atmospheric refraction increase or decrease the apparent size of the sun's disc when it is near the horizon? Why?

4. Assuming the horizontal parallax of the sun at 8.8″, what is the horizontal parallax of Mars when nearest us at a distance of 0.378 astronomical units?

5. What is the greatest apparent diameter of the earth as seen from Mars?

6. What is the horizontal parallax of Jupiter when at a distance of 6 astronomical units?

7. What is the lowest latitude where twilight can last all night? Can it do so at New York? At London? At Edinburgh?

CHAPTER IV.

PROBLEMS OF PRACTICAL ASTRONOMY, LATITUDE, TIME, LONGITUDE, AZIMUTH, AND THE RIGHT ASCENSION AND DECLINATION OF A HEAVENLY BODY.

100. There are certain problems of Practical Astronomy which have to be solved in obtaining the fundamental facts from which we deduce our knowledge of the earth's form and dimensions, and other astronomical data.

The first of these problems is that of the Latitude.

DEFINITION OF LATITUDE.

The latitude (*astronomical*) of a place (Art. 30) is simply *the altitude of the celestial pole* (*Polhöhe*), or, what comes to the same thing, as is evident from Fig. 7 (Art. 33), it is the *declination of the zenith.* It may also be defined, from the mechanical point of view, as *the angle between the plane of the earth's equator and the observer's plumb-line or vertical.*

Neither of these definitions assumes anything as to the form of the earth. This *astronomical* latitude is seldom identical with the *geocentric*, or even with the *geodetic* or *chartographical* latitude of a place — the latitude used in accurate mapping. It is, however, the only kind of latitude which can be *directly* determined from astronomical observations, and its determination is one of the most important operations of Economic Astronomy.

101. Determination of Latitude. — First: *By Circumpolars.* The most obvious method of determining the latitude is to observe, with the meridian circle or some analogous instrument, the altitude of a circumpolar star at its upper culmination, and again, twelve hours later, at its lower. Each of the observations must be corrected for *refraction*, and then the *mean of the two corrected altitudes will be the latitude.*

This method has the advantage of being an *independent* one; *i.e.*, it does not require any data (such as the declination of the stars used) to be accepted on the authority of previous observers. But to obtain much accuracy it requires considerable time and a large fixed instrument. In low latitudes the refraction is also very troublesome.

102. Second: *By the meridian altitude or zenith distance of a body of known declination.*

In Fig. 37 the semicircle $AQPB$ is the meridian, Q and P being respectively the equator and the pole, and Z the zenith. QZ is the *declination of the zenith*, or the observer's latitude $(= PB = \phi)$. Suppose now that we observe Zs $(= \zeta_s)$, the zenith distance of a star s (south of the zenith), as it crosses the meridian, and that its declination Qs $(= \delta_s)$ is known; then evidently $\phi = \delta_s + \zeta_s$.

In the same way, if the star were at n, between zenith and pole, $\phi = \delta_n - \zeta_n$.

If we use the meridian circle, we can always select stars that pass near the zenith where the refraction will be small; moreover, we can select them in such a way that some will be as much north of the zenith as others are south, and thus *eliminate* the refraction errors. But we have to get our star declinations out of catalogues made by previous observers, and so the method is not an *independent* one.

103. **At Sea** the latitude is usually obtained by *observing with the sextant the sun's maximum altitude*, which of course occurs at noon. Since at sea it is seldom that one knows beforehand precisely the moment of local noon, the observer takes care to begin to observe the sun's altitude some ten or fifteen minutes earlier, repeating his observations every minute or two. At first the altitude will keep increasing, but immediately after noon it will begin to decrease. The observer uses therefore the *maximum*[1] altitude obtained, which, corrected for refraction, parallax, semidiameter, and dip of the horizon, will give him the true latitude of his ship, by the formula $\phi = \delta \pm \zeta$.

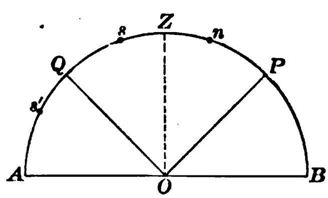

FIG. 37. — Determination of Latitude.

104. Third: *By Circum-meridian Altitudes.* — If the observer knows his time with reasonable accuracy, he can obtain his latitude from observations made when the body is *near* the meridian, with practically the same precision as at the moment of meridian passage. It would take us a little

[1] On account of the sun's motion in declination, and the northward or southward motion of the ship itself, the sun's maximum altitude is usually attained not *precisely* on the meridian, but a few seconds earlier or later. This requires a slight correction to the deduced latitude, explained in books on Navigation or Practical Astronomy.

too far to explain the method of reduction, which is given with the necessary tables in all works on Practical Astronomy. The great advantage of this method is that the observer is not restricted to a single observation at each meridian-passage of the sun or of the selected star, but can utilize the half-hours preceding and following that moment. The meridian-circle cannot be used, as the instrument must be such as to make extra-meridian observations possible. Usually the sextant or universal instrument is employed. This method is much used in the French and German geodetic surveys.

105. Fourth: *The Zenith Telescope Method.*—(Sometimes known as the American method, because first practically introduced by Captain Talcott of the United States Engineers, in a boundary survey in 1845.)

The essential characteristic of the method is the *micrometric* measurement of the *difference* between the nearly equal zenith distances of two stars which culminate within a few minutes of each other, one north and the other south of the zenith, and not very far from it: such pairs of stars can always be found. When the method was first introduced, a special instrument, known as the zenith telescope, was generally employed, but at present a simple transit instrument, with declination micrometer, and a delicate level attached to the telescope tube, is ordinarily used.

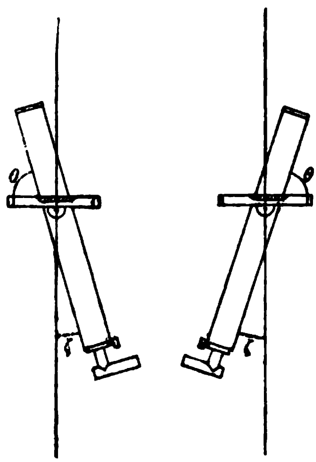

FIG. 38.—Principle of the Zenith Telescope.

The telescope is set at the proper altitude for the star which first comes to the meridian, and the "latitude level," as it is called, is set horizontal; as the star passes through the field of view its distance north or south of the central wire is measured by the micrometer. The instrument is then reversed, and so set by turning the telescope up or down (*without, however, disturbing the angle θ* (Fig. 38) *between the level and telescope*), that the level is again horizontal. After this reversal and adjustment, the telescope tube is then evidently elevated at exactly the same angle, ζ, as before, but on the opposite side of the zenith. As the second star passes through the field, we measure with the micrometer *its* distance north or south of the centre of the field; the comparison of the two micrometer measures gives the *difference* of the two zenith distances.

From Fig. 37 we have

for star *south* of zenith, $\phi = \delta_s + \zeta_s$;
for star *north* of zenith, $\phi = \delta_n - \zeta_n$.

Adding the two equations and dividing by 2, we have

$$\phi = \left(\frac{\delta_s + \delta_n}{2}\right) + \left(\frac{\zeta_s - \zeta_n}{2}\right).$$

The declinations, δ_s and δ_n, are given in the star catalogue, and the micrometer gives $(\delta_s - \delta_n)$. Usually, also, small corrections, seldom reaching 1″, must be addended for *differential* refractions, and for level-change, if any.

The great advantage of the method consists in its dispensing with a *graduated circle*, and in avoiding almost wholly the errors due to *refraction*. Forty years ago it was not always easy to find accurate determinations of the declinations of the stars employed, but at present this difficulty has practically disappeared, so that this method of determining the latitude is now not only the most convenient and rapid, but is quite as precise as any, if the level is sufficiently sensitive. Evidently the accuracy depends upon the exactness with which the level measures the slight, but inevitable, difference between the inclinations of the instrument when pointed on the two stars. In Dr. Chandler's "Almucantar," an instrument used for the same purpose, the telescope preserves its constant inclination *automatically*, by being mounted upon a base which floats in mercury, thus dispensing with the level.

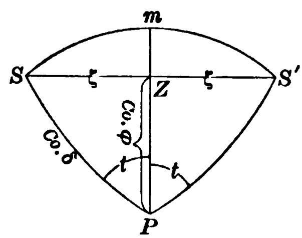

FIG. 39.
Latitude by Prime Vertical Transits.

106. Fifth: *By the Prime Vertical Instrument* (p. 43).—We observe simply the moment when a known star passes the prime vertical on the eastern side, and again upon the western side. Half the interval will give the *hour-angle* of the star when on the prime vertical; *i.e.*, the angle ZPS in Fig. 39, where Z is the zenith, P the pole, and SZS' the prime vertical. The distance PS of the star from the pole is the complement of the star's declination; and PZ is the complement of the observer's latitude. Since the prime vertical is perpendicular to the meridian at the zenith, the triangle PZS will be right-angled at Z; and from Napier's rule of circular parts (taking ZPS as the middle part) we shall have

$$\cos ZPS = \tan PZ \cot PS,$$

or

$$\cos t = \cot \phi \tan \delta;$$

whence

$$\tan \phi = \tan \delta \sec t.$$

If δ nearly equals ϕ, t will be small, and a considerable error in the observation of t will then produce very little change in its secant or in the computed latitude.

The observations are not so convenient and easy as in the case of the zenith telescope, and the number of stars available is less; but the method presents the great advantage of requiring nothing but an ordinary transit instrument, without any special outfit of micrometer and latitude level. It also entirely evades the difficulties caused by refraction.

107. Sixth: *By the Gnomon.*—The ancients had no instruments such as we have hitherto described, and of course could not use any of the preceding methods of finding the latitude. They were, however, able to make a very respectable approximation by means of the simplest of all astronomical instruments, *the gnomon*. This is merely a vertical shaft or column of known height erected on a perfectly horizontal plane; and the observation consists in noting the length of the shadow cast at noon at certain times of the year.

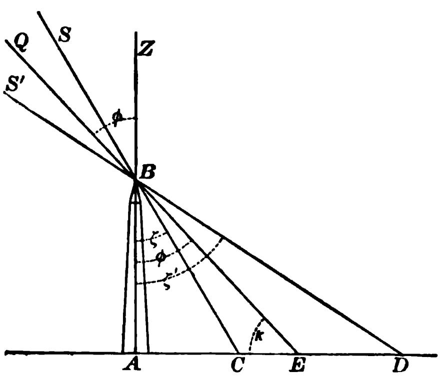

FIG. 40.—Latitude by the Gnomon.

Suppose, for instance, that on the *day of the summer solstice*, at noon, the length of the shadow is AC, Fig. 40. The height AB being given, we can easily compute in the right-angled triangle the angle ABC, which equals SBZ, the sun's zenith distance when farthest north. Again observe the length AD of the shadow at *noon of the shortest day in winter*, and compute the angle ABD, which is the sun's corresponding zenith distance when farthest south. Now, since the sun travels equal distances north and south of the celestial equator, the mean of the two results will give the angular distance between the equator and the zenith; *i.e.*, the *declination of the zenith*, which (Art. 100) is the latitude of the place.

The method is an independent one, like that by the observation of circumpolar stars, requiring no data except those which the observer determines for himself. Evidently, however, it does not admit of much accuracy, since the penumbra at the end of the shadow makes it impossible to measure its length very precisely.

It should be noted that the ancients, instead of designating the position

of a place by means of its latitude, used its *climate* instead; the climate (from κλίμα) being the *slope* of the plane of the celestial equator, the angle AEB, which is the complement of the latitude.

For the use of the gnomon in determining the obliquity of the ecliptic and the length of the year, see Art. 176. Many of the Egyptian obelisks are known to have been used for astronomical observations and were probably erected mainly for that purpose.

For numerous other methods of determining the Latitude, see Chauvenet's Practical Astronomy.

108. Variation of Latitude and Motion of the Poles. — It has long been doubted whether latitudes are strictly constant. They cannot be so if the axis of the earth shifts its position within the globe, for then the poles must also move, and the latitudes of places will change correspondingly. Some have supposed that in the past there have been great changes of this kind, and have sought thus to explain certain geological epochs, as for instance the glacial and the carboniferous. But thus far no confirmatory evidence of such displacement has appeared; nor is there yet any absolute evidence of certain slow, continuous, "secular" changes which have been strongly suspected.

Theoretically any alteration in the arrangement of the matter of the earth, by elevation, subsidence, transportation, or denudation, must almost necessarily disturb the axis to some extent. The question is merely whether we can observe with sufficient accuracy to detect the changes. Within the past few years this limit has been reached, and we now have conclusive proof of slight, but unquestionable, periodical "*variations of latitude.*" The first satisfactory evidence of such variations was obtained from observations made at Berlin (by Küstner) and at other German stations in 1888 and 1889. The result has since been abundantly confirmed by observations in Russia, France, England, the United States, and in the Sandwich Islands. Moreover, Dr. S. C. Chandler of Cambridge (U. S.), by a brilliant and laborious series of investigations, finds the same variations exhibited clearly in almost every extended body of reliable observations made since 1750. From the whole mass of evidence he concludes that the movement of the pole is composed of two motions, one an *annual* revolution in a narrow ellipse about 30 feet long, but varying in form and position, the other, a revolution in a circle about 26 feet in diameter, with *a period of about 428 days:* both motions are *counter-clock-wise.* The resultant motion appears very irregular, and varies greatly from year to year.

Fig. 40* (at the end of the chapter, page 95) represents the actual motion from 1890 to 1895 as deduced by Albrecht from all available observations.

The polar displacements produce also slight changes of *Azimuth* in geodetic lines, as has been actually observed at Pulkowa; and a minute *tide* (only a fraction of an inch), which theory indicates as a necessary consequence of shiftings of the earth's axis, has been detected in the Pacific and Atlantic oceans, and on the coast of Holland. The annual component of this polar motion is very likely due to meteorological causes which follow the seasons, such as the deposit and melting of snow and ice. The explanation of the 428 day component is not yet entirely clear, and its discussion would take us too far.

It is likely also that irregular disturbances, due to various causes, occasionally modify the regular periodic motions.

TIME AND ITS DETERMINATION.

109. One of the most important problems presented to the astronomer is the determination of *Time*. By universal consent the apparent rotation of the heavens is made to furnish the standard, and the determination of time is effected by ascertaining by observation the ***hour-angle of the object selected to mark the beginning of the day by its transit across the meridian.*** In practice three kinds of time are now recognized, viz., ***sidereal time, apparent solar time,*** and ***mean solar time.***

(For definition of hour-angle, see Art. 24.)

110. *Sidereal Time.* — As has already been explained (Art. 26), the sidereal time at any moment is *the hour-angle of the vernal equinox at that moment;* or, what comes to the same thing, though it sounds differently, it is *the time marked by a clock which is so set and adjusted as to show noon,* or $0^h\ 00^m\ 00^s$, *at each transit of the vernal equinox.* The sidereal *day,* thus defined, is the time intervening between two successive transits of the same *star;* at least, it is so within the hundredth part of a second, though on account of the precession of the equinoxes (and the proper motions of the stars) the agreement is not absolute, the difference amounting to about one day in twenty-six thousand years.

111. *Apparent Solar Time.* — Just as sidereal time is the hour-angle of the vernal equinox, so at any moment ***the apparent solar time is the hour-angle of the sun.*** It is the time ***shown by the sun-dial,*** and its noon is when the sun crosses the meridian. On account

of the annual eastward motion of the sun among the stars (due to the earth's orbital motion), this day is about four minutes longer than the sidereal; *i.e.*, while the earth's revolution brings our meridian back to a given *star* in just twenty-four (sidereal) hours, it takes $\frac{1}{365}$ of a day longer in the sun's case. Moreover, because the sun's motion in right ascension is not uniform, the apparent solar days are not all of the same length, nor, consequently, its hours, minutes, or seconds. December 23d is fifty-one seconds longer from (apparent) noon to noon than September 16th. For this reason, apparent solar time is not satisfactory for scientific use, and has long been discarded in favor of mean solar time.

112. *Mean Solar Time.*—A *"fictitious sun"* is therefore imagined, which moves *uniformly and in the celestial equator*, completing its annual course in exactly the same time as that in which the actual sun makes the circuit of the ecliptic. It is mean noon when this "fictitious sun" crosses the meridian, and at any moment *the hour-angle of this "fictitious sun" is the mean time* for that moment.

Sidereal time will not answer for business purposes, because its noon (the transit of the vernal equinox) occurs at all hours of night and daylight in different seasons of the year. Apparent solar time is scientifically unsatisfactory, because of the variation in the length of its days and hours. And yet we have to live by the sun; its rising and setting, daylight and night, control our actions. In mean solar time we find a satisfactory compromise, an invariable time unit, and still an agreement with sun-dial time close enough for convenience. It is the time now used for all purposes except in certain astronomical work. The difference between apparent time and mean time, never amounting to more than about a quarter of an hour, is called the *equation of time*, and will be discussed hereafter in connection with the earth's orbital motion, Chap. VI.

The nautical almanac furnishes data by means of which the sidereal time may be deduced from the corresponding solar, or *vice versa*, by a very brief and simple calculation. See Appendix, Art. 1000.

113. In practice the problem of determining the time always takes the form of *ascertaining the error of a time-piece;* that is, the amount by which a clock or watch is fast or slow of the time it ought to show. The methods most in use by astronomers are the following:—

First. *By means of the transit instrument.* Since the right ascension of a star is the sidereal time of its passage across the meridian

(Art. 26), it is obvious that the difference between the right ascension of a known star and the time shown by a sidereal clock at the instant when the star crosses the middle wire of an accurately adjusted transit instrument, is the error of the clock at that moment. Practically, it is usual to observe a number of stars (from eight to ten), reversing the instrument once at least, so as to eliminate the collimation error (Art. 60). With a good instrument a skilled observer can determine this clock error or "correction" within about one-thirtieth of a second of time, provided proper means are taken to ascertain and allow for his "personal equation."

114. *Personal Equation.* — It is found that every observer has his own peculiarities of time observation with a transit, and his "*personal equation*" is the amount to be added (algebraically) to the time observed by him, in order to get the actual moment of transit as it would be recorded by some supposable arrangement, which should automatically register the moment when the star's image was bisected by the wire.

This personal equation differs for different observers, but is reasonably (though never strictly) constant for one who has had much practice. In the case of observations with the chronograph, it is usually less than $\pm 0^s.2$. It can be determined by an apparatus in which an artificial star, resembling the real stars as much as possible in appearance, is made to traverse the field of view and to telegraph its arrival at certain wires, while the observer notes the moments for himself.

One of the most important problems of practical astronomy now awaiting solution is the contrivance of some practical method of time observation free from this annoying human element. Attempts are being made to utilize photography, and with fair prospects of success.

If mean time is wanted, it can be deduced from the sidereal time by the data of the almanac.

The sun can also be observed instead of the stars, the moment of the sun's transit being that of apparent noon; but this observation, for many reasons, is far less accurate and satisfactory than observations of the stars.

115. Second. *The method of equal altitudes.* — If we observe with a sextant in the forenoon the time shown by the chronometer when the sun attains the height indicated by a certain reading of the sextant, and then in the afternoon, the time when the sun again reaches the same

altitude, the moment of apparent noon will be half-way between the two observed times; provided, of course, that the chronometer runs uniformly during the interval, and also provided that proper correction is made for the sun's slight motion in declination — a correction easily computed.

The advantage of this method is that the errors of graduation of the sextant have no effect, nor is it necessary for the observer to know his latitude except approximately.

Per contra, there is, of course, danger that the afternoon observations may be interfered with by clouds; and, moreover, both observations must be made at the same place.

A modification of this method is now coming into extensive use, in which two different stars of known right ascension and of nearly the same declination are used, at equal altitudes east and west of the meridian.

116. Third. *By a single altitude of the sun, the observer's latitude being known.* — This is the method usual at sea. The altitude of the sun having been measured with the sextant, and the corresponding time shown by the chronometer having been accurately noted, we compute the hour-angle of the sun, P, from the triangle ZPS (Fig. 41), and this hour-angle corrected for the equation of time, gives the true mean time at the observed moment. The difference between this and that shown by the chronometer is *the error of the chronometer*. In the triangle ZPS all three of the sides are given, viz.: PZ is the complement of the latitude ϕ, which is supposed to be known; PS is the complement of the declination δ, which is found in the almanac, as is also the equation of time; while ZS or ζ, is the complement of the sun's altitude, as measured by the sextant, and corrected for semidiameter, refraction, and parallax. The formula is

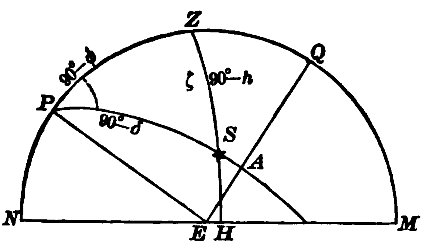

FIG. 41.— Determination of Time by a Single Altitude.

$$\sin \tfrac{1}{2} P = \left(\frac{\sin \tfrac{1}{2}[\zeta + (\phi-\delta)] \sin \tfrac{1}{2}[\zeta - (\phi - \delta)]}{\cos \phi \cos \delta}\right)^{\frac{1}{2}}.$$

In order to accuracy, it is desirable that the sun should be on the prime vertical, or as near it as practicable. It should *not* be near the

meridian. Any slight error in the assumed latitude produces no sensible effect upon the result, if the sun is exactly east or west at the time the observation is taken. The disadvantage of the method is that any error of graduation of the sextant vitiates the result.

In some cases a person is so situated that it is necessary to determine his time roughly, without instruments; and this can be done within about a half a minute by establishing a noon-mark, which is nothing but a line drawn exactly north and south, with a plumb-line, or some vertical edge, like the edge of a door-frame or window sash, at its southern extremity. The shadow will then always fall upon the meridian line at *apparent* noon.

117. The Civil and the Astronomical Day. — The *astronomical* day begins at mean noon.[1] The *civil* day begins at midnight, twelve hours earlier. Astronomical mean time is reckoned round through the whole twenty-four hours, instead of being counted in two series of twelve hours each. Thus, 10 A.M. of Wednesday, May 2, *civil reckoning*, is Tuesday, May 1, 22^h, by *astronomical reckoning.*

LONGITUDE.

118. Having now methods of obtaining the true local time, we can attack the problem of longitude, which is perhaps the most important of all the economic problems of astronomy. The great observatories at Greenwich and at Paris were established simply for the purpose of furnishing the observations which could be made the basis of the accurate determination of longitude at sea.

The longitude of a place on the earth is *the angle at the pole between the meridian of Greenwich and the meridian passing through the observer's place;* or it is the arc of the equator intercepted between these meridians; or, what comes to the same thing, since this arc is measured by the time required for the earth to turn sufficiently to bring the second meridian into the same position held by the first, it is simply *the difference of their local times,* — the amount by which the noon at Greenwich is earlier or later than at the observer's place. It is now usually reckoned in hours, minutes, and seconds, instead of degrees.

Since it is easy for the observer to find his own local time by the methods which have been given, the knot of the problem is really this: *being at any place, to find the corresponding local time at Greenwich without going there.*

---

[1] There is a proposition to make the astronomical day correspond with the civil, which may take effect in 1900. But practical astronomers dislike to have to change dates at midnight, in the midst of their work.

The methods of finding the longitude may be classed under three different heads:

First, By means of signals simultaneously observable at the places between which the difference of longitude is to be found.

Second, By making use of the moon as a clock-hand in the sky.

Third, By purely mechanical means, such as chronometers and the telegraph. This is the modern method, and the best wherever available.

119. Under the first head we may make use of

[A] *A Lunar Eclipse.* — When the moon enters the shadow of the earth, the phenomenon is seen at the same moment, no matter where the observer may be. By noting, therefore, his own local time at the moment, and afterwards comparing it with the time at which the phenomenon was observed at Greenwich, he will obtain his longitude from Greenwich. Unfortunately, the edge of the earth's shadow is so indistinct that the progress of events is very gradual, so that sharp observations are impossible.

[B] *Eclipses of the satellites of Jupiter* may be used in the same way, with the advantage that they occur very frequently, — almost every night, in fact; but the objection to them is the same as to the lunar eclipses, — they are not sudden.

[C] *The appearance and disappearance of meteors* may be and has been used to determine the difference of longitude between places not more than two or three hundred miles apart, and gives very accurate results. (Now superseded by the telegraph.)

[D] *Artificial signals*, such as flashes of powder and rockets, can be used between two stations not too far distant. Early in the century the difference of longitude between the Black Sea and the Atlantic was determined by means of a chain of signal stations on the mountain tops; so also, later, the difference of longitude between the eastern and western extremities of the northern boundary of Mexico. This method is now superseded by the telegraph.

120. Second, *the moon regarded as a clock.*

Since the moon revolves around the earth once a month, it is, of course, continually changing its place among the stars; and as the laws of its motion are now well known, and as the place which it will occupy is predicted for every hour of every Greenwich day three years in advance in the nautical almanac, it is possible to deduce the corresponding Greenwich time by any observation which will determine the place of the moon among the stars. The almanac

place, however, is the place at which the moon would be seen by an observer *at the centre of the earth*, and consequently the actual observations are in most cases complicated with very disagreeable reductions for parallax before they can be made available.

The simplest lunar method is,

[A] *That of Moon Culminations.* — We merely observe with a transit instrument the time when the moon's bright limb crosses the meridian of the place; and immediately after the moon we observe one or more stars with the same instrument, to give us the error of our clock. As the moon is observed on the meridian, its parallax does not affect its right ascension, and accordingly, by a simple reference to the almanac, we can ascertain the Greenwich time at which the moon had the particular right ascension determined by the observation. The method has been very extensively used, and would be an admirable one were it not for the effects of personal equation.

It seldom happens that the personal equation of an observer is the same for such an object as the limb of the moon as it is for a star; and since the moon's motion among the stars is very slow, the effect of such a difference is multiplied by about 30 (roughly the number of days in a month) in its effect upon the longitude deduced.

[B] *Lunar-Distances.* — At sea it is, of course, impossible to observe the moon with a transit instrument, but we can observe its distance from the stars near its path by means of a sextant. The distance observed will not be the same that it would be if the observer were at the centre of the earth, but by a mathematical process called "clearing a lunar" the distance as seen from the centre of the earth can be easily deduced, and compared with the distance given in the almanac. From this the longitude can be determined. Any error, however, in measuring a lunar-distance entails an error about thirty times as great in the resulting longitude, and the method is at present very little used, the moon having been superseded by the chronometer for such purposes.

[C] *Occultations.* — Occasionally, in its passage through the sky, the moon over-runs a star, or "*occults*" it. The star vanishes instantaneously, and, of course, at the moment of its disappearance the distance from the centre of the moon to the star is precisely equal to the apparent semi-diameter of the moon; we thus have a "lunar-distance" self-measured.

Observations of this kind furnish one of the most accurate methods

of determining the difference of longitude between widely separated places, the only difficulty arising from the fact that the edge of the moon is not smooth, but more or less mountainous, so that the distance of a star from the moon's centre is not always the same at the moment of its disappearance.

[D] *In the same way a solar eclipse may be employed by observing the moment when the moon's limb touches that of the sun.*

It will be noticed that these two last methods (the methods of occultation and solar eclipse) do not belong in the same class with the method of lunar eclipse, because the phenomena are not seen at the same instant at different places, but the calculation of longitude depends upon the determination of the moon's place in the sky at the given time, as seen from the earth's centre.

There are still other methods, depending upon measurements of the moon's position by observations of its altitude or azimuth. In all such cases, however, every error of observation entails a vastly greater error in the final results. Lunar methods (excepting occultations) are only used when better ones are unavailable.

121. Finally we have what may be called the *mechanical methods* of determining the longitude.

[A] *By the chronometer;* which is simply an accurate watch that has been set to indicate Greenwich time before the ship leaves port. In order to find the longitude by the chronometer, the sailor has to determine its "error" upon local time by an observation of the altitude of the sun when near the prime vertical, as indicated on page 78. If the chronometer indicates true Greenwich time, *the error deduced from the observation will be the longitude.* Usually, however, the indication of the chronometer face requires correction for the rate and run of the chronometer since leaving port.

Chronometers are only imperfect instruments, and it is important, therefore, that several of them should be used to check each other. It requires three at least, because if only *two* chronometers are carried and they disagree, there is nothing to indicate which one is the delinquent.

On very long voyages the errors of chronometers are cumulative, and the uncertainty accumulates much more rapidly than in proportion to the time; *i.e.*, if the error to be feared in the use of a chronometer in longitude determinations at the end of a week is about two seconds of time, at the end of the month it would be, not eight seconds, but very likely twenty or thirty, owing to the possible *changes of its rate* during the voyage.

If, therefore, a ship is to be at sea, without making port, more than three

or four months at a time, the method becomes untrustworthy, and it may be necessary to recur to lunar distances; for voyages of less than a month the method is now, practically, all that could be desired.

[B] But the method which, wherever it is applicable, has superseded all others, is that of *The Telegraph*. When we wish to find the longitude between two stations connected by telegraph, the process is usually as follows: The observers at both stations, after ascertaining that they both have clear weather, proceed to determine their own local time by extensive series of star observations with the transit instrument. Then, at an agreed-upon time, the observer at Station A "switches his clock" into the telegraphic circuit, so that its beats are communicated along the line and received upon the chronograph of the other, say the western station. After the eastern clock has thus sent its signals, say for two minutes, it is switched out of the circuit, and the western observer now switches his clock into the circuit, and its beats are received upon the eastern chronograph. The operation is closed by another series of star observations.

We have now upon each chronograph sheet an accurate comparison of the two clocks, showing the amount by which the western clock is slow of the eastern. If the transmission of electric signals were instantaneous, the difference shown upon the two chronograph sheets would agree precisely. Practically, however, there will always be a small discrepancy amounting to twice the time occupied in the transmission of the signals; but the mean of the two differences will be the true difference of longitude of the places after the proper corrections have been applied. *Especial care must be taken to determine with accuracy, or to eliminate, the personal equations of the observers.*

It is customary to make observations of this kind on not less than five or six evenings in cases where it is necessary to determine the difference of longitude with the highest accuracy. The *astronomical* difference of longitude between two places can thus be telegraphically determined within about the one-hundredth part of a second of time; *i.e.*, within about ten feet or so, in the latitude of the United States.

It may be noted here that the time occupied by the transmission of electric signals in longitude operations is not to be taken as the real measure of "the velocity of the electric fluid" upon the wires, as was once supposed. The time apparently consumed in the transmission is simply the time required for the current at the receiving station (which current probably *begins* at the very instant the key is touched at the other end of the line) to become *strong enough* to do its work in making the signal; and this time depends upon a multitude of circumstances.

122. Local and Standard Time. — In connection with time and longitude determinations, a few words on this subject will be in place. Until recently it has always been customary to use only *local* time, each observer determining his own time by his own observations. Before the days of the telegraph, and while travel was comparatively slow and infrequent, this was best; but the telegraph and railway have made such changes that, for many reasons, it is better to give up the old system of local times in favor of a system of standard time. It facilitates all railway and telegraphic business in a remarkable degree, and makes it practically easy for every one to keep accurate time, since it can be daily wired from some observatory to every telegraph office.

According to the system that is now established in this country, there are five such standard times in use, — the colonial, the eastern, the central, the mountain, and the Pacific, — which differ from Greenwich time by exactly four, five, six, seven, and eight hours respectively, *the minutes and seconds being identical everywhere.* At most places only one of these times is employed; but in cities where different systems join each other, there are two standard times in use, differing from each other by exactly one hour, and from the local time by about half an hour. In some such places the local time also maintains its place.

In order to determine the standard time by observation, it is only necessary to determine the local time by one of the methods given, and correct it according to the observer's longitude from Greenwich.

123. Where the Day Begins. — If we imagine a traveller starting from Greenwich on Monday noon, and journeying westward as swiftly as the earth turns to the east under his feet, he would, of course, keep the sun exactly on the meridian all day long, and have continual noon. But what noon? It was Monday when he started, and when he gets back to London, twenty-four hours later, it is Tuesday noon there, and there has been no intervening sunset. When does Monday noon become Tuesday noon? The convention is that *the change of date occurs at the* 180*th meridian from Greenwich.* Ships crossing this line *from the east* skip one day in so doing. If it is Monday forenoon when the ship reaches the line, it becomes Tuesday forenoon the moment it passes it, the intervening twenty-four hours being dropped from the reckoning on the log-book. *Vice versa,* when a vessel crosses the line *from the western side,* it counts the same day twice, passing from Tuesday forenoon back to Monday, and having to do its Tuesday over again.

This 180th meridian passes mainly over the ocean, hardly touching land anywhere. There is a little irregularity in the date upon the different islands near this line. Those which received their earliest European inhabitants *via* the Cape of Good Hope have, for the most part, the Asiatic date. belonging to the west side of the 180th meridian; while those that were approached *via* Cape Horn have the American date.

When Alaska was transferred from Russia to the United States, it was necessary to drop one day of the week from the official dates.

THE PLACE OF A SHIP AT SEA.

124. The determination of the place of a ship at sea is commercially of such importance that, at the risk of a little repetition, we collect together here the different methods available for its determination. The methods employed are necessarily such that observations can be made with the sextant and chronometer, the only instruments available under the circumstances.

The **Latitude** is usually obtained by observations of the sun's altitude at noon, according to the method explained in Art. 103.

The **Longitude** is usually found by determining the error upon local time of the chronometer, which carries Greenwich time. The necessary observations of the sun's altitude should be made when the sun is near the prime vertical, as explained in Art. 116.

In the case of long voyages, or when the chronometer has for any reason failed, the longitude may also be obtained by measuring a lunar-distance and comparing it with the data of the nautical almanac.

By these methods separate observations are necessary for the latitude and for the longitude.

125. Sumner's Method.—Recently a new method, first proposed by Captain Sumner, of Boston, in 1843, has been coming largely into use. In this method, each observation of the sun's altitude, with the corresponding chronometer time, is made to define the position of the ship upon a certain line, called *the circle of position.* Two such observations will, of course, determine the exact place of the vessel at one of the intersections of the two circles.

At any moment the sun is vertically over some point upon the earth's surface, which may be called the *sub-solar point.* An observer there would have the sun directly overhead. Moreover, if at any point on the earth an observer measures the altitude of the sun with his sextant, *the zenith distance of the sun* (which is the complement of this altitude) *will be his distance from the sub-solar point at the moment of observation, reckoned in degrees of a great circle.*

If, then, I take a terrestrial globe, and, opening the dividers so as to cover an arc equal to this observed zenith distance of the sun, put one foot of the dividers upon the sub-solar point, and sweep a

circle on the surface of the globe around that point, the observer must be *somewhere on the circumference of that circle;* and moreover, if to the observer the sun is in the *southwest*, he himself must be in the opposite direction from this sub-solar point; *i.e.*, *northeast* of it. In other words, the *azimuth of the sun* at the time of observation informs him upon *what part of the circle* he is situated.

Suppose a similar observation made at the same place a few hours later. The sub-solar point, and the zenith distance of the sun, will have changed; and we shall obtain a new circle of position, with its centre at the new sub-solar point. The observer must be at one of its two intersections with the first circle — which of the two intersections is easily determined from the roughly observed azimuth of the sun.

If the ship moves between the two observations, the proper allowance must be made for the motion. This is easily done by shifting upon the chart that part of the first circle of position where the ship was situated, carrying the line forward parallel to itself, by an amount just equal to the ship's run between the two observations, as shown by the log. The intersection with the second circle then gives the ship's place *at the time of the second observation.*

The only problem remaining is to find the position of the "sub-solar point" at any given moment. Now, the *latitude* of this point is obviously the *declination of the sun* (which is found in the almanac). If the sun's declination is zero, the sun is vertically over some point upon the equator. If its declination is + 20°, it is vertically over some point on the twentieth parallel of north latitude, etc.

In the next place, its *longitude* is equal to the *Greenwich apparent solar time* at the moment of observation; and this is given by the chronometer (which keeps Greenwich mean solar time), by simply adding or subtracting the equation of time; so that, by looking in his almanac and at his chronometer, the observer has the position of the sub-solar point immediately given him.

Suppose, for example, that on May 20 (the sun's declination being + 20°), at 11 A.M., Greenwich *apparent* time (*i.e.*, May 19, 23^h by astronomical reckoning), according to the chronometer, the sun is observed to have an altitude of 40° by a ship in the North Atlantic. The sub-solar point will then be (Fig. 42) at a point in Africa having a latitude of + 20°, and an east longitude of 15° — at *A* in the figure. And the radius of the "circle of position," *i.e.*, the distance from *A* to *C* — will be 50°.

Again, a second observation is made three hours later, when the sun's altitude is found to be 65°. The sub-solar point will then be at *B*, latitude

20°, longitude 30° W., and the radius of the circle of position BC will be 25°, C being the ship's place.

126. It is, however, seldom convenient to carry a large globe, and in practice the usual procedure is the following. The latitude of a ship is always known within a few degrees by the "dead-reckoning"; suppose that it is known to be about 50° 30′. From the first observation calculate (by the methods of Art. 121) what the *longitude* would be if the latitude were 50°, and also if it were 51°. Mark the two points on the charts and connect them by a straight line, which will be (very nearly) a portion of the first circle of position. In the same way obtain a second "position line" from the second observation. The intersection of the two lines will give the ship's place, the first position line being moved forward, parallel to itself, by the amount of the ship's motion in the interval between the two observations.

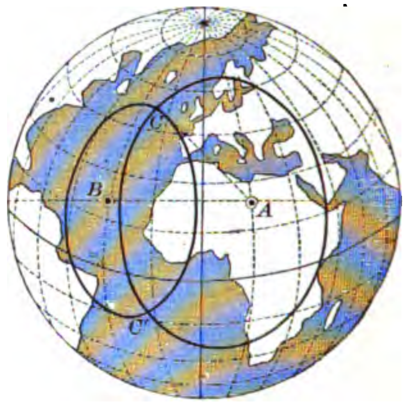

FIG. 42. — Sumner's Method.

The peculiar advantage of the method is, that a single observation is used for all it is worth, giving *accurately* the position of a line upon which the ship is somewhere situated, and *approximately* (by the rough observation of the sun's azimuth) the part of that line upon which its place will be found. In approaching the American coast, for instance, if an observation be taken in the forenoon, the ship's position circle will lie nearly parallel to the coast, and then a single observation will give approximately the distance of the ship from land, which may be all the sailor wishes to know. The observations need not be taken at any particular time. We are not limited to observations at noon, or to the time when the sun is near the prime vertical. It is to be noted, however, *that everything depends upon the chronometer,* as much as in the ordinary chronometric determination of longitude.

127. Determination of Azimuth. — A problem, important, though not so often encountered as that of latitude and longitude determina-

tions, is that of determining *the azimuth, or true bearing, of a line upon the earth's surface.* The process is this: With a theodolite having an accurately graduated horizontal circle the observer points alternately upon the pole star and upon a distant signal erected for the purpose; the signal being an artificial star consisting of a small hole in a plate of metal, with a bull's-eye lantern or other light behind it. It is desirable that it should be at least a mile away from the observer, so that any small displacement of the instrument will be harmless. The theodolite must be carefully adjusted for collimation, and especial pains must be taken to have the axis of the telescope perfectly level.

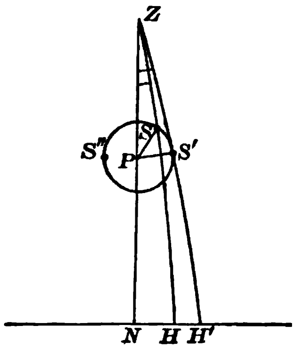

FIG. 43.— Determination of Azimuth.

The next morning by daylight the observer measures the angle or angles between the night-signal and the objects whose azimuth is required.

If the pole star were exactly at the pole, the mere difference between the two readings of the circle, obtained when the telescope is pointed on the star and on the signal, would directly give the azimuth of the signal. As this is not the case, however, the *time* at which each observation of the pole star is made must be noted, and the azimuth of the star must be computed for that moment. This can easily be done, as the right ascension and declination of this star are given in the almanac for every day of the year.

Recurring to the Z.P.S. [zenith-pole-star] triangle, N (Fig. 43) being the north point of the horizon, P the pole, and NZ the meridian, we at once see that the side PS is the complement of the star's declination; the side PZ is the complement of the observer's latitude (which must be known); and the angle at P is the difference between the right ascension of the pole star and the sidereal time of the observation; [$(t-a)$ if the star is west of the meridian at the time, and $(a-t)$ if it is east.] This will come out in hours, of course, and must be reduced to degrees before making the computation. We thus have two sides of the triangle, viz., PS and PZ, with the included angle at P, from which to compute the angle Z at the zenith. This is the star's azimuth.

The pole star is used because, being so near the pole, any slight error in the assumed latitude of the place or in the sidereal time of the observation will hardly produce any effect upon the result, especially if the star be caught between five and six hours before or after its upper culmination, at

a time when it changes its azimuth very slowly (near S' or S'' in the figure). The sun, or any other heavenly body whose position is given in the almanac, can also be used as a reference point in the same way, provided sufficient pains is taken to secure an accurate observation of the time at the instant when the pointing is made. The altitude should not exceed thirty degrees or so. But the results are usually rough compared with those obtained by means of the pole star.

DETERMINATION OF THE POSITION OF A HEAVENLY BODY.

128. The position of a heavenly body is defined by its right ascension and declination. These quantities may be determined —

(1) **By the meridian circle,** provided the body is bright enough to be seen by the instrument and comes to the meridian in the night-time. If the instrument is in exact adjustment, the *sidereal time when the object crosses the middle wire of the reticle of the instrument is directly* (according to Art. 27) *the right ascension of the object.*

The reading of the circle of the instrument, corrected for refraction and parallax if necessary, gives the *polar distance* of the object, if the polar point of the circle has been determined (Art. 66); or it gives the *zenith distance* of the object if the nadir point has been determined (Art. 67). In either case the *declination* can be immediately deduced, being the complement of the polar distance, and equal to the latitude of the observer, minus the distance of the star south of the zenith. One complete observation, then, with the meridian circle, determines both the right ascension and declination of the object.

It is often better to use the instrument "differentially," *i.e.*, to observe some standard star, whose place is already accurately known, along with the object whose place is to be determined. We thus obtain the difference of their Right Ascensions and Declinations, and slight errors in the graduation and adjustment of the instrument affect the final result far less than in an "absolute" determination.

If a body (a comet, for instance) is too faint to be observed by the telescope of the meridian circle, which is seldom very powerful, or if it does not come to the meridian during the night, we usually accomplish our object —

129. (2) **By the Equatorial,** determining the position of the body by measuring the *difference of right ascension and declination* between it and some neighboring star, whose place is given in a star catalogue, and of course has been determined by the meridian circle of some observatory.

In measuring this difference of right ascension and declination, we usually employ a filar micrometer fitted like the reticle of a meridian circle. It carries a number of wires which lie north and south in the field of view, and these are crossed at right angles by one or more wires which can be moved by the micrometer screw. *The difference of right ascension* between the star and the object to be determined is measured by simply observing with the chronograph the transits of the two objects across the north and south wires; *the difference of declination*, by bisecting each object with one of the micrometer wires as it crosses the middle of the field of view. The observed difference must be corrected for refraction and for the motion of the body, if it is appreciable.

Other less complicated micrometers are also in use. One of them, called the *ring micrometer*, consists merely of an opaque ring supported in the field of view either by being cemented to a glass plate or by slender arms of metal. The observations are made by noting the transits of the comparison star and of the object to be determined across the outer and inner edges of the ring. If the radius of the ring is known in seconds of arc, we can from these observations deduce the differences both of right ascension and declination. The results are less accurate than those given by the wire micrometer, but the ring micrometer has the advantage that it can be used with any telescope, whether equatorially mounted or not, and requires no adjustment.

There are also many other methods of effecting the same object.

130. To Compute the Time of Sunrise or Sunset. — To solve this problem, it is only necessary to work out the Z.P.S. triangle and find the hour-angle P, having given precisely the same data as in finding the time by a single altitude of the sun (Art. 116). PZ is the observer's co-latitude, PS is the complement of the sun's declination (given by the almanac); and the true distance from the zenith to the centre of the sun at the moment when its upper edge is at the horizon is $90° 50'$, which is made up of $90°$,+ $16'$ (the mean semi-diameter of the sun), plus $34'$ (the mean refraction at the horizon). The resulting hour-angle P, corrected for the equation of time, gives the mean time (*local*) at which the sun's upper limb touches the horizon, under the average circumstances of temperature and barometric pressure. If it is very cold, with the barometer standing high, sunrise will be accelerated, or sunset retarded, by a considerable fraction of a minute. If the sun rises or sets over the sea-horizon, and the observer's eye is at any considerable elevation above the sea-level, the dip of the horizon must also be added to the $90° 50'$ before making the computation.

The beginning and end of twilight may be computed in the same way by merely substituting $108°$ for $90° 50'$.

131. To Compute the Time of the Rising or Setting of a Star, or of the Moon. — In the case of a star we compute its hour-angle at the horizon just as for sunrise, only using 90° 34′ for the zenith-distance instead of 90° 50′. The hour-angle *added to* the star's Right Ascension gives the *sidereal time* of its setting; by *subtracting* the hour-angle we get the sidereal time of its rising. The sidereal times are then converted into local mean-time by the data given in the Almanac. (Appendix, Art. 1000.)

The rapid motion of the moon complicates the problem in her case, and we have to use a method of approximation. We begin by estimating the Greenwich time of moon-rise as nearly as we can without actual calculation. We then take out from the Almanac the moon's Right Ascension and Declination for that moment (the Almanac gives the data for every hour). With the declination and the latitude of the place we compute the moon's hour-angle, taking the zenith-distance as 89° 53′, since the horizontal parallax of the moon (57′) is to be deducted from the 90° 50′ which we used in the case of the sun. The hour-angle thus computed is then subtracted from the moon's Right Ascension, and we thus get an *approximate sidereal* time of moon-rise, which must be converted into mean time. If the time originally assumed by estimation does not differ from this computed result by more than fifteen minutes or so, the latter may be taken as correct within a fraction of a minute. But if the difference is greater, we must have recourse to the Almanac again, must look out afresh the Right Ascension and Declination of the moon corresponding to the approximate time, as computed, and then repeat the calculation with the new data. A third computation is never necessary.

Exercises on Chapter IV.

(In cases where corrections for refraction are given they are to be taken from Table VIII, Appendix, taking into account the temperature and barometric pressure if given among the data.)

1. Given the following meridian circle observations on Beta Ursæ Minoris at its upper and lower culminations respectively, viz.:

55° 48′ 06.0″, Temp. 30° F., Barometer 30.1 inches.
24° 58′ 56.4″ " 25° F. " 30.1 "

The nadir reading (Art. 67) was 270° 01′ 06.8″ in both cases. Required the latitude of the place and the declination of the star.

Ans. Lat. 40° 20′ 57.8″.
Dec. 74° 34′ 40.1″.

2. Given the meridian altitude of the sun's lower limb, 62° 24′ 45″, the height of the observer's eye above the sea-level being 16 feet (Art. 81).

The sun's declination was + 20° 55′ 10″, and its semidiameter, 15′ 47″. Its parallax at the observed altitude was 5″, and the mean refraction may be used. Required the latitude of the ship. *Ans.* + 48° 19′ 03″.

3. How much will a sidereal clock gain on a mean solar clock in 10 hours and 30 minutes?

Ans. 1 min. 43.5 sec.

4. How many times will the seconds hand of a sidereal clock overtake that of a solar clock in a solar day if they start together?

Ans. 236 times.

5. At what intervals do the coincidences occur?

Ans. 6 min. 5.242 sec.

6. In determining longitudes by telegraph will it or will it not make a difference whether sidereal or solar clocks are used by the observers?

7. A ship leaving San Francisco on Tuesday, October 12, reaches Yokohama after a passage of exactly sixteen days. On what day of the month and of the week does she arrive?

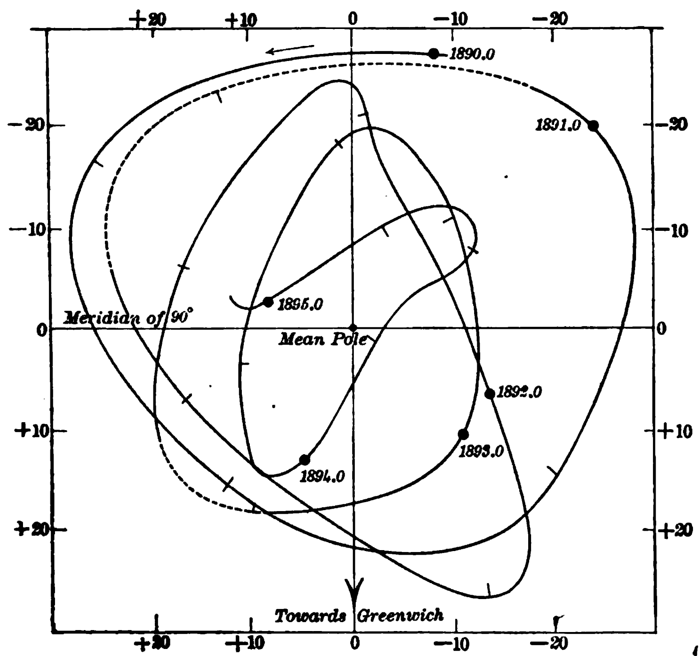

FIG. 40.* — Path of the Earth's Pole from 1890 to 1895. (Albrecht.)

In this figure the scale is hundredths of a second of arc, each of which is very approximately one foot. The zero at the bottom indicates the

direction of Greenwich from the "mean pole," and the zero at the left hand (nearly) that of Chicago and New Orleans. The position of the pole is marked for each third month, the dotted portions of the curve indicating times during which no actual observations were available.

It is to be borne in mind that although the curve is based on observations at more than a dozen different stations, yet the possible error of the plotted result for the place of the pole at a given moment may easily be four or five feet in error, and the absolute correctness of the curve must not be too implicitly accepted.

CHAPTER V.

THE EARTH AS AN ASTRONOMICAL BODY.

APPROXIMATE DIMENSIONS. — PROOFS OF ITS ROTATION. — ACCURATE DETERMINATION OF ITS FORM AND SIZE BY GEODETIC OPERATIONS AND PENDULUM OBSERVATIONS. — ASTRONOMICAL, GEODETIC AND GEOCENTRIC LATITUDE. — DETERMINATION OF THE EARTH'S MASS AND DENSITY.

132. Having discussed the methods of making astronomical observations, we are now prepared to consider the earth in its astronomical relations; *i.e.*, those facts relating to the earth which are ascertained by astronomical methods, and are similar to the facts which we shall have to consider in the case of the other planets. The facts are broadly these: —

1. *The earth is a great ball, about* 7918 *miles in diameter.*
2. *It rotates on its axis once in twenty-four sidereal hours.*
3. *It is flattened at the poles,* the polar diameter being nearly *twenty-seven miles,* or one *two hundred and ninety-fifth part* less than the equatorial.
4. *It has a mean density between* 5.5 *and* 5.6 *as great as that of water, and a mass represented in tons by six with twenty-one ciphers after it (or six sextillions of tons, according to the French numeration).*
5. *It is flying through space in its orbital motion around the sun, with a velocity of about nineteen miles a second;* i.e., *about seventy-five times as swiftly as any cannon-ball.*

I.

133. The Earth's Approximate Form and Size. — It is not necessary to dwell upon the ordinary proofs of its globularity. We merely mention them. 1. It can be circumnavigated. 2. The appearance of vessels coming in from sea indicates that the surface is everywhere convex. 3. The fact that the sea-horizon, as seen from an eminence, is everywhere depressed to the same extent below the level line, shows that the surface is approximately spherical. 4. The fact that as one goes from the equator toward the north, the elevation of

the pole increases proportionally to the distance from the equator proves the same thing. 5. *The shadow of the earth, as seen upon the moon at the time of a lunar eclipse, is that which only a sphere could cast.*

We may add as to the smoothness and globularity of the earth, that if the earth be represented by an 18-inch globe, the difference between the polar and equatorial diameter would only be about one-sixteenth of an inch, the highest mountains upon the earth's surface would be represented by about one-eightieth of an inch, and the average elevation of the continents would be hardly greater than that of a film of varnish. The earth is really relatively smoother and rounder than most of the balls in a bowling-alley.

134. The best method of ascertaining the size of the earth — in fact the only one of real value — is by measuring arcs of the meridian in order to ascertain *the number of miles or kilometres in one degree*, from which we immediately get the circumference of the earth. This measure involves two distinct operations. One — the measure of the number of miles — is purely *geodetic*; the other — the determination of the number of degrees, minutes, and seconds between the two stations — is purely *astronomical.*

We have to find by *astronomical* observation the angle between two radii drawn from the centre of the earth to the two stations (regarding the earth as spherical); or, what is the same thing, the *angular distance in the sky between their respective zeniths.* The two stations being on the same meridian, all that is necessary is to measure their *latitudes* by any of the methods which have been given in Chapter IV. and take the difference. This will be the angle wanted. If, for instance, the distance between the two stations was found by measurement to be 120 miles, and the difference of latitude was found by astronomical observations to be $1° 44'.2$, we should get 69.27 miles for one degree. Three hundred and sixty times this would be the circumference of the earth, a little less than 25,000 miles, and the diameter would be found by dividing this by π, which would give 7920 miles.

135. Eratosthenes of Alexandria seems to have understood the matter as early as 250 B.C. His two stations were Alexandria and Syene in Upper Egypt. At Syene he observed that at noon of the longest day in summer there was no shadow at the bottom of a well, the sun being then vertically overhead. On the other hand, the gnomon at Alexandria, on the same day, by the length of the shadow, gave him $\frac{1}{50}$ of a circumference, or $7° 12'$ as

the distance of the sun from the zenith at that place, which, therefore, is the difference of latitude between Alexandria and Syene.

The weak place in his work was in the measurement of the distance between the two places. He states it as 5000 stadia, thus making the circumference of the earth 250,000 stadia; but we do not know the length of his stadium, nor does he give any account of the means by which he measured the distance, if he measured it at all. There seem to have been as many different stadia among the ancient nations as there were kinds of "feet" in Europe at the beginning of this century.

The first really valuable measure of the arc of a meridian was that made by Picard in Northern France in 1671 — the measure which served Newton so well in his verification of the idea of gravitation.

136. An approximate measure of the diameter is easily obtained. Erect upon a level plain three rods in line, a mile apart, and cut off their tops at the same level, carefully determined with a surveyor's levelling instrument. It will then be found that the line AC, Fig. 44, joining the extremities of the two terminal rods, *when corrected for refraction*, passes about eight inches below B, the top of the middle rod.

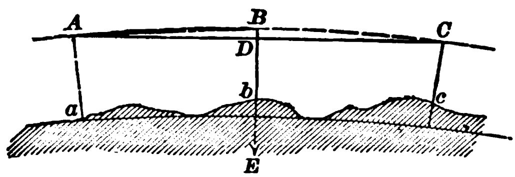

FIG. 44. — Curvature of the Earth's Surface.

Suppose the circle ABC completed, and that E is the point on the circumference opposite B, so that BE equals the diameter of the earth ($= 2R$).

By geometry, $BD : BA = BA : BE$,

whence $$BE = \frac{BA^2}{BD}, \text{ or } R = \frac{BA^2}{2BD}.$$

Now BA is one mile, and $BD = \frac{2}{3}$ of a foot, or $\frac{1}{7920}$ of a mile.

Hence $2R = \dfrac{1^2}{\frac{1}{7920}}$, or 7920 miles: a very fair approximation.

On account of refraction, however, the result cannot be made *exact* by any care in observation. The *observed* value of BD (uncorrected) ranges from 4.5 inches to 6.5, according to the state of the weather.

II.

137. The Rotation of the Earth. — At the time of Copernicus the only argument in favor of the earth's rotation[1] was that the hypoth-

[1] The word *rotate* denotes a spinning motion like that of a wheel on its axis. The word *revolve* is more general in its application, and may be applied either to

esis was *more probable* than that the heavens themselves revolved. All phenomena *then known* would be sensibly the same on either supposition. A little later, analogy could be adduced, for when the telescope was invented, we could *see* that the sun, moon, and several of the planets are rotating globes.

At present we are able to adduce experimental proofs which absolutely demonstrate the earth's rotation, and some of them even make it visible.

138. 1. *The Eastward Deviation of Bodies falling from a Great Height.*—The idea that such a deviation ought to occur was first suggested by Newton. Evidently, since the top of a tower, situated anywhere but at the pole of the earth, describes every day a larger circle than its base, it must move faster. A body which is dropped from the top, retaining its excess of eastward motion as it descends, must therefore strike *to the east* of the point which is vertically under its starting-point, provided it is not deflected in its fall by the resistance of the air or by air-currents. Fig. 45 illustrates the principle. A body starting from A, the top of the tower, reaches the earth at D (BD being equal very approximately to AA'), while during its fall the *bottom* of the tower has only moved from B to B'. The experiments are delicate, since the deviation is very small, and it is not easy to avoid the effect of air-currents. It is also extremely difficult to get balls so perfectly spherical that they will not sheer off to one side or the other in falling.

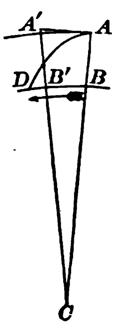

FIG. 45.
Eastward Deviation of a Falling Body.

The best experiments of this kind so far have been those of Benzenberg, performed at Hamburg in 1802, and those of Reich, performed in 1831, in an abandoned mine shaft near Freiberg, in Saxony. The latter obtained a free fall of 520 feet, and from the mean of 106 trials, the eastern deviation observed was 1.12 inches, while theory would make it 1.08. The experiment also gave a southern deviation of 0.17 of an inch, unexplained by theory. It seems to indicate the probable error of observation. The balls in falling sometimes deviated two or three inches one side or the other from the average.

describe such a spinning motion, or (and this is the more usual use in astronomy) to describe the motion of one body around another, as that of the earth around the sun.

The formula given by Worms in his treatise on "The Earth and its Mechanism," is

$$x = \frac{4\pi t(H - \frac{1}{2}\Delta)\cos\phi}{3T};$$

where x is the deviation, t is the number of seconds occupied in falling, T the number of seconds in a sidereal day, H the height fallen through, and Δ the difference between H and the height through which a body would fall in t seconds if there were no resistance (so that $\Delta = \frac{1}{2}gt^2 - H$). Finally, ϕ is the latitude of the place of observation. In latitude 45° a fall of 576 feet should give, neglecting the resistance of the air, a deviation of 1.47 inches. The resistance would increase it a little.

It will be noted that *at the pole*, where the cosine of the latitude equals zero, *the experiment fails*. The largest deviation is obtained at the equator.

139. 2. *Foucault's Pendulum Experiment.* — In 1851 Foucault, that most ingenious of French physicists, devised and first executed an experiment which actually shows the earth's rotation to the eye. From the dome of the Pantheon in Paris he suspended a heavy iron ball about a foot in diameter by a wire more than 200 feet long (Fig. 46). A circular rail some twelve feet across, with a little ridge of sand built upon it, was placed under the pendulum in such a way that a pin attached to the swinging ball would just scrape the sand and leave a mark at each vibration. The ball was drawn aside by a cotton cord and allowed to come absolutely to rest; then the cord was burned, and the pendulum set to swinging in a true plane; but this plane seemed to *deviate slowly towards the right*, cutting the sand in a new place at each swing and shifting at a rate which would carry it completely around in about thirty-two hours if the pendulum did not first come to rest. In fact, the floor of the Pantheon was seen turning under the plane of the pendulum's vibration. The experiment created

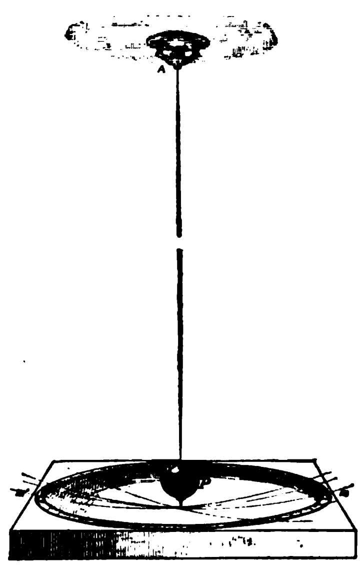

FIG. 46. — Foucault's Pendulum Experiment.

great enthusiasm at the time, and has since been very frequently performed, and always with substantially the same results.

140. The approximate theory of the experiment is very simple. Such a pendulum, consisting of a round ball hung by a *round* wire or else suspended on a *point, so as to be equally free to swing in any plane* (unlike the common clock pendulum in this freedom), being set up at the pole of the earth, would appear to shift around in twenty-four hours. Really, the plane of vibration remains invariable and the earth turns under it, the plane of vibration in this case being unaffected by the motion of the earth. This can be easily shown by setting up a similar apparatus, consisting of a ball hung by a thread, upon a table, and then turning the table around with as little jar as possible. The plane of the swing will remain unchanged by the motion of the table.

It is easy to see, further, that at the equator there would be no such tendency to shift. In any other latitude the effect will be intermediate, and the time required for the pendulum to complete the revolution of its plane will be *twenty-four hours divided by the sine of the latitude.* The northern edge of the floor of a room (in the northern hemisphere) is nearer the axis of the earth than its southern edge, and therefore is carried more slowly eastward by the earth's rotation. Hence it must *skew around* continually, like a postage stamp gummed upon a whirling globe anywhere except at the globe's equator. The southern extremity of every north and south line on the floor continually works toward the east faster than the northern extremity, causing the line itself to shift its direction accordingly, compared with the direction it had a few minutes before. A free pendulum, set at first to swing along such a line, must therefore apparently deviate continually at the same rate in the opposite direction. In the *northern* hemisphere its plane moves *dextrorsum; i.e.*, with the hands of a watch: in the *southern*, its motion is *sinistrorsum.*

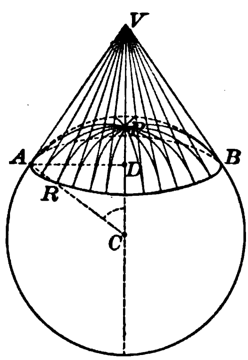

FIG. 47.
Explanation of the Foucault Pendulum Experiment.

141. Suppose a parallel of latitude drawn through the place in question, and a series of tangent lines drawn toward the north at points an inch or so apart on this parallel. All these tangents would meet at some point, *V*, Fig. 47, which is on the earth's axis produced; and taken together these tangents

would form a cone with its point at V. Now if we suppose this cone cut down upon one side and opened up (technically, "*developed*"), it would give us a sector of a circle, as in Fig. 48, and the angle V, reckoned around from A to A' through B, is the sum total of all the angles between all the adjacent meridian tangents touching the earth on that parallel (— a reentrant angle, greater than 180°, in the figure). Now, *first*, the circumference of the parallel (Fig. 47), or the arc ABA', which measures the angle V in Fig. 48, equals $2\pi \times AD$; and, since the angle DAC (Fig. 47) equals the latitude, $AD = R \cos\phi$ (ϕ being the latitude). Hence $ABA' = 2\pi R \times \cos\phi$. *Second*, the radius of the sector in Fig. 48 is the same as AV, the side of the cone in Fig. 47; and since in Fig. 47 the angle $AVD = \phi$, we have $AV = R \cot\phi$, and the circumference, $ABA'm = 2\pi R \cot\phi$.

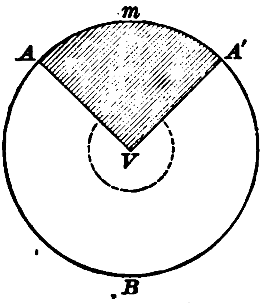

FIG. 48. — Developed Cone.

Hence, *finally*, $\dfrac{V}{360°} = \dfrac{ABA'}{ABA'm} = \dfrac{2\pi R \cos\phi}{2\pi R \cot\phi} = \sin\phi$, and $V = 360° \sin\phi$, *i.e.*, the total angle described by the plane of the pendulum in a day $= 360° \times \sin$ of the latitude.

At the pole the cone produced by the tangent lines becomes a little "button," a complete circle. At the equator it becomes a *cylinder*, and the angle is zero.

It is worth noting that the azimuthal motion of any star at the horizon in a minute of time is $15' \times \sin\phi$, — *the same at all parts of the horizon.* See Appendix, Art. 1001.

In order to make the experiment successfully, many precautions must be taken. It is specially important that the pendulum should vibrate in a true plane, without any lateral motion. To secure this end, it must be carefully guarded against all jarring motion and air-currents. To diminish the effect of all such disturbances, which will always occur to a certain extent, the pendulum should be very heavy and very long, and of course the suspended ball must be truly round and smooth. Ordinary clock-work cannot be used to keep the pendulum in vibration, since it must be free to swing in every plane. Usually, the apparatus once started is left to itself until the vibrations cease of their own accord; but Foucault contrived a most ingenious electrical apparatus, which we have not space to describe, by means of which the vibration could be kept up for days at a time without receiving any hurtful disturbance whatever.

It will be noticed that this experiment is most effective precisely where the experiment of falling bodies fails. This is best near the pole, the other at the equator.

142. 3. *By the Gyroscope*, an experiment also due to Foucault, and proposed and executed soon after the pendulum experiment.

The instrument shown in Fig. 49 consists of a wheel so mounted in gimbals that it is free to turn in every direction, and so delicately balanced that it will stay in any position if undisturbed. If the wheel be set to rotating rapidly, *it will maintain the direction of its axis invariable, unless acted upon by extraneous force.* If, then, we set the axis horizontal and arrange a microscope to watch a mark upon one of the gimbals, it will appear slowly to shift its position as the earth revolves, in the same way as the plane of the pendulum behaves.

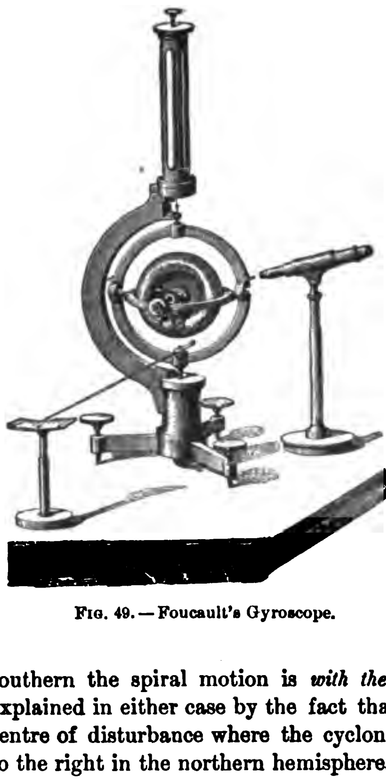

FIG. 49.—Foucault's Gyroscope.

143. 4. There are many other phenomena which depend upon and really demonstrate the earth's rotation. We merely mention them:—

a. The Deviation of Projectiles. In the northern hemisphere a projectile always deviates towards the right; in the southern hemisphere toward the left.

b. The Trade Winds.

c. The Vorticose Revolution of the Wind in Cyclones. In the northern hemisphere the wind in a cyclone moves spirally towards the centre of the storm, whirling *counter clock-wise,* while in the southern the spiral motion is *with the hands of a watch.* The motion is explained in either case by the fact that currents of air, setting out for the centre of disturbance where the cyclone is formed, deviate like projectiles, to the right in the northern hemisphere, and towards the left in the southern hemisphere, so that they do not meet squarely in the centre of disturbance.

d. The Ordinary Law of Wind-change; that is, in the northern hemisphere the north wind, under ordinary circumstances, changes to a northeast, a northeast wind to an east, east to southeast, etc. When the wind changes in the opposite direction, it is said to "*back*" around. In the southern hemisphere it of course *usually* backs around, much to the disconcertment of the early Australian settlers.

It might seem at first that the rotation of the earth, which occupies twenty-four hours, is not a very rapid motion. A point on the equa-

tor, however, has to move nearly one thousand miles an hour, which is about fifteen hundred feet per second, and very nearly the speed of a cannon-ball.

144. Invariability of the Earth's Rotation. — It is a question of great importance whether the day changes its length. Theoretically it must almost necessarily do so. The friction of the tides and the deposits of meteoric matter upon the earth both tend to lengthen it; while on the other hand, the earth's loss of heat by radiation and consequent shrinkage must tend to shorten it. Then geological changes, the elevation and subsidence of continents, and the transportation of matter by rivers, act, some one way, some the other. At present it can only be said that the change, if any has occurred since astronomy became accurate, has been too small to be detected. The day is certainly not longer or shorter by $\frac{1}{100}$ of a second than in the days of Ptolemy, and *probably* has not changed by $\frac{1}{1000}$ of a second. The criterion is found in comparing the *times* at which celestial phenomena, such as eclipses, transits of Mercury, etc., occur. For changes in the *position of the axis*, see Art. 108.

III.

145. The Earth's Form, more accurately stated, *is that of a spheroid of revolution, having an equatorial radius of* 6,377,377 *metres, and a polar radius of* 6,355,270 *metres, according to Listing* (1873); *or of* 6,378,206.4 *and* 6,356,583.8 respectively, *according to Clarke.*[1] It must be understood, also, that this statement is only a second *approximation* (the first being that the earth is a globe). Owing to mountains and valleys, etc., the earth's surface does not strictly correspond to that of any geometrical solid whatever.

The flattening at the poles is the necessary consequence of the earth's rotation, and might have been cited in the preceding section as proving it.

146. There are three ways of determining the form of the earth: one, by *measurement of distances upon its surface in connection with the latitudes and longitudes of the points of observation.* This gives not only the *form*, but the *dimensions.* The second method is by the observation of the *varying force of gravity at various points,* — observations which are made by means of a pendulum apparatus of some

[1] This is Clarke's spheroid of 1866, and is adopted by the United States Coast and Geodetic Survey. See Appendix, page 601, for his spheroid of 1878.

kind, and determine *only the form*, but not the size of the earth. The third method is by means of certain purely astronomical phenomena, known as "precession" and "nutation" (to be treated of hereafter), and by certain irregularities in the motion of the moon. Observations of the occultations of stars at widely distant stations can also be utilized for the same purpose. All the methods of this third class, like the pendulum method, give only the *form* of the earth.

147. 1. *Measurements of Arcs of Meridian in Different Latitudes.* — To determine the size of the earth regarded as a sphere, a *single* arc of meridian in any latitude is sufficient. Assuming, however, that the earth is not a sphere, but a spheroid with elliptical meridians, we must measure at least *two* such arcs, one of which should be near the equator, the other near the pole.

The *astronomical work* consists simply in finding with the greatest possible accuracy the *difference of latitude* between the terminal stations of the meridian arc. The *geodetic work* consists in measuring their *distance* from each other in miles, feet, or metres, and it is this part of the work which consumes the most time and labor. The process is generally that known as triangulation.

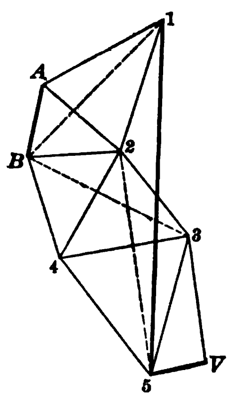

FIG. 50. — A Triangulation.

Two stations are selected for the extremities of a *base line* six or seven miles long, and the ground between them is levelled as if for a railroad. The distance between these stations (*A* and *B* in Fig. 50) is then carefully measured by an apparatus especially designed for the purpose and with an error not to exceed half an inch or so in the whole distance. A third station, 1, is then chosen, so situated that it will be visible from both *A* and *B*, and all the angles of the triangle *AB* 1 are measured with great care by a theodolite. A fourth station, 2, is then selected, such that it will be visible from *A* and 1 (and if possible from *B* also), and the angles of the triangle *A* 1 2 are measured in the same way. In this manner the whole ground between the two terminal stations is covered with a network of triangulation, the two terminal stations themselves being made two of the triangulation points. Knowing *one distance and all the angles* in this system, it is possible to compute with great accuracy the exact length of the line 1 5 and its direction.

The sides of the triangles are usually from twenty-five to thirty miles in length, though in a mountainous country not infrequently much longer ones are available. Generally speaking, the fewer the

stations necessary to connect the extremities of the arc, and the longer the lines, the greater will be the ultimate accuracy. In this way it is possible to measure distances of 200 or 300 miles with a probable error not exceeding two or three feet.

Many arcs of meridians have been measured in this way, — not less than twenty or thirty in different parts of the earth, the most extensive being the so-called Anglo-French arc, extending more than twelve degrees in length; the Indian arc, nearly eighteen degrees long; and the great Russo-Scandinavian arc, more than twenty-five degrees in length, and reaching from Hammerfest to the mouth of the Danube. One short arc has been measured in South America and one in South Africa.

In a general way, it appears that the higher the latitude the longer the arc. Thus, near the equator the length of a degree has been found to be 362,800 feet in round numbers, while in northern Sweden, in latitude 66°, it is 365,800 feet; in other words, the earth's surface is *flatter near the poles.* It is necessary to travel 3000 feet further in Sweden than in India to increase the latitude one degree, as measured by the elevation of the celestial pole.

The following little table gives the length of a degree of the meridian at different latitudes: —

At the equator			one	degree	=	68.704	miles.
At	latitude	20°	"	"	=	68.786	"
"	"	40°	"	"	=	68.993	"
"	"	60°	"	"	=	69.230	"
"	"	80°	"	"	=	69.386	"
"	"	90°	"	"	=	69.407	"

The difference between the equatorial and polar degree of latitude is more than seven-tenths of a mile, or over 3500 feet, while the probable error of measurement cannot exceed more than a foot or two to the degree.

It will be understood, of course, that the length of a degree at the pole is obtained by *extrapolation* from the measures made in lower latitudes.

148. The deduction of the exact form of the earth from such measurements is an abstruse problem. Owing to errors of observation and local deviations in the direction of gravity, the different arcs do not give strictly accordant results, and the best that can be done is to find the result *which most nearly satisfies all the observations.*

If we assume that the form is that of an *exact spheroid of revolution,* with all the meridians true ellipses and all exactly alike, the problem is simplified somewhat, though still too complicated for discussion

here. Theory indicates that the form of a revolving mass, fluid enough to yield to the forces acting in such a case, *might, and probably would, be such a spheroid;* but other forms are also theoretically possible, and some of the measurements rather indicate that the equator of the earth is not a true circle, but an oval flattened by nearly half a mile. On the whole, however, astronomers are disposed to take the ground that since no regular geometrical solid whatsoever can *absolutely* represent the form of the earth, we may as well assume a regular spheroid for the standard surface, and consider all variations from it as local phenomena, like hills and valleys.

149. Each measurement of a degree of latitude gives the "*radius of curvature,*" as it is called, of the meridian at the degree measured. The length of a degree from 44° 30′ to 45° 30′, multiplied by 57.29 (the number of degrees in a radian), gives the radius of the "*osculatory circle,*" which would just fit the curve of the meridian at that point. Having a table giving the actual length of each degree of latitude, we could construct the earth's meridian graphically as follows: —

Draw the line AX, Fig. 51. On it lay off Aa, equal to the radius of curvature of the first measured degree (that is, 57.3 times the length of the degree), and with a as centre, describe an arc AB, making the angle AaB just one degree. Next produce the line Ba to b, making Bb the radius of curvature of the second degree, and draw this second degree-arc; and so proceed until the whole ninety have been drawn. This will give one-quarter of the meridian, and of course the three other quarters are all just like it. a, b, c, etc., are called the "centres of curvature" of the different degrees.

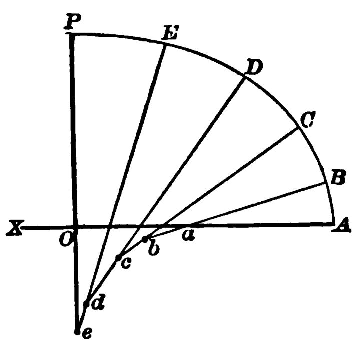

FIG. 51.
Radii of Curvature of the Meridian.

If we assume the curve to be an ellipse, then the equatorial semidiameter, AO, and the polar, PO, are given respectively by the two formulas, $AO = \sqrt[3]{qp^2}$ and $PO = \sqrt[3]{q^2p}$, q and p being the radii of curvature (Aa and Pe in the figure) at the equator and pole.

150. The "*ellipticity*" or "*oblateness*" of an ellipse is the fraction found by dividing the difference of the polar and equatorial diameters by the equatorial, and is expressed by the equation

$$d = \frac{A - B}{A}.$$

In the case of the earth this is $\frac{1}{295}$, according to Clarke's spheroid, of 1866. Until within the last few years Bessel's smaller value, viz., $\frac{1}{299}$, was generally adopted. Listing's larger value, $\frac{1}{288}$, is now preferred by some.

The *ellipticity* of an ellipse must not be confounded with its *eccentricity*. The latter is

$$e = \frac{\sqrt{A^2 - B^2}}{A},$$

and is always a much larger numerical quantity than the ellipticity. In the case of the earth's meridian, it is $\frac{1}{12.1}$ as against $\frac{1}{295}$. Its symbol is usually e.

151. *Arcs of longitude* are also available for determining the earth's form and size. On a spherical earth a degree of longitude measured along any parallel of latitude would be equal to one degree of the equator multiplied by the cosine of the latitude. On an oblate or orange-shaped spheroid (the surface of which lies wholly within the sphere having the same equator) the degrees of longitude are evidently everywhere shorter than on the sphere, the difference being greatest at a latitude of 45°.

In fact, *arcs in any direction between stations of which both the latitude and longitude are known* can be utilized for the purpose; and thus the extensive surveys that have been made in different countries have given us a pretty accurate knowledge of the earth's dimensions. It is very desirable that in some way the chain of actual measurements should be extended from the eastern continent to the western, but the immense difficulties of so doing are obvious.

At present the distance from a point on the earth's surface (say the observatory at Washington) to any other point in the opposite hemisphere (say the observatory at the Cape of Good Hope) is uncertain to perhaps the extent of a quarter of a mile.

152. 2. *Pendulum Experiments.* — Since (Physics, p. 75),

$$t = \pi\sqrt{\frac{l}{g}}, \text{ therefore, } g = \frac{\pi^2 l}{t^2};$$

we can therefore measure the variations of the force of gravity, g, at different parts of the earth, either by taking a pendulum of invariable length and determining t, the time of its vibration; or by measuring the length, l, of a pendulum which will vibrate seconds. Extensive surveys of this sort have been made, and are still in progress; and it is found that the *force of gravity at the pole exceeds that at the equator by about $\frac{1}{190}$ part.* In other words, a person who

weighs 190 pounds at the equator (*by a spring balance*) would, if carried to the pole, show 191 pounds by the same balance.

The apparatus most used at present for the purpose of measuring the force of gravity is a modification of the so-called Kater's pendulum. The pendulum itself now usually employed, as constructed by Repsold, consists of a brass tube about three inches in diameter and about four feet long: the two ends are alike in form, but one end is weighted and the other is light. Two parallel knife edges are inserted through the rod at right angles, one near the heavy end and the other at just the same distance from the lighter one, and the weights and dimensions of the apparatus are so adjusted that the *time of vibration will be very approximately the same whether the pendulum is swung heavy end up or light end up, and will be not far from one second.* The distance between the knife edges will then, according to the theory of the pendulum, be very nearly equal to the length of a simple pendulum vibrating in the same time; and the small difference can be accurately calculated when we know the exact time of vibration, each end up. The knife edges swing on agate planes which are fastened upon a firm support; and great pains must be taken to have the support really firm. Professor Peirce of our Coast Survey a few years ago detected important errors in a majority of the earlier pendulum observations, due to insufficient care in this respect.

152*. In 1891 Professor Mendenhall, then superintendent of the United States Coast Survey, greatly improved the apparatus by substituting for the seconds pendulum a half-seconds one, and enclosing it in a tight case exhausted of air. This renders the instrument much more manageable and portable, and avoids almost entirely the troublesome and uncertain correction for the resistance of the air. Two little mirrors, one attached to the pendulum itself and the other fixed near it in the case, give the means of observing the pendulum swing by watching the reflection of a flash produced electrically every second by the clock or chronometer which furnishes the time. With this apparatus the determinations, however, are merely *relative*, the pendulum being used simply as "invariable," without inversion. A somewhat similar, but less elaborate arrangement, with a half-seconds pendulum, was still earlier introduced in Europe by Von Sterneck.

153. The observations consist in comparing the pendulum with a clock, either by noting the "*coincidences*," or by an electrical record automatically made on a chronograph. The observations need to be carefully corrected for *temperature* (which, of course, affects the distance between the knife edges), for the *length of arc* through which the pendulum is swinging, and for the *resistance of the air*. The observations determine the "*force of gravity*" (French "*pesanteur*") at the station. This "force of gravity," however, thus determined, is not simply the earth's *attraction*, but includes also the effects of the centrifugal force, due to the earth's rotation, which we must consider and allow for.

EFFECT OF CENTRIFUGAL FORCE DUE TO EARTH'S ROTATION.

154. At the equator the centrifugal force acts vertically in direct opposition to gravity, and is given by the well-known formula

$$C = \frac{V^2}{R}$$

(see Physics, p. 17), in which V is the velocity of the earth's surface at the equator, and R the earth's radius. Since V is equal to the earth's circumference divided by the number of seconds in a sidereal day, we have

$$V = \frac{2\pi R}{t}, \text{ and } C = \frac{4\pi^2 R}{t^2}.$$

Now R, the radius of the earth, equals 20,926,000 feet; and t equals 86,164 mean-time seconds. C, therefore, comes out 0.111 feet, which is $\frac{1}{289}$ of g, g being $32\frac{1}{6}$ feet.

We may remark in passing that if the rate of rotation were seventeen times as great, C would be 17^2, or 289 times greater than now, and would equal gravity; so that on that supposition bodies at the equator would weigh absolutely nothing, and any greater velocity of rotation would send them flying.

At any other latitude, since $MN = OQ \cos MOQ$,[1] the centrifugal force, c, equals $C \cos \phi$, acting at right angles to the axis of the earth and parallel to the plane of the equator. Now, this centrifugal force c is not *wholly* effective in diminishing the weight of a body, but only that portion of c (MR in Fig. 52) which is directed vertically. c is MT in the figure, and MR is equal to c multiplied by the cosine of ϕ, which finally gives us $C \times \cos^2 \phi$ for the amount by which the centrifugal force diminishes gravity at a station whose latitude is ϕ.

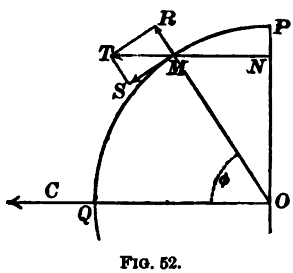

FIG. 52.
The Earth's Centrifugal Force.

Every determination, therefore, of the "force of gravity," obtained by the pendulum, needs to be increased by the quantity

$$\frac{g}{289} \times \cos^2 \phi,$$

[1] This is not *exact*, since MN in an oblate spheroid is less than $OQ \times \cos MOQ$; but the difference is unimportant in the case of the earth.

in order to get the real value of the earth's *gravitational attraction* at the point of observation.

The other component of c (viz. MS) acts at right angles to gravity and parallel to the earth's surface, and is given by the formula

$$C \cos \phi \sin \phi = \tfrac{1}{2} C \sin 2\phi.$$

The direction of still water is determined by the resultant of the earth's attraction combined with this deflecting force acting towards the equator; so that this surface is not perpendicular to a line drawn towards the centre of the earth anywhere excepting at the equator and the poles.

155. Having a series of pendulum observations, we can then form a table showing the force of gravity at each station; and correcting this by adding the amount of the centrifugal force at each place, we shall have the force of the earth's attraction. This is greater the nearer each station is to the centre of the earth; but unfortunately there is no simple relation connecting the force with the distance. The attraction depends not only on the distance from the centre of the earth, but also upon the form of the earth and the constitution of its interior, and the arrangement of its strata of different density. We may safely assume, however, that the earth is made up *concentrically*, so to speak; the strata of equal density being arranged like the coats of an onion. On this hypothesis Clairaut, in 1742, demonstrated the relation given below, which is always referred to as Clairaut's equation.

Let w be the loss of weight between the equator and the pole, and C the centrifugal force at the planet's equator, both being expressed as fractions of the equatorial force of gravity, and let d be the ellipticity of the planet.

Then, as Clairaut proved,

$$d + w = 2\tfrac{1}{2} \times C;$$

whence

$$d = 2\tfrac{1}{2}C - w.$$

In the case of the earth,

$$C = \frac{1}{289}, \text{ and } w = \frac{1}{190},$$

whence

$$d = 2\tfrac{1}{2} \times \frac{1}{289} - \frac{1}{190},$$

which gives

$$d = \frac{1}{292.8}.$$

But the different results obtained from pendulum observations range all the way from $\frac{1}{282}$ to $\frac{1}{299}$.

155*. As regards the purely astronomical methods, the one which depends on *precession* and *nutation* requires assumptions respecting the distribution of matter within the earth which render the result somewhat uncertain. Harkness deduces by it a value of $\frac{1}{297}$.

The *lunar perturbation* from which the oblateness of the earth can be calculated is very small (only about 8″), and hardly well enough determined as yet. According to Harkness the values obtained from it range between $\frac{1}{295}$ and $\frac{1}{311}$.

The observations of star-occultations during lunar eclipses are not yet sufficiently numerous to furnish a reliable value.

Considering all the data it can only be said that the oblateness probably lies between $\frac{1}{290}$ and $\frac{1}{300}$, and probably nearer the latter limit than the other. Harkness, in his "adjusted" system of astronomical constants, gives as his final result $\frac{1}{300.2 \pm 3.0}$.

156. Astronomical, Geographical, and Geocentric Latitudes. — The astronomical latitude of a place has been defined as *the elevation of the pole*, or, what comes to the same thing, it is *the angle between the plane of the equator and the direction of gravity* at that place, however that direction may be affected by local causes.

The *geocentric* latitude, on the other hand, is the angle made at the centre of the earth (as the word implies) between the plane of the equator and a line drawn from the observer to the centre of the earth, which line of course does not coincide with the direction of gravity, since the earth rotates, and is not spherical.

The *geographical* or *geodetic* latitude of a station is the angle formed with the plane of the equator by a line drawn from the station *perpendicular to the surface of the standard spheroid.*

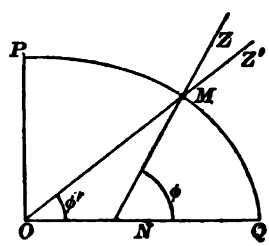

FIG. 53.
Astronomical and Geocentric Latitude.

If the earth's surface were *strictly spheroidal, and there were no local variations of gravity,* the astronomical latitude and the geographical latitude would coincide — and they never differ greatly; but the geocentric latitude differs from them by a very considerable quantity — as much as 11′ in latitude 45°. The geocentric latitude is but little used except in certain astronomical calculations where parallax is involved.

In Fig. 53, the angle MOQ is the geocentric latitude of M, while MNQ is the geographical latitude. MNQ is also the astronomical latitude, unless there is some local disturbance of the direction of gravity. The angle OMN, which is the difference between the geocentric and astronomical latitudes, is called "*the angle of the vertical.*"

157. It will be noticed that the astronomical latitude of a place is the only one of these three latitudes which is *determined directly by observation.* In order to know the *geocentric* and *geographical* latitudes of a place, we must know the form and dimensions of the earth, which are ascertained only by the help of observations made elsewhere.

The geocentric degrees are longer near the equator than near the poles, and it is worth noticing that if we form a table giving the length of each degree of *geographical* latitude from the equator to the pole, the same table, read *backwards*, gives the length of *geocentric* degrees.

Since the earth is ellipsoidal instead of spherical, it is evident that lines of "level" on the earth's surface are affected by the earth's rotation. If this rotation were to cease, the direction of gravity would be so much changed that the Gulf of Mexico would run up the Mississippi River, because the distance from the centre of the earth to the head of the river is less by some thousands of feet than the distance from the mouth of the river to the centre of the earth.

158. Station Errors. — The irregularities in the direction of gravity are by no means insensible as compared with the accuracy of modern astronomical observation, and the difference between the astronomical latitude and longitude of a place and the geographical latitude and longitude of the same place constitute what is called the "*station error.*" In the eastern part of the United States these station errors, according to the Coast Survey observations, average about $1\frac{1}{2}''$. Errors of from $4''$ to $6''$ are not uncommon, and in mountainous countries, as for instance in the Caucasus and in Northern India, these errors occasionally amount to $30''$ or $40''$. They are not "errors" in the sense that the astronomical latitude of the place has not been determined correctly, but are merely the effects of the irregular distribution of matter in the crust of the earth in altering the direction of gravity. Pendulum observations show local variations in the *force* of gravity quite proportional to the deviations which the station-errors show in its *direction.*

IV.

159. The Earth's Mass and Density. — The '*mass*' of a body is the *quantity of matter* that it contains, the unit of mass being the quantity of matter contained in a certain arbitrary body which is taken as a standard. For instance, a "kilogram" is the quantity of matter

contained in the block of platinum preserved at Paris as the standard of mass. A pound is similarly defined by reference to the prototypes at Washington and London.

Two masses of matter are defined as equal which *require the same expenditure of energy to give them the same velocity;* or, *vice versa, those are equal which, when they have the same velocity, possess the same energy, and, in giving up their motion and coming to rest, do the same amount of work* (*i.e., they have the same "inertia"*).

Masses can therefore be compared by subjecting them to the action of some given force (stress), and comparing the *energies* developed in them when they have moved equal *distances*, or the *velocities* attained at the end of a given *time*.

160. Proportionality of Mass to Weight. — Newton showed by his experiments with pendulums of different substances, that at any given point the attraction of the earth for a body of any kind of matter is proportional to the mass of that body; the attraction being measured as a pull or "stress" in this case, and called "*the weight*" of the body. In other and more common language, *the mass of a body is proportional to its weight* (we must not say it *is* its weight), provided the weighing of the bodies thus compared is done, in cases where scientific accuracy is essential, at the same place on the earth's surface. Practically, therefore, we *usually measure the masses of bodies by simply weighing them.*[1] It is to be carefully observed, however, that the words "kilogram," "pound," "ton," etc., have also a secondary meaning, as denoting units of pull and push, — of "*stress,*" speaking strictly and technically, — or of "force," as that much abused word is very generally used.

It is, from a literary point of view, just as proper to speak of a *stress or a pull* of a hundred pounds[2] as of a *mass* of a hundred pounds, but the word "pound" means an entirely different thing in the two cases. At the surface of the earth the relation between the ideas, however, is so close that the way in which the ambiguity came about is perfectly obvious, and it is hardly probable that language will ever change so as to remove it. To a certain extent it is admittedly unfortunate, and the student must always be on his guard against it. At the earth's surface a *mass* of 100 pounds always "*weighs*" very nearly 100 pounds; but, to anticipate slightly, at an elevation

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[1] See note at the end of chapter, page 123.

[2] The scientific and unambiguous unit of stress is the *dyne*, which equals the weight at Paris of $\frac{1}{980.94}$ of a gram, — nearly 1.02 milligrams.

of 4000 miles above the surface, the same mass would "weigh" only 25 pounds; at the distance of the moon about half an ounce; while on the surface of the sun it would "weigh" nearly 2800 pounds (of stress).

161. Gravity. — The law of gravitation discovered by Newton declares that *any particle of matter attracts any other particle with a force* ("*stress*," if the bodies are prevented from moving) *proportional inversely to the square of the distance between them, and directly to the product of their masses;* or, as a formula, we may write,

$$F = G\frac{M_1 \times M_2}{d^2},$$

in which M_1 and M_2 are the two masses, and d the distance between them, while G is a constant numerical factor which depends upon the system of units employed.

It is known as the "Newtonian Constant" or the "Constant of Gravitation," being supposed to maintain the same value throughout the universe. According to the most recent determination, — that of Boys in 1893 (Art. 166), — its value in the C. G. S. system (centimetre-gram-second) is **666×10^{-10} dynes**; *i.e.*, two balls, each having a mass of one gram, and with their centres one centimetre apart, would attract each other with a force (*stress*) of 666 ten-thousand-millionths of a dyne.

The "*acceleration*" of a particle due to the attraction of a mass, M, at distance, d, is given by the equation, $f = G' \frac{M}{d^2}$; and when two masses, M_1 and M_2 (which are free to move) attract each other their *relative* acceleration is the sum of the two accelerations which each produces in the other. It is therefore given by the formula, $f = G' \frac{M_1 + M_2}{d^2}$. (Note that in this we have the *sum* of the masses, instead of their *product*.) In the C. G. S. system G' is numerically identical with G, f being measured in cm. per sec.

We must not imagine the word "*attract*" to mean too much. It merely states the fact that there is a *tendency* for the bodies to move toward each other, without including or implying any explanation of the fact. So far, no explanation has appeared which is less difficult to comprehend than the fact itself.

162. When the distance between attracting bodies is large as compared with their own magnitude, then, reckoning the distance between their centres of mass as their true distance, the formula is sensibly true for them as it would be for mere particles. When, however, the distance is not thus great, the calculation of the attraction becomes a very serious problem, involving what is known as a "double integra-

tion." We must find the attraction of each particle of the first body upon each particle of the other body, and take the sum of all these infinitesimal stresses. Newton, however, showed that if the *bodies are spheres, either homogeneous or of concentric structure, then they attract and are attracted precisely as if the matter in them were wholly collected at their centres.* The earth, for instance, attracts a body at its surface very nearly as if it were all collected at its own centre, 4000 miles distant; not exactly so, because the earth is not strictly spherical; but in what follows we shall neglect this slight inaccuracy.

163. In order, then, to find the mass of the earth in kilograms, pounds, or tons, we must find some means of accurately comparing its attraction for some object on its own surface with the attraction of the same object by some body of known mass, at a measured distance. The difficulty lies in the fact that the attraction produced by any body, not too large to be handled conveniently, is so excessively small that only the most delicate operations serve to detect and measure it.

The first successful attack upon the problem was made in 1774 by Maskelyne, the Astronomer Royal, by means of what is now usually referred to as: —

164. 1. "THE MOUNTAIN METHOD," because, in fact, the earth in this operation is weighed against a mountain.

Two stations were chosen on the same meridian, one north and one south of the mountain Scheballien, in Scotland. In the first place, a careful topographical survey was made of the whole region, giving the precise distance between the stations, as well as the exact dimensions of the mountain, which is a "hog-back" of very regular contour. From the known dimensions of the earth and the measured distance, the difference of the *geographical* latitudes of the two places M and N (Fig. 54) can be accurately computed; *i.e.*, the angle which the plumb lines at M and N would have made if there were no mountain there. In this case it was 41″. The next operation was to observe the *astronomical* latitude at each station. This astronomical difference of latitude, *i.e.*, the angle which the plumb lines actually do make, was found to be 53″, the plumb lines at M and N being drawn inward out of their normal position by the attraction of the mountain to the

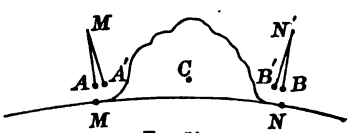

FIG. 54.
The Mountain Method of Determining the Earth's Density.

extent of 6″ on each side; so that the astronomical difference of latitude was increased by 12″ over the geographical.

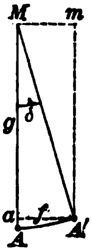

FIG. 55.

Now, in such a case the ratio of gravity to the deflecting force, according to the laws of the composition of forces, is that of aM to aA' in the figure (Fig. 55), or the ratio of 1 to the tangent of the deflection, δ; that is, calling the deflecting force f, we have $\frac{g}{f} = \cot\ \delta, = \cot\ 6''$ in this case.

By the law of gravitation, the earth's attracting force at its surface is given by the formula

$$g = G\frac{E}{R^2},$$

where E is the mass of the earth (the unknown quantity of our problem), and R its radius, 4000 miles. Similarly, if C in the figure is the centre of attraction of the mountain, we have

$$f = G\frac{m}{d^2},$$

m being the mass of the mountain, and d the distance from C to the station. Combining this with the preceding, we get

$$\frac{E}{m} = \left(\frac{g}{f}\right)\left(\frac{R}{d}\right)^2, \text{ or } \frac{E}{m} = \cot\ 6''\left(\frac{R}{d}\right)^2.$$

We thus get the *ratio of the earth's mass to that of the mountain*; and provided we can find the mass of the mountain in tons or any other known unit of mass, the problem will be completely solved. By a careful geological survey of the mountain, with deep borings into its strata, the mass of the mountain was determined as accurately as it could be (though here is the weakest point of the method), and thus the *mass* of the earth was finally computed.

Now, knowing the diameter of the earth, its volume in cubic feet is easily found, and from the volume and the known number of mass-pounds ($62\frac{1}{2}$ nearly) in a cubic foot of water, the weight the earth would have, if composed of water, follows. Comparing this with the mass actually found, we get the density, which in this experiment came out 4.71.

A repetition of the work in 1832 at Arthur's Seat, near Edinburgh, gave 5.32, and several later determinations have since been made by this method, giving results in near agreement with that stated in Art. 132.

165. 2. Much more trustworthy results, however, are obtained by the method of the Torsion Balance, first devised by Michell, but first employed by Cavendish in 1798. A light rod, carrying two small balls at its extremities, is suspended horizontally at its centre by a long fine metallic wire. If it be allowed to come to rest, and then a very slight deflecting force be applied, the rod will be pulled out of position by an amount depending on the stiffness and length of the wire, as well as the force itself. When the deflecting force is removed, the rod will vibrate back and forth until brought to rest by the resistance of the air. The "*torsional coefficient*," as it is called (*i.e.*, the stress corresponding to a torsion of one revolution), can be accurately determined by observing *the time of vibration* when the dimensions and weight of the rod and balls are known. If, now, two large balls A and B are brought near the smaller ones, as in Fig. 56, a deflection will be produced by their attraction, and the small balls will move from a and b to a' and b'. By shifting the large balls to the other side at A' and B', we get an equal deflection in the opposite direction, *i.e.*, to a'' and b'', and the difference between the two positions assumed by the small balls, *i.e.*, $a'a''$ and $b'b''$, will be twice the deflection.

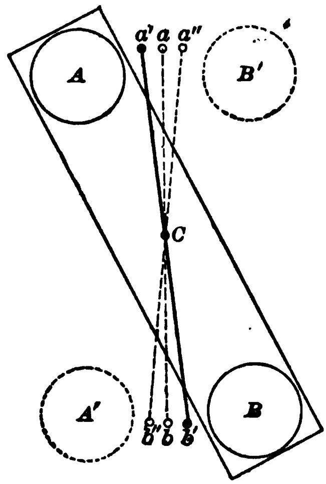

FIG. 56. — Plan of the Torsion Balance.

It is not necessary, nor even best, to wait for the balls to come to rest. We note the extremities of their swing. The middle point of the swing gives the point of rest, and the time occupied by the swing is the time of vibration, which we need in determining the coefficient of torsion. We must also measure accurately the distance, Aa' and Bb', between the centre of each of the large balls and the point of rest of the small ball when deflected.

The *earth's attraction* on each of the small balls of course equals *the ball's weight*. The *attractive force of the large ball* on the small one near it is found directly from the experiment. If the deflection, for instance, is 1° and the coefficient of torsion is such that it takes *one grain* to twist the wire around one whole revolution, then the deflecting force, which we will call f as before, will be $\frac{1}{360}$ of a grain.

Call the mass of the large ball B, and let d be the measured distance from its centre to that of the deflected ball. We shall then have

$$f = G\frac{B}{d^2};^{1}$$

also, w being the weight of the small ball,

$$w = G\frac{E}{R^2},$$

whence we get, very much as in the preceding case,

$$E = B\,\frac{w}{f}\left(\frac{R^2}{d^2}\right).$$

The method differs from the preceding in that we use a large ball of metal instead of a mountain, and measure its deflecting force by a laboratory experiment instead of comparing astronomical observations with geodetic measurements.

166. In the earlier experiments by this method the small balls were of lead, about two inches in diameter, at the extremities of a light wooden rod, five or six feet long, enclosed in a case with glass ends, and their position and vibration was observed by a telescope looking directly at them from a distance of several feet. The attracting masses, B, were balls also of lead, about one foot in diameter, mounted on a frame pivoted in such a way that they could be easily brought to the required positions.

Great difficulty was caused by air currents in the case, and it was necessary to enclose the whole apparatus in a small room of its own which was covered with tin-foil on the outside, and to avoid going near the room or allowing any radiant heat to strike it for hours before the observations. Baily, in England, and Reich, in Germany, between 1838 and 1842, made very extensive series of observations of this kind. Baily obtained 5.66 for the earth's density, and Reich 5.48.

The experiment was repeated in 1872 by Cornu, in Paris, with a modified apparatus.

The horizontal bar was in this case only half a metre long, of aluminium, with small platinum balls at the end. For the large balls, glass globes were used, which could be pumped full of mercury or emptied at pleasure. The whole was enclosed in an air-tight case, and the air exhausted by an air-pump. The deflections and vibrations were observed by means of a tele-

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[1] Note that G is *directly* given by the observations; $G = f\dfrac{d^2}{B}$.

scope watching the image of a scale reflected in a small mirror attached to the aluminium beam near its centre, according to the method first devised by Gauss and now so generally used in galvanometers and similar apparatus. Cornu obtained 5.56 as the result, and showed that Baily's figure required a correction which, when applied, would reduce it to 5.55.

A still more recent and elaborate repetition of the experiment was made by Boys at Oxford in 1890–1893. The beam, only about half an inch long, was suspended in a partial vacuum by a torsion-fibre of *quartz*. The attracted balls were of gold, a quarter of an inch in diameter, and the attracting balls were of lead, $4\frac{1}{4}$ and $2\frac{1}{4}$ inches in diameter — two sets. His result for the density of the earth was 5.527. Still more recently (in 1897) Braun of Mariaschein (Bohemia) publishes a result obtained by the same general method and in perfect agreement with that of Boys.

167. 3. POTSDAM OBSERVATIONS. — During 1886 and 1887 another series of observations was made by Wilsing, at Potsdam, with apparatus similar in principle to the torsion balance, except that the bar carrying the balls to be attracted was *vertical*, and turned on knife edges very near its centre of gravity. The knife edges, like those of an ordinary balance, rested upon agate planes, and the centre of gravity of the apparatus was so adjusted that one vibration of the pendulum, under the influence of gravity alone, would occupy from two to four minutes. The deflecting weights in this case were large cylinders of cast iron, suspended in such a way that they could be brought opposite the small balls, first on one side and then on the other. The whole was set up in a basement, and carefully and very effectually guarded against all changes of temperature, the arrangements being such that all manipulations and observations could be effected from the outside without entering the room. The deflections and vibrations were observed by a reflected scale, as in Cornu's observations. The result obtained was 5.59. Several other methods have been used; of less scientific value, however.

168. *a.* The mass of the earth can be deduced by ascertaining *the force of gravity at the top of a mountain* and *at its base, by means of pendulum experiments.* The mass of the mountain must be determined by a survey, just as in the Schehallien method, which makes the method unsatisfactory. At the top of a mountain the height of which is h, and the distance of its centre of attraction from the top is d, gravity will be made up of two parts, one the attraction of the earth at a distance from its centre equal to $R + h$, and the other the attraction of the mountain alone considered. Calling the mass of the mountain m, and gravity at its summit g' (g being the force of gravity at the earth's surface), we shall have the proportion

$$g : g' = \frac{E}{R^2} : \left[\frac{E}{(R+h)^2} + \frac{m}{d^2}\right],$$

the second fraction in the last term of the proportion being the attraction of the mountain. When g and g' are ascertained by the pendulum experiments, E remains as the only unknown quantity, and can be readily found. Observations of this kind were made by Carlini, in 1821, on Mt. Cenis, and the result was 4.95: also by Mendenhall on Fusiyama in 1890, and by Preston on Mauna Kea in 1892, the results being respectively 5.77 and 5.57.

169. *b. By means of pendulum observations at the earth's surface compared with those at the bottom of a mine of known depth.* This method was employed by Airy in 1854, at Harton Colliery, 1200 feet deep; result, 6.56. In this case the principle involved is somewhat different. *At any point within a hollow, homogeneous, spherical shell, gravity is zero,* as Newton has shown. The attraction balances in all directions. If, then, we go down into a mine, the effect on gravity is the same as if a shell composed of all that part of the earth above our level had been removed. At the same time our distance from the earth's centre has been decreased by d, the depth of the mine.

At the surface $$g = G\frac{E}{R^2}, \text{ as before.}$$

At the bottom of the mine $$g' = G\frac{E - \text{“shell”}}{(R-d)^2}.$$

Comparing the two equations, we find E in the terms of the shell, since the ratio of g to g' is given by pendulum observations. Obviously, however, the mass of the "shell" is difficult to determine with accuracy. And it is by no means homogeneous, so that there is no great reason for surprise at the discordant result. g' was found to be actually greater than g, showing that although at the centre of the earth the attraction necessarily becomes zero, yet as *we descend below the surface, gravity increases for a time* down to some unknown but probably not very great depth, where it becomes a maximum.

170. *c. By experiments with a common balance.* If a body be hung from one of the scale-pans of a balance, its apparent weight will obviously be increased when a large body is brought very near it underneath; and this increase can be measured. Poynting in England and Jolly in Germany have recently used this method, and have obtained results agreeing very fairly with those got from the torsion balance. A series of observations by a modification of this method, and on a very large scale, has been carried out at Berlin, and the result published late in 1896 by Richarz, is 5.505, in excellent accordance with Poynting who in 1891 got 5.493.

171. Constitution of the Earth's Interior. — Since the average density of the earth's crust does not exceed three times that of water, while the mean density of the whole earth is about 5.53 (taking the average of all the most trustworthy results), it is obvious that at the centre the density must be very much greater than at the surface, — very likely as high as eight or ten times that of water,

and equal to the density of the heavier metals. There is nothing in this that might not have been expected. If the earth were ever fluid, it is natural to suppose that in the solidification the densest materials would settle towards the interior.

Whether the interior of the earth is solid or fluid it is difficult to say with certainty. Certain tidal phenomena, to be discussed hereafter, have led Sir William Thomson (now Lord Kelvin) and the younger Darwin to conclude that the earth as a whole is solid throughout, and "more rigid than steel," volcanic centres being mere pustules in the general mass. To this many geologists demur.

As regards the temperature at the earth's centre, it is hardly an astronomical question, though it has very important astronomical relations. We can only take space to say that the temperature appears to increase from the surface downward at the rate of about one degree Fahrenheit for every fifty or sixty feet, so that at the depth of a few miles the temperature must be very high.

171.* (Note to Art. 160.) **Measurement of Mass by Means of Inertia.** — It is quite possible to measure masses without weighing. In Fig. 126, *B* is a receptacle carried at the end of a horizontal arm *A*, which is itself attached to an axis *MN*, exactly vertical and free to turn on pivots at top and bottom. A spiral spring *S*, like the hair spring of a watch, is connected with this axis so that if *A* is disturbed it will oscillate back and forth at a rate which depends upon the stiffness of the spring and the total moment of inertia of the apparatus. If we put into *B* one standard "pound" (of mass), it will vibrate a certain number of times a minute; if *two* pounds, it will vibrate *more slowly*; if *three*, still more slowly; and so on: and this time of vibration can be determined and tabulated. To determine now the mass of a body *X*, we have only to put it into the receptacle *B*, set the apparatus vibrating, and count the number of swings in a minute. Referring to our table, we find what number of "pounds" in *B* would have given the same rate of vibration. We know then that the "*inertia*" of *X* is the same as that of this number of "pounds," and therefore its *mass* is the same.

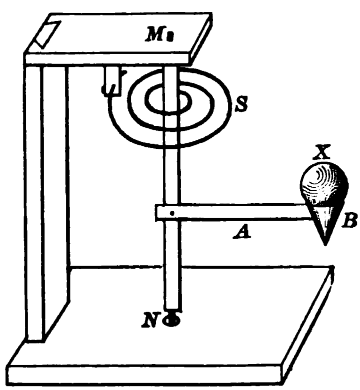

FIG. 53*.

This determination is independent of all considerations of *weight:* the apparatus would give the same results on the surface of the moon, or on that

of Jupiter, as on the earth. It is obvious, however, that an instrument of this sort could not compete in accuracy or convenience with a well-made balance, because of the friction of the pivots, the resistance of the air, etc. We introduce it simply to assist in separating the idea of *mass* from that of *weight.*

Exercises on Chapter V.

1. Does the transportation of sediment by the Mississippi tend to lengthen or to shorten the day?

2. If the diameter of the earth were doubled, keeping its mass unchanged, how would its density and the weight of bodies at its surface be affected?

3. If its diameter were trebled, keeping its density unchanged, how much would its mass and the weight of bodies at its surface be increased?

4. Supposing the earth to be homogeneous, how great (approximately) would be the force of gravity a thousand miles below its surface?

5. Assuming the earth to be homogeneous, at what depth (approximately) would a pendulum which at the surface of the earth vibrates seconds vibrate in a second and a quarter? *Ans.* 1440 miles.

6. Given two spheres one of which has a mass m times greater than the other: on what point on the line joining their centres are their attractions equal?

Solution. Let d be the distance between their centres, and x the distance of the point of equilibrium from the smaller body: then the attraction of the larger body at that point is $G\dfrac{m}{(d-x)^2}$, that of the smaller being $G\dfrac{1}{x^2}$. Canceling the Gs, and taking the square roots, we have $\dfrac{\sqrt{m}}{(d-x)} = \dfrac{1}{x}$; from which we have *Ans.* $x = \dfrac{d}{1+\sqrt{m}}$.

7. Assuming the moon's mass as $\frac{1}{81}$ of the earth's, where is the equilibrium point on the line of centres?

Ans. At a point one-tenth of the distance from the moon to the earth.

8. Assuming the distance of the sun as 93 000000 miles, and its mass as 330000 times that of the earth, where is the point at which their attractions balance on the line of centres?

Ans. Distance from the earth $= \dfrac{93\,000000}{1+\sqrt{330000}} = \dfrac{93\,000000}{575.456} = 161600$ miles.

Note. — This distance is much less than the distance of the moon from the earth, so that at the time of new moon the sun's attraction upon the moon very much exceeds that of the earth. Compare Art. 439.

CHAPTER VI.

THE APPARENT MOTION OF THE SUN AMONG THE STARS, AND THE EARTH'S ORBITAL MOTION. — THE EQUATION OF TIME, PRECESSION, NUTATION, AND ABERRATION. — VARIOUS KINDS OF "YEAR." — THE CALENDAR.

172. The Annual Motion of the Sun. — The apparent *annual motion of the sun* must have been one of the earliest noticed of all astronomical phenomena. Its discovery antedates history.

As seen by the people in Europe and Asia, the sun, starting in the spring, mounts higher in the sky each day at noon for three months, until it reaches its greatest elevation at the summer solstice, and then descends towards the south, reaching in the autumn the same noonday elevation it had in the spring. It keeps on its southward course to a winter solstice in December, and then returns to its original height at the end of a year, marking and causing the seasons by its course. A year, the interval between the successive returns of the sun to the same position, was very early found to consist of a little more than three hundred and sixty days.

Nor is this all. The sun's motion is *not merely a north-and-south motion*, but it also moves *eastward*[1] *among the stars;* for in the spring the stars which are rising in the eastern horizon at sunset are different from those which are found there in the summer or winter. In the spring, the most conspicuous of the eastern constellations at sunset are Leo and Boötes; a little later, Virgo appears; in the summer, Ophiuchus and Libra; still later, Scorpio; and in midwinter, Orion and Taurus are in the eastern sky at evening.

173. So far as mere appearances go, everything would be explained by assuming that the earth is at rest and the sun moving around it; but equally by the converse supposition, — for if the earth as seen from the sun appears at any point in the heavens, the sun as seen from the earth must appear in exactly the opposite point, and

[1] Of course these two motions of the sun are not independent, but only "components" of its motion in the ecliptic (Art. 175).

must keep opposite, moving through the same path in the sky (but six months behind), and always in the same "angular direction," if we may use the expression. (Just as two opposite teeth on a gear-wheel move in the same *angular direction*, though at any moment they are moving in opposite *linear* directions.)

174. That it is really the earth which moves, and not the sun, is absolutely demonstrated by three phenomena too minute and delicate for pre-telescopic observations, but accessible to modern methods. One of them is *the aberration of light;* a second, *the regular annual backward and forward shift of the lines in star-spectra;* the third, *the annual parallax of the fixed stars.* These can be explained only by the actual motion of the earth.

175. The Ecliptic. — By observing with a meridian circle daily the declination of the sun, and the difference between its right ascension and that of some star (Flamsteed used α Aquilæ for the purpose), we shall obtain a series of positions of the sun's centre which can be plotted on a celestial globe ; and we can thus make out the path of the sun among the stars, and find the place where it cuts the celestial equator, and the angle it makes. This path turns out to be a *great circle,* as is shown by its cutting the equator at two points just 180° apart (the so-called equinoctial points or equinoxes), and makes an angle with it of approximately 23½°. This great circle is called the ECLIPTIC, because, as was early discovered, eclipses happen only when the moon is crossing it. It may be defined as *the trace of the plane of the earth's orbit upon the celestial sphere, i.e.,* the great circle formed by the intersection of the infinitely extended plane of the earth's orbit with the celestial sphere.

176. Definitions. — The angle which the ecliptic makes with the equator is called the *Obliquity of the ecliptic,* and the points midway between the equinoxes are called the *Solstices* (*sol-stitium*), because at these points the sun "*stands,*" or stops moving in declination for a short time.

Two circles parallel to the equator, drawn through the solstices, are called the *Tropics* (Greek τρέπω), or "*turning-lines,*" because there the sun turns from its northward motion to a southward, or *vice versa.* The obliquity is, of course, simply equal to the *sun's maximum declination,* or greatest distance from the equator, which is reached in June and December.

The ancients were accustomed to determine it by means of the gnomon[1] (Art. 107). The length of the shadow at noon on the solstitial days determines the zenith distance of the sun on those days, and the difference of the zenith distances at the two solstices is twice the angle desired. The gnomon also determined for the ancients the length of the year, it being only necessary to observe the interval between days in the spring or autumn, when the shadow had the same length at noon.

177. The Zodiac and its Signs. — A belt 16° wide, 8° on each side of the ecliptic, is called the *Zodiac*. The name is said to be derived from *ζώον*, a living creature, because the constellations in it (except Libra) are all figures of animals. It was taken of that particular width by the ancients simply because the moon and the then known planets never go further than 8° from the ecliptic.

This belt is divided into the so-called SIGNS, each 30° in length, having the following names and symbols: —

Spring	Aries,	♈	Autumn	Libra,	♎
	Taurus,	♉		Scorpio,	♏
	Gemini,	♊		Sagittarius,	♐
Summer	Cancer,	♋	Winter	Capricornus,	♑
	Leo,	♌		Aquarius	♒
	Virgo,	♍		Pisces,	♓

The symbols are for the most part conventionalized pictures of the objects. The symbol for Aquarius is the Egyptian character for water. The origin of the signs for Leo, Virgo, and Capricornus is not quite clear. It has been suggested that ♌ is simply a "*cursive*" form for Λ, the initial of Λέων; ♍ for Παρ (Παρθένος), and ♑ for Τρ (Τράγος).

CELESTIAL LATITUDE AND LONGITUDE.

178. Since the moon and all the principal planets always keep within the zodiac, the ecliptic is a very convenient circle of reference, and was used as such by the ancients. Indeed, until the invention of pendulum clocks, it was on the whole more convenient than the equator, and more used.

The two points in the heavens 90° distant from the ecliptic are called the *Poles of the ecliptic*. The northern one is in the constellation

[1] The Chinese claim to have made an observation of this kind about 1100 B.C., and the result given is very nearly what it should have been at that time. (The obliquity changes slightly in centuries.) If their observation is genuine, it is probably the oldest of all astronomical records.

of Draco, about half-way between the stars δ and ζ Draconis. Now, suppose a set of great circles drawn, like meridians, through these poles of the ecliptic, and hence perpendicular to that circle; these are *Circles of latitude* or *secondaries to the ecliptic.* The LONGITUDE of a star or any other heavenly body is, then, *the angle made at the pole of the ecliptic, between the circle of latitude, which passes through the vernal equinox, and the circle of latitude passing through the body;* or, what comes to the same thing, it is *the arc of the ecliptic included between the vernal equinox and the foot of the circle of latitude passing through the body.* Celestial longitude is always reckoned *eastward* from the vernal equinox, completely around the ecliptic, so that the longitude of the sun when 10° *west* of the vernal equinox would be written as 350°, and not as −10°.

The LATITUDE of a star is simply *its distance north or south of the ecliptic measured on the star's circle of latitude.*

179. It will be seen that *longitude differs from right ascension in being reckoned on the ecliptic instead of on the equator*, nor can it be reckoned in *time*, but only in degrees, minutes, and seconds. *Latitude differs from declination in that it is reckoned from the ecliptic instead of from the equator.*

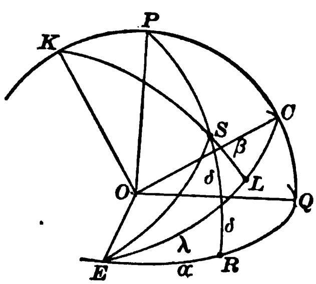

FIG. 57.

Relation between Celestial Latitude and Longitude, and Right Ascension and Declination.

The relation between right ascension and declination on the one hand, and longitude and latitude on the other, may be made clearer by the accompanying diagram (Fig. 57), in which EC is the ecliptic and EQ the equator, E being the vernal equinox. S being a star, its right ascension (α) is ER and its declination (δ) is SR; its longitude (λ) is EL, and its latitude (β) is SL. P and K are the poles of the equator and ecliptic respectively, and the circle $KPCQ$ is the *Solstitial Colure*, so called.

The student can hardly take too great care to avoid confusion of celestial latitude and longitude with right ascension and declination or with *terrestrial* latitude and longitude. It is, of course, unfortunate that latitude in the sky should not be analogous to latitude upon the earth, or celestial longitude to terrestrial. The terms right ascension and declination are, however, of comparatively recent introduction, and found the ground preoccupied, celestial latitude and longitude being much older.

180. Conversion of λ and β into α and δ, or Vice Versa. — Right ascension and declination can, of course, always be converted into longitude and latitude by a trigonometrical calculation. We proceed as follows: In the triangle ERS, right-angled at R, we have given ER and RS (α and δ), from which we find the hypothenuse ES and the angle RES. Next in the triangle ELS, right-angled at L, we have the hypothenuse ES and the angle LES, which is equal to $RES-LEQ$ (LEQ being ω, the obliquity of the ecliptic). Hence we easily find EL and LS.

181. The Earth's Orbit in Space. — The *ecliptic is not the earth's orbit*, and must not be confounded with it. It is a *great circle* of the infinite celestial sphere, the *trace* made upon the sphere by the plane of the earth's orbit, as was stated in its definition. The fact that it is a great circle gives us no information about the earth's orbit, except that *the orbit all lies in one plane passing through the sun.* It tells us nothing as to its real form and size.

By reducing the observations of the sun's right ascension and declination through the year to longitude and latitude (the latitude will always be zero, of course, except for some slight perturbations) and combining them with observations of the sun's apparent diameter, we can, however, ascertain the real form of the earth's orbit and the law of its motion in this orbit. But the *size* of the orbit — the scale of miles — cannot be fixed until we can find the sun's distance.

182. To find the Form of the Orbit, we may proceed thus: Take a point S for the sun and draw from it a line SO, Fig. 58, directed towards the vernal equinox as the origin of longitudes. Lay off from S indefinite lines, making angles with SO equal to the earth's[1] longitude on each of the days observed through the year; *i.e.*, the angle OS 10, is the longitude at the time of the 10th observation; and so on. We shall thus get a sort of "spider," showing the *directions* as seen from the sun on those days.

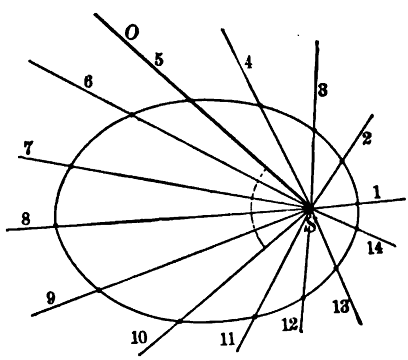

FIG. 58.

Determination of the Form of the Earth's Orbit.

Next, as to *relative distances.* While the apparent diameter of the sun does not tell us its real distance from the earth, unless we first

[1] The *earth's* longitude is the observed longitude of the sun + 180.°

know the sun's real diameter in miles, the changes in the apparent diameter do inform us as to the *relative* distance of the earth at different times, since the nearer we are, the larger the sun appears, — the distance being inversely proportional to the apparent diameter (Art. 6). If, then, we lay off on the arms of our "spider" distances inversely proportional[1] to the number of seconds of arc in the sun's measured diameter at each date, these distances will be *proportional* to the true distance of the earth from the sun, and the curve joining the points thus obtained will be a true map of the earth's orbit, though without any scale of miles upon it.

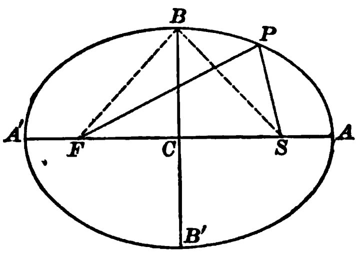

FIG. 59. — The Ellipse.

When the operation is performed, we find that the orbit is an ellipse of small eccentricity (about one-sixtieth), with the sun, not in the centre, but at one focus.[2]

183. For the benefit of any who may not have studied conic sections we define the ellipse. It is a curve such that *the sum of the two distances from any point on its circumference to two points within, called the foci, is always constant,* and equal to what is called the major-axis of the ellipse. $SP + PF = AA'$, in Fig. 59. AC is called the semi-major-axis, and is usually denoted by A or a. BC is the semi-minor-axis, denoted by B or b. The eccentricity, denoted by e, is the fraction $\frac{SC}{AC}$.

Since BS is equal to A, $SC = \sqrt{A^2 - B^2}$; and $e = \frac{\sqrt{A^2 - B^2}}{A}$.

The points where the earth is nearest to and most remote from the sun are called respectively *perihelion* and *aphelion*, and the line that joins them is, of course, the major-axis of the orbit. This line, considered as indefinitely produced in both directions, is called the *line of apsides*, — the major-axis being a limited piece or "sect" of the line of apsides.

184. The variations of the sun's diameter are too small to be detected without a telescope (amounting only to about three per cent), so that the ancients were unable to perceive them. Hipparchus, however, about

[1] The distances to be laid off are found by dividing some arbitrarily chosen constant (say 10000″) by the number of seconds of arc in each measured diameter of the sun. [2] See note on page 154.

150 B.C., discovered that the earth is not in the centre of the circular orbit which he supposed the sun to describe around it. Everybody assumed, on *à priori* grounds, never disputed until the time of Kepler, that the sun's orbit must be a circle and described with a uniform motion, because a circle is the only "perfect" curve, and uniform motion the only perfect motion. Obviously, however, the sun's *apparent* motion is not uniform, because it takes 186 days for the sun to pass from the vernal equinox to the autumnal through the summer months, and only 179 days to return during the winter. Hipparchus explained this difference by the hypothesis that the earth is out of the centre of the sun's path.

185. To find the Eccentricity of the Orbit. — Having the greatest and least apparent diameters of the sun, the eccentricity, e, is easily found. In Fig. 59, since, by definition, $e = CS \div CA$, we have $CS = CA \times e$, or Ae. The perihelion distance AS is therefore equal to $A \times (1-e)$, and the aphelion distance SA' to $A\ (1+e)$. Suppose now that the greatest and least measured diameters of the sun are p and q. This gives us the proportion $p:q = A\ (1+e) : A\ (1-e)$, since the diameters are *inversely* proportional to the distances. From this we get

$$e = \frac{p-q}{p+q}.$$

The actual values of p and q are $32'\ 36''.4$ and $31'\ 31''.8$, which give $e = 0.01678$: this is about $\frac{1}{60}$, as has been stated.

186. To find the Law of the Earth's Motion. — By comparing the measured apparent diameter with the differences of longitude from day to day, we can also deduce the *law* of the earth's motion. On making a table of daily motions and apparent diameters, we find that these *daily motions* vary *directly as the squares of the diameters;* from which it directly follows that the earth moves in such a way that its *radius-vector describes areas proportional to the times* (a law which Kepler first brought to light in 1609). The radius-vector is the line which joins the earth to the sun at any moment.

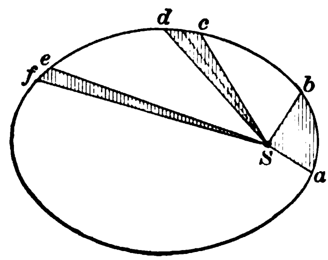

FIG. 60.
Equable Description of Areas.

187. Consider a small elliptical sector, dSc (Fig. 60), described by the earth in a unit of time. Regarding it as a triangle, its area is given by the formula $\frac{1}{2}\,Sc \times Sd \sin cSd$; and calling this angle θ (which will be very small), and considering that in so short a time

Sd and Sc would remain sensibly equal, each being equal to R (the radius-vector at the middle point of the arc), this formula becomes,

$$\text{Area of sector} = \tfrac{1}{2}\, R^2\theta.$$

Now, calling the sun's apparent diameter D, we have

$$R \text{ varies as } \frac{1}{D} \text{ or } R = \frac{k}{D},$$

(k being a constant, and depending on the sun's diameter in miles);

whence $$R^2 = \frac{k^2}{D^2}.$$

But our measurements show that $\theta = k_1 D^2$, k_1 being another constant.

Substitute these values of R^2 and θ in the formula for the area, and we have

$$\text{Area of sector} = \tfrac{1}{2}\frac{k^2}{D^2} \times k_1 D^2 = \tfrac{1}{2}\, k^2 k_1,$$

a constant; that is, the area described by the radius-vector in a unit of time is always the same. The planet near perihelion moves so much faster, that the areas aSb, cSd, and eSf are all equal to each other, if the arcs are described in the same time.

188. Kepler's Problem. — As the case stands so far, this is a mere fact of observation; but as we shall see hereafter, and as was demonstrated by Newton, the fact shows that the earth moves under the action of a force *always directed in line with the sun*. In such a case the "equable description of areas" is a necessary mechanical consequence. It is true in every case of elliptical motion, and enables us to find the position of the earth or any planet in its orbit at any time, when we once know the time of its orbital revolution (technically the period), and the time when it was at perihelion. Thus, the angle ASP (Fig. 61), which is called the *Anomaly* of the planet, must be such that the area of the elliptical sector ASP will be that portion of the whole ellipse which is represented by the fraction $\frac{t}{T}$, t being the number of days since the planet last passed the perihelion, and T the number of days in the whole period. For instance, if the earth last passed perihelion on Dec. 31 (which it did), its place on May 1 must be such that the

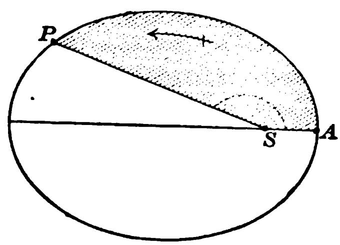

FIG. 61. — Kepler's Problem.

sector ASP will be $\frac{121}{365}$ of the whole of the earth's orbit; since from December 31 to May 1 is 121 days. The solution of this problem, known as "*Kepler's problem*," leads to transcendental equations, and lies beyond our scope.

See Watson's "Theoretical Astronomy," pp. 53 and 54, or any other similar work; also Appendix, Arts. 1002 and 1003.

189. Anomaly and Equation of the Centre. — The angle ASP, which has been termed simply the "*Anomaly*," is strictly the *true* Anomaly, as distinguished from the *mean* Anomaly. The *former* may be defined as *the angle actually made at any time by the radius-vector of a planet with the line of apsides*, the angle being reckoned from the perihelion point completely around in the direction of the planet's motion. The *mean* Anomaly is what the Anomaly *would be at the given moment if the planet had moved with uniform* angular *velocity, completing the orbit in the same period, and passing perihelion at the same time*, as it actually does. The difference between the two anomalies is called the *Equation of the Centre.* This is zero at perihelion and aphelion, and a maximum midway between them. In the case of the sun, its greatest value is nearly 2°, the sun getting alternately that amount ahead of, and behind, the position it would occupy if its apparent daily motion were uniform.

THE SEASONS.

190. The Seasons. — The earth in its motion around the sun always keeps its axis parallel to itself, for the mechanical reason that a revolving body necessarily maintains the direction of its axis invariable, unless disturbed by extraneous force, as is very prettily illustrated by the gyroscope. Fig. 61* shows the way in which the north pole of the earth is inclined with reference to the sun at different seasons of the year.

About March 20 the earth is so situated that the plane of its equator passes through the sun, the sun's declination being zero on that day. At that time, the line which separates the illuminated portions of the earth passes through the two poles (as shown in Fig. 62 *B*), and day and night are everywhere equal. The same is again true of the 22d of September, when the sun is at the autumnal equinox on the opposite side of the orbit.

About the 21st of June the earth is so situated that its *north* pole is inclined towards the sun by about 23½°, which is the sun's northern declination on that date. As shown in Fig. 62 *A*, the south pole is then in the obscure half of the earth's globe.

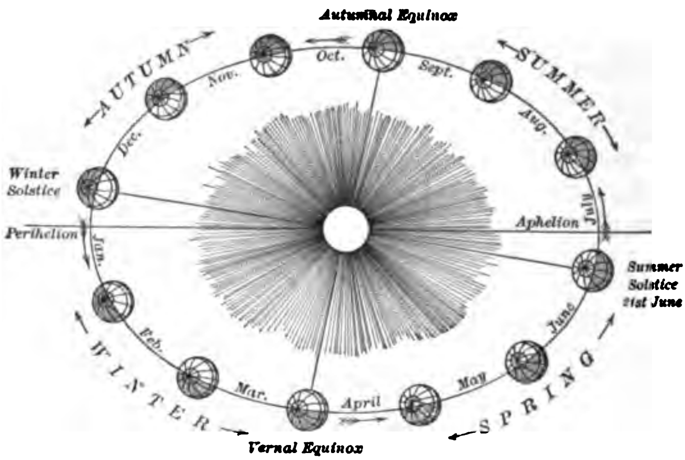

Fig. 61*. — The Seasons.

The north pole then receives sunlight all day long; and in all portions of the northern hemisphere the day is longer than the night, the difference between the day and night depending upon the latitude of the place, while in the southern hemisphere the

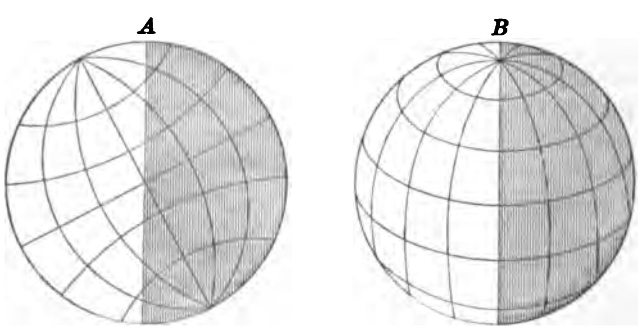

Fig. 62. — Position of Pole at Solstice and Equinox.

days are shorter than the nights. At the time of the winter solstice these conditions are reversed. At the equator (of the earth) the day and night are equal at all times of the year. The sun when in northern declination of course always rises at a point on the horizon *north of east*, and sets at a point north of west, so that for a portion of the time each day it shines on the north side of a house.

191. Diurnal Phenomena near the Pole. — At the north pole, where the celestial pole is in the zenith, and the diurnal circles are parallel with the horizon, the sun will maintain the same elevation all day long, except for the slight change caused by the variation of its declination in twenty-four hours. The sun will appear on the horizon at the date of the vernal equinox (in fact, about three days before, on account of refraction), and slowly wind upward in the sky until it reaches its maximum elevation of $23\frac{1}{2}°$ on June 21. Then it will retrace its course until a day or two after the autumnal equinox, when it sinks out of sight.

At points between the north pole and the polar circle the sun will appear above the horizon earlier in the year than March 20, and will rise and set daily until its declination becomes equal to the observer's distance from the pole, when it will make a complete circuit of the heavens, touching the horizon at midnight at the northern point; and after that never setting again until it reaches the same declination in its southward course after passing the solstice. From that time it will again rise and set daily until it reaches a southern declination just equal to the observer's polar distance, when the long night begins; to continue until the sun, having passed the southern solstice, returns again to the same declination at which it made its appearance in the spring. At the polar circle itself (or, more strictly speaking, owing to refraction, about one-half a degree south of it) the "*midnight sun*" will be seen on just one day in the year, the day of the summer solstice; and there will also be one absolutely sunless day, viz., the day of the winter solstice. The same remarks apply in the southern hemisphere, by making the obvious changes.

192. Effects on Temperature. — The changes in the duration of "*insolation*" (exposure to sunshine) at any place involve changes of temperature and other climatic conditions, thus producing the seasons. Taking as a standard the amount of heat received in twenty-four hours on the day of the equinox, it is clear that the surface of the soil at any place in the northern hemisphere will receive more than this average amount of heat whenever the sun is north of the celestial equator, for two reasons.

1. Sunshine lasts more than half the day.
2. The *mean elevation* of the sun during the day is greater than

when it is at the equinoxes, since it is higher at noon, and in any case reaches the horizon at rising and setting. Now, the more obliquely the rays strike, the less heat they bring to each square inch of surface. A beam of sunshine of given cross-section, *ABCD*, is spread over a larger area when it strikes obliquely than when it is vertical, as is obvious from Fig. 62*, and its heating efficiency is correspondingly diminished. If Q is the amount of heat per square inch brought by the ray when falling perpendicularly, as on the surface AC, then on Ac (on which it strikes at the angle h, equal to the sun's altitude) the amount per square inch will be $Q \times \sin h$, since $AB = Ab \times \sin h$. This difference in favor of the more nearly vertical rays is exaggerated by the absorption of heat in the atmosphere, because rays that are nearly horizontal have to traverse a much greater thickness of air before reaching the ground.

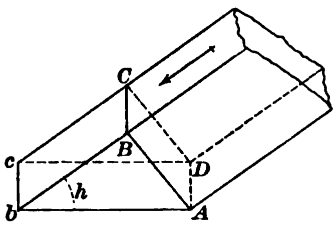

Fig. 62*.
Effect of Sun's Elevation on Amount of Heat Imparted to the Soil.

For these two reasons, at a place in the northern hemisphere, the temperature rises rapidly as the sun comes north of the equator, thus giving us our summer.

193. Time of Highest Temperature. — We, of course, receive the most heat *per diem* at the time of the summer solstice; but this is not the hottest time of the summer, for the obvious reason that the weather is then all the time *getting hotter*, and the maximum will not be reached *until the increase ceases;* that is, not until the *amount of heat lost in the night* equals that *stored up by day*.

If the earth's surface threw off the same amount of heat hourly whether it were hot or cold, then this maximum would not come until *the autumnal equinox*. This, however, is not the case. The soil loses heat faster when warm than it does when cold, the loss being nearly proportional to the difference between the temperature of the soil and that of surrounding space (Newton's law of cooling); and so the time of the maximum is made to come not far from the end of July, or the first of August, in our latitude. For similar reasons the minimum temperature of winter occurs about February 1, about half-way between the solstice and the vernal equinox. Since, however, our weather is not entirely "made on the spot where it is used," but is affected by winds and currents that come from great distances, the actual time of the maximum temperature cannot be determined by any mere astronomical considerations, but varies considerably from year to year.

194. Difference between Seasons in Northern and Southern Hemispheres. — Since in December the distance of the earth from the sun is about three per cent less than it is in June, the earth (as a whole) receives hourly about six per cent more heat in December than in June, the heat varying inversely as the *square* of the distance. For this reason the southern summer, which occurs in December and January, is *hotter* than the northern. It is, however, seven days *shorter*, because the earth moves more rapidly in that part of its orbit. The total amount of heat per acre, therefore, received during the summer is sensibly the same in each hemisphere, the shortness of the southern summer making up for its increased warmth.

195. The southern *winter*, however, is both longer and colder than the northern; and it has been vigorously maintained by certain geologists, that, on the whole, the mean annual temperature of the hemisphere which has its winter at the time when the earth is in aphelion is lower than that of the opposite one. It has been attempted to account for the glacial epochs in this way. It is certain that at present, at any place in the southern hemisphere, the difference between the maximum temperature of summer and the minimum of winter must be greater than in the case of a station in the northern hemisphere, similarly situated as to elevation, etc. We say "at present" because, on account of certain slow changes in the earth's orbit, to be spoken of immediately, the state of things will be reversed in about ten thousand years, the northern summer being then the hotter and shorter one.

196. Secular Changes in the Orbit of the Earth. — The orbit of the earth is not absolutely unchangeable in *form* or *position*, though it is so in the long run as regards the *length of its major axis* and the *duration of the year*.

197. 1. *Change in Obliquity of the Ecliptic.* — The ecliptic slightly and very slowly shifts its position among the stars, thus altering the latitudes of the stars and the angle between the ecliptic and equator, *i.e.*, the obliquity of the ecliptic. This obliquity is at present about 24′ less than it was 2000 years ago, and is still *decreasing* about half a second a year. It is computed that this diminution will continue for about 15,000 years, reducing the obliquity to $22\frac{1}{2}°$, when it will begin to increase. The whole change, according to J. Herschel, can never exceed about 1° 20′ on each side of the mean.

198. 2. *Change of Eccentricity.* — At present the eccentricity of the earth's orbit (which is now 0.0168) is also slowly diminishing.

According to Leverrier, it will continue to decrease for about 24,000 years, until it becomes 0.003, and the orbit will be almost circular. Then it will increase again for 40,000 years, until it becomes 0.02.

In this way the eccentricity will oscillate backwards and forwards, always, however, remaining between zero and 0.07; but the oscillations are not equal either in amount or time, and so cannot properly be compared to the "vibrations of a mighty pendulum," which is rather a favorite figure of speech.

199. 3. *Revolution of the Apsides of the Earth's Orbit.* — The line of apsides of the orbit, which now stretches in both directions towards the constellations of Sagittarius and Gemini, is also slowly and steadily moving eastward, and at a rate which, if it were constant, would carry it around the circle in about 108,000 years.

200. These so-called "*secular*" changes are due to the action of the other planets upon the earth. Were it not for their attraction, the earth would keep her orbit with reference to the sun strictly unaltered from age to age, except that possibly in the course of millions of years the effects of falling meteoric matter and the attraction of the nearer fixed stars might make themselves felt.

Besides these secular perturbations of the earth's orbit, the earth itself is continually being slightly disturbed in its orbit. On account of its connection with the moon, it oscillates each month a few hundred miles above and below the true plane of the ecliptic, and by the action of the other planets it is sometimes set forwards or backwards to the extent of a few thousand miles. Of course every such change produces a corresponding slight change in the apparent position of the sun.

201. Equation of Time. — We have stated a few pages back (Art. 111), that the interval between the successive passages of the sun across the meridian is somewhat variable, and that for this reason apparent solar, or sun-dial, days are unequal. On this account mean time has been adopted, which is kept by a "*fictitious*," or "*mean*," sun moving uniformly in the equator at the same average rate as that of the real sun in the ecliptic. The hour-angle of this mean sun is, as has been already explained, *the local mean time* (or clock time); while the hour-angle of the real sun is the *apparent* or *sun-dial time*. The *Equation of Time* is the difference between these two times, reckoned as *plus* when

the sun-dial is *slower* than the clock, and *minus* when it is faster. It is the *correction which must be added* (algebraically) *to apparent time in order to get mean time.* As it is the difference between the two hour-angles, it may also be defined as *the difference between the right ascensions of the mean sun and the true sun;* or as a formula we may write: $E = a_t - a_m$, in which a_m is the right ascension of the mean sun,[1] and a_t of the true sun.

The principal causes of this difference are two:—

202. 1. *The Variable Motion of the Sun in the Ecliptic, due to the Eccentricity of the Earth's Orbit.*—Near perihelion, which occurs about December 31, the earth's orbital motion is most rapid according to the law of "equal areas" (Art. 186). This makes the sun's apparent eastward motion (in longitude) correspondingly greater, and hence at this time the apparent solar days exceed the sidereal by more than the average amount, making the sun-dial days longer than the mean. (The average solar day, it will be remembered, is $3^m\ 56^s$ longer than the sidereal.) The sun-dial will therefore lose time at this season, and will continue to do so for about three months, until the angular motion of the sun falls to its mean value. Then it will gain until aphelion, when, if the clock and the sun were started together at perihelion, they will once more be together. During the next half of the year the action will be reversed. Thus, twice a year, so far as the eccentricity of the earth's orbit is concerned, the clock and sun would be together at perihelion and aphelion, while half-way between they would differ by about *eight minutes;* the equation of time (so far as due to this cause only) being about + 8 minutes in the spring, and − 8 minutes in the autumn.

203. 2. *The Inclination of the Ecliptic to the Equator.*—Even if the *sun's (apparent) motion in longitude* (*i.e.*, along the ecliptic) were uniform, its motion in *right ascension* would be variable. If the true and fictitious suns started together at the equinox, they would indeed be together at the solstices and at the other equinox, because it is just 180° from equinox to equinox, and the solstices are ex-

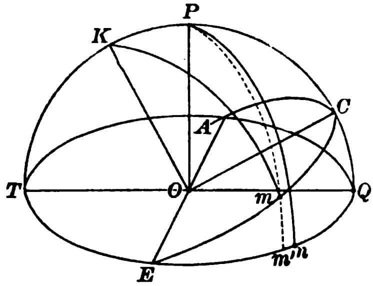

FIG. 63.
Effect of Obliquity of Ecliptic in producing Equation of Time.

[1] a_m always also equals the sun's mean *longitude*.

actly half-way between them. But at *intermediate points*, between the equinoxes and solstices, they would not be together on the same hour-circle. This is best seen by taking a celestial globe and marking *on the ecliptic* a point, *m*, half-way between the vernal equinox and the solstice, and also marking a point, *n*, *on the equator*, 45° from the equinox. It will at once be seen that the former point, *m* in Fig. 63,[1] is *west* of *n*, so that *m* in the daily westward motion of the sky will come to the meridian first; in other words, when the sun is half-way between the vernal equinox and the summer solstice, the *sun-dial* is *faster* than the clock, and the equation of time is *minus*. The difference, measured by the arc *m'n*, amounts to nearly ten minutes; and of course the same thing holds, *mutatis mutandis*, for the other quadrants.

204. Combination of the Effects of the Two Causes. — We can represent graphically these two components of the equation of time and the result of their combination as follows (Fig. 64): —

The central horizontal line is a scale of dates one year long, the letters denoting the beginning of each month. The dotted curve

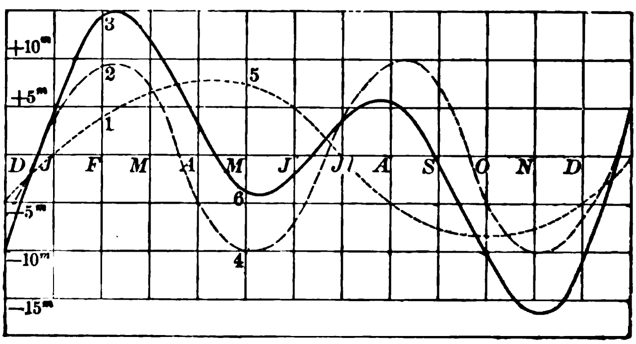

FIG. 64. — The Equation of Time.

shows the equation of time due to the *eccentricity* of the earth's orbit, above considered. Starting at perihelion on December 31, this component is then zero, rising from there to a value of about $+8^m$ on April 2, falling to zero on June 30, and reaching a second maximum of -8^m on October 1. In the same way the broken-line curve

[1] Fig. 63 represents a celestial globe viewed from the *west* side, the axis being vertical, and *K*, the pole of the ecliptic, on the meridian, while *E* is the vernal equinox.

denotes the effect of the *obliquity of the ecliptic*, which, by itself alone considered, would produce an equation of time having *four* maxima of, approximately, 10^m each, about the 6th of February, May, August, and November (alternately + and —), and reducing to zero at the equinoxes and solstices.[1]

The full-lined curve represents their combined effect, and is constructed by making its ordinate at each point equal to the sum (algebraic) of the ordinates of the two other curves. At the 1st of February, for instance, the distance, $F\,3$, in the figure $= F\,1 + F\,2$. So, also, $M\,6 = M\,4 + M\,5$; the components, however, in this case have opposite signs, so that the *difference* is actually taken.

The equation of time is zero *four* times a year, viz., on April 15, June 14, September 1, and December 24. The maxima are February 11, $+14^m\ 32^s$; May 14, $-3^m\ 55^s$; July 26, $+6^m\ 12^s$, and November 2, $-16^m\ 18^s$. But the dates and amounts vary slightly from year to year.

The two causes above discussed are only the *principal* ones effective in producing the equation of time, but all other causes combined never alter the equation by more than a few seconds.

205. Precession of the Equinoxes. — The length of year was found in two ways by the ancients: —

1. By the gnomon, which gives the time of the equinox and solstice; and

2. By observing the position of the sun with reference to the stars, — their rising and setting at sunrise or sunset.

Comparing the results of observations made at long intervals, Hipparchus (120 B.C.) found that the two do not agree; the former year (from equinox to equinox) being $20^m\ 23^s$ shorter than the other (according to modern data). The equinox moves *westward* on the ecliptic, as if it *advanced to meet the sun* on each annual return. He therefore called its motion the "*precession*" of the equinoxes.

On comparing the *latitudes* of the stars in the time of the ancient astronomers with the present latitudes, we find that they have changed very slightly indeed; and we know therefore that the ecliptic maintains its position sensibly unaltered. On the other hand, the *longitudes* of the stars have been found to increase regularly at the rate of about $50''.2$ annually, — fully 30° in the last 2000 years. Since longitudes are reckoned from the equinox (the

[1] The fact that our afternoons begin to lengthen about December 8 is due to the rapid decrease of the equation of time then in progress.

intersection between the ecliptic and equator), and since the ecliptic does not move, it is evident that the motion must be in the *celestial equator;* and accordingly we find that both the *right ascension* and the *declination* of the stars are constantly changing.

206. Motion of the Pole of the Equator around the Pole of the Ecliptic. — The obliquity of the ecliptic, which equals the distance in the sky between the pole of the equator and the pole of the ecliptic (Art. 178), has remained nearly constant. Hence the pole of the equator must be describing a circle around the pole of the ecliptic in a period of about 25800 years (360° divided by 50″.2). The pole of the ecliptic has remained nearly fixed among the stars, but the other pole has changed its position materially. At present the pole star is about 1¼° from the pole. At the time of the star catalogue of Hipparchus it was 12° distant from it, and during the next two centuries it will approach to within about 30′, after which it will recede.

207. If upon a celestial globe we take the pole of the ecliptic as a centre, and describe about it a circle with a radius of 23½°, we shall get the approximate track of the celestial pole, and shall find that the circle passes very near the star α Lyræ, which will be the pole star about 12000 years hence. Reckoning backwards, we find that some 4000 years ago α Draconis was the pole star; and it is a curious circumstance that certain of the tunnels in the pyramids of Egypt face exactly to the north, and slope at such an inclination that this star at its lower culmination would have been visible from their lower end at the date when the pyramids are supposed to have been built. It is probable that these passages were arranged to be used for the purpose of observing the transits of this star.

Because of the changes in the position of the ecliptic (Art. 197) the track of the pole among the stars is not a perfect circle; its centre is not fixed.

208. Effect of Precession upon the Signs of the Zodiac. — Another effect of precession is that the *signs* of the zodiac do not now agree with the *constellations* which bear the same name. The sign of Aries is now in the constellation of Pisces; and so on, each sign having "backed," so to speak, into the constellation west of it.

209. Physical Cause of Precession. — The *physical cause* of this slow conical rotation of the earth's axis around the pole of the ecliptic lies in the two facts that *the earth is not exactly spherical*, and that *the attractions of the sun and moon*[1] *act upon the equatorial*

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[1] The *planets*, by their action upon the plane of the earth's orbit (Art. 197), slightly disturb the equinox *in the opposite direction.* This effect amounts to about 0″.16 annually.

ring of matter which projects above the true sphere, tending to draw the plane of the equator into coincidence with the plane of the ecliptic by their greater attraction on the nearer portions of the ring. The action is just what it would be if a spheroidal ball of iron of the shape of the earth had a magnet brought near it. The magnet, as illustrated in Fig. 65, would tend to draw the plane of the equator into the line *CM* joining its pole with the centre of the globe, because it attracts the nearer portion of the equatorial protuberance at *E* more strongly than the remoter at *Q*. If it were not for the earth's rotation, this attraction would bring the two planes of the ecliptic and equator together; but since the earth is spinning on its axis, we get the same result that we do with the whirling

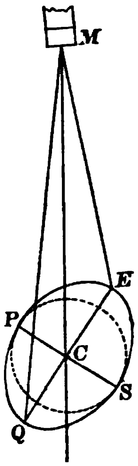

FIG. 65.
Effect of Attraction on a Spheroid.

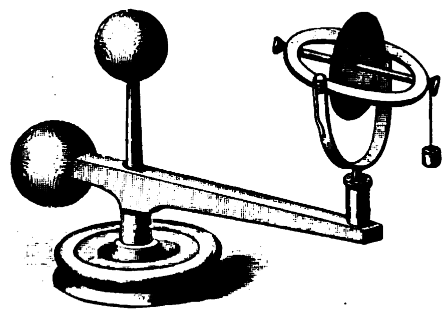

FIG. 66.
Precession illustrated by the Gyroscope.

wheel of a gyroscope by hanging a weight at one end of the axis. We then have the result of the combination of two rotations at right angles with each other, one the whirl of the wheel, the other the "tip" which the weight tends to give the axis. (Physics, pp. 53–54.)

210. In this case, if the wheel of the gyroscope is turning swiftly *clock-wise*, as seen from above (Fig. 66), the weight at the (lower) end of the axis will make the axis move slowly around, *counter-clock-wise*, without at all changing its inclination. If we regard the horizontal plane passing through the gyroscope as representing the ecliptic, and the point in the ceiling vertically above the gyroscope as the pole of the ecliptic, the line of the axis of the wheel produced upward would describe on the ceiling a circle around this imaginary ecliptic pole, acting precisely as does the pole of the earth's axis in the sky. The swifter the wheel's rotation, the slower would be this

"precessional" motion of its axis; and of course, the rate of motion also depends upon the magnitude of the suspended weight.

211. A full treatment of the subject would be too complicated for our pages. An elementary notion of the way the action takes place, correct as far as it goes, is easily obtained by reference to Fig. 67. Let XY be the axis of the gyroscope, the wheel being seen in section edge-wise, and the eye being on the same level as the centre of the wheel; the wheel turning so that the point B is coming towards the observer. Now, suppose a weight hung on the lower end of the axis. If the wheel were not turning, the point B would come to some point F in the same time it now takes to reach C (that is, after a quarter of a revolution). By combination of the two motions it will come to a point K at the end of the same time, having crossed the horizontal plane AD at L; and this can be effected only by a backward "skewing around" of the whole wheel, axis and all. This does not, of course, explain why the inclination of the axis does not change under the action of the weight, but is only a very partial illustration, showing merely why the plane of the wheel regresses. A complete discussion would require the consideration of the motion of every point on the wheel by a thorough and difficult analytical treatment.

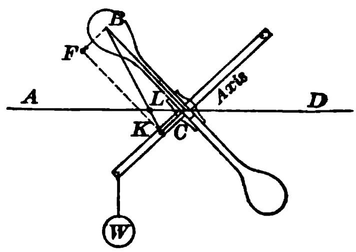

FIG. 67.
Regression of the Gyroscope Wheel.

The motions of the earth's axis, discussed in Art. 108, do not displace the celestial pole with reference to the stars, and must not be confounded with precession.

212. Why Precession is so Slow. — The slowness of the precession depends on three things: (*a*) the enormous "moment of rotation" of the earth — a point on the equator moves with the speed of a cannon ball; (*b*) the smallness of the mass (compared with that of the whole earth) of the protuberant ring to which precession is due; and (*c*) the minuteness of the force which tends to bring this ring into coincidence with the ecliptic, a force which is not constant and persistent, like the weight hung on the gyroscope axis, but very variable.

213. The Equation of the Equinox. — Whenever the sun is in the plane of the equator (which is twice a year, at the time of the equinoxes), the *sun's* precessional force disappears entirely, its attraction then having no tendency to draw the equator out of its position. The moon's action, on account of her proximity, is still more powerful than that of the sun; on the average two and a half times as great. Now, the moon crosses the celestial equator twice every month, and at those times *her* action ceases.

There is still another cause for variation in the effectiveness of the *moon's* attraction. As we shall see hereafter, she does not move in the ecliptic, but in a path which cuts the ecliptic at an angle of about 5°, at two points called the *Nodes;* the *ascending* node being the point where she crosses the ecliptic from south to north. These nodes move westward on the ecliptic (Art. 455), making the circuit once in about nineteen years. Now, when the ascending node of the moon's orbit is at B (Fig. 68), near the autumnal equinox F, its inclination to the equator will be, as the figure shows, *less* than the obliquity of the ecliptic by about 5°; *i.e.*, it will be only about 18°. On the other hand, nine and a half years later, when the node has backed around to a point A, near the vernal equinox, the inclination of the moon's orbit to the equator will be nearly 28°. When the node is in this position, the moon will produce nearly twice as much precessional movement each month as when the node was at B.

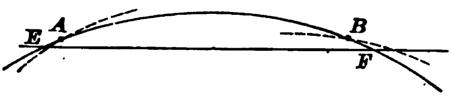

FIG. 68.
Variation in the Inclination of Moon's Orbit to Equator.

The precession, therefore, is not uniform, but variable, almost ceasing at some times and at others becoming rapid. This causes what is called the *equation of the equinox.*

214. Nutation. — Not only does the precessional force vary in amount at different times, but in most positions of the disturbing body with respect to the earth's equator there is a *slight thwartwise component of the force, tending directly to accelerate or retard* the precessional movement of the pole — just as if one should gently draw the weight W (Fig. 66) horizontally. The consequence is what is called *Nutation* or "nodding." The axis of the earth, instead of moving smoothly in a circle, nods in and out a little with respect to the pole of the ecliptic, describing a wavy curve resembling that shown in Fig. 69, but with nearly 1400 indentations in the entire circumference traversed in 26,000 years.

215. We distinguish three of these nutations. (*a*) The *Lunar Nutation*, depending upon the motion of the moon's nodes. This has a period of a little less than nineteen years, and amounts to 9″.2. (*b*) The *Solar Nutation*, due to the changing declination of the sun. Its period is a year, and its amount 1″.2. (*c*) The *Monthly Nutation*, precisely like the solar nutation, except that it is due to the *moon's* changes of declination during the month. It is, however, too small to be certainly measured, not exceeding one-tenth of a second.

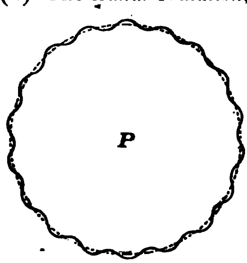

FIG. 69. — Nutation.

Nutation was detected by Bradley in 1728, but not fully explained until 1748.

Neither* precession *nor* nutation *affects the* latitudes *of the stars, since they are not due to any change in the position of the ecliptic, but only to displacements of the earth's axis. The* longitudes *alone are changed by them.

The *right ascension* and *declination* of a star are both affected.

216. The Three Kinds of Year. — In consequence of the motion of the equinoxes caused by precession, the *sidereal* year and the equinoctial or "*tropical*" year do not agree in length. Although the sidereal year is the one which represents the earth's *true* orbital revolution around the sun, it is not used as the year of chronology and the calendar, because the *seasons* depend on the sun's place in relation to the equinoxes. The tropical year is the year usually employed, unless it is expressly stated to the contrary. The length of the Sidereal year is $365^d\ 6^h\ 9^m\ 9^s$; that of the Tropical year is about 20^m less, $365^d\ 5^h\ 48^m\ 46^s$.

The third kind of year is the *anomalistic* year, which is the time from perihelion to perihelion again. As the line of apsides of the earth's orbit moves always slowly towards the east, this year is a little longer than the sidereal. Its length is $365^d\ 6^h\ 13^m\ 48^s$.

217. The Calendar. — The natural units of time are *the day*, *the month*, and *the year*. The day, however, is too short for convenient use in designating extended periods of time, as for instance in expressing the age of a man. The month meets with the same objection, and for all chronological purposes, therefore, the year is the unit practically employed. In ancient times, however, so much regard was paid to the month, and so many of the religious beliefs and observances connected themselves with the times of the new and full moon, that the early history of the calendar is largely made up of attempts to fit the month to the year in some convenient way. Since the two are incommensurable, the problem is a very difficult, and indeed strictly speaking, an impossible, one.

In the earliest times matters seem to have been wholly in the hands of the priesthood, and the calendar then was predominantly *lunar*, with months and days intercalated from time to time to keep the seasons in place. The Mohammedans still use a purely lunar calendar, having a year of twelve lunar months, and containing alternately 354 and 355 days. In their reckoning the seasons fall, of course, continually in different months, and their calendar gains about one year in thirty-three upon the reckoning of Christian nations.

218. The Metonic Cycle. — Among the Greeks the discovery of the so-called lunar or Metonic cycle by Meton, about 433 B.C., considerably simplified matters. This cycle consists of 235 *synodic months* (from new moon to new again), which is very approximately equal to 19 common years of $365\frac{1}{4}$ days.

235 months equal $6939^d\ 16^h\ 31^m$; 19 tropical years equal $6939^d\ 14^h\ 27^m$; so that at the end of the 19 years, the new and full moon recur again on the same days of the year, and at the same time of day within about two hours. The calendar of the phases of the moon, for instance, for 1889 is the same as for 1870 and 1908, except that intervening leap-years may change the dates by one day.

The "*Golden number*" of a year is its number in this Metonic cycle, and is found by adding 1 to the "date-number" of the year and dividing by 19. The remainder, unless zero, is the "golden number" (if it comes out zero, 19 is taken instead). Thus the golden number for 1888 is found by dividing 1889 by 19, and the remainder 8 is the golden number of the year.

This cycle is still employed in the ecclesiastical calendar in finding the time of Easter.

The still more accurate **Callipic cycle** consists of 76 years, or *four* Metonic cycles, and so takes account of the leap-years.

219. Julian Calendar. — Until the time of Julius Cæsar the Roman calendar seems to have been based upon the lunar year of twelve months, or 355 days, and was substantially like the modern Mohammedan calendar, with arbitrary intercalations of months and days made by the priesthood and magistrates from time to time in order to bring it into accordance with the seasons. In the later days of the Republic, the confusion had become intolerable. Cæsar, with the help of the astronomer Sosigenes, whom he called from Alexandria for the purpose, reformed the system in the year 45 B.C., introducing the so-called "Julian calendar," which is still used either in its original shape or with a very slight modification. He gave up entirely the attempt to coordinate the month with the year, and adopting $365\frac{1}{4}$ days as the true length of the tropical year, he ordained that every fourth year should contain an extra day, the *sixth day before the Kalends of March on that year being counted twice*, whence the year was called "*bissextile*." Before his time the year had begun in March (as indicated by the Roman names of the months, — September, seventh month; October, eighth month, etc.), but he ordered it to begin on the 1st of January, which in that year (45 B.C.) was on the day of the new moon next following the winter solstice. In introducing the change it was necessary to make the preceding year 445 days long, and it is still known in

the annals as "the year of confusion." He also altered the name of the month Quintilis, calling it "July" after himself.

There was some irregularity in the bissextile years for a few years after Cæsar's death, from a misunderstanding of his rule for the intercalary day: but his successor Augustus remedied that, and to put himself on the same level with his predecessor, he took possession of the month Sextilis, calling it "August"; and to make its length as great as that of July, he robbed February of a day.

From that time on, the Julian calendar continued unbrokenly in use until 1582; and it is still the calendar of Russia and of the Greek Church.

220. The Gregorian Calendar. — The Julian calendar is not quite correct. The true length of the tropical year is 365 days 5 hours 48 minutes and 46 seconds, and this leaves a difference of 11 minutes and 14 seconds by which the Julian calendar year is the longer, being exactly $365\frac{1}{4}$ days. As a consequence, the date of the equinox comes gradually earlier and earlier by about three days in 400 years. ($400 \times 11\frac{7}{30}^{m} = 4493$ minutes $= 3^{d}\ 2^{h}\ 53^{m}$.) In the year 1582, the date of the vernal equinox had fallen back 10 days to the 11th of March, instead of occurring on the 21st of March, as at the time of the Council at Nice, 325 A.D. Pope Gregory, therefore, acting under the advice of the Jesuit astronomer, Clavius, ordered that the day following October 4 in the year 1582 should be called not the 5th, but the 15th, and that the rule for leap-year should be slightly changed so as to prevent any such future displacement of the equinox. The rule now stands: *All years whose date-number is divisible by four without a remainder are leap-years, unless they are century years* (1700, 1800, etc.). *The century years are not leap-years unless their date-number is divisible by* 400, *in which case they are:* that is, 1700, 1800, and 1900 are not leap years; but 1600, 2000, and 2400 are.

221. Adoption of the New Calendar. — The change was immediately adopted by all Catholic nations; but the Greek Church and most of the Protestant nations, rejecting the Pope's authority, declined to accept the correction. In England it was at last adopted in the year 1752, at which time there was a difference of eleven days between the two calendars. (The year 1600 was a leap-year according to the Gregorian system as well as the Julian, but 1700 was not.) Parliament in 1751 enacted that the day following the 2d of September, in the year 1752, should be called the 14th instead of the 3d; and also that this year (1752), and all subsequent years, should begin on the first of January.

The change was made under very great opposition, and there were violent riots in consequence in different parts of the country, especially at Bristol, where several persons were killed. The cry of the populace was, "Give us back our fortnight," for they supposed they had been robbed of eleven days, although the act of Parliament was carefully framed to prevent any injustice in the collection of interest, payment of rents, etc.

At present, since the year 1800 was not a leap-year according to the Gregorian calendar, while it was so according to the Julian, the difference between the two calendars amounts to twelve days; thus in Russia the 19th of August would be reckoned as the 7th. In Russia, however, for scientific and commercial purposes *both* dates are very generally used, so that the date mentioned would be written August $\frac{7}{19}$. When Alaska was annexed to the United States, its calendar had to be altered by *eleven* days. (See Art. 123.)

After February 28, 1900 (Gregorian), the difference will become *thirteen* days, and will remain so until A.D. 2100.

222. The Beginning of the Year. — The beginning of the year has been at several different dates in the different countries of Europe. Some have regarded it as beginning at Christmas, the 25th of December; others, on the 1st of January; others still, on the 1st of March; others, on the 25th; and others still, at Easter, which may fall on any day between the 22d of March and the 25th of April.

In England previous to the year 1752 the legal year commenced on the 25th of March, so that when the change was made, the year 1751 necessarily lost its months of January and February, and the first twenty-four days of March. Many were slow to adopt this change, and it becomes necessary, therefore, to use considerable care with respect to English dates which occur in the months of January, February, or March about that period. The month of February, 1755, for instance, would by some writers be reckoned as occurring in 1754. Confusion is best avoided by writing February $\frac{1754}{1755}$.

The so-called **Fictitious Year,** used in the reduction of star-places (Art. 797), begins at the moment when the sun's *mean longitude* is 280°. This always occurs sometime during the 31st of December.

223. First and Last Days of the Year. — Since the ordinary civil year consists of 365 days, which is 52 weeks and one day, the last day of each common year falls on the same day as the first; so that any given date will fall one day later in the week than it did on the preceding year, unless a 29th of February has intervened, in which case it will be *two* days later; that is, if the 3d of January, 1889, falls on Thursday, the same date in 1890 will fall on Friday.

223*. Julian Period and Julian Epoch. — The Julian Period consists of 7980, (28 × 19 × 15), Julian years, each containing exactly 365¼ days, and its starting-point or "Epoch" is January 1, 4713 B.C.,

the Julian date of January 1, 1 A.D., being J.E. 4714. It was proposed by J. Scaliger in 1582 as a universal harmonizer of the different systems of chronological reckoning then in use, and its adoption has brought order out of confusion. It is extensively employed in astronomical calculations, the date of any phenomenon being expressed beyond all ambiguity either by the (Julian) year and day, or still more simply, as day number so and so of the Julian era. Thus the date of the solar eclipse of August 9, 1896, is J.E., year 6609, 222d day, or simply *Julian-day* 2 413781, and this is perfectly definite to every astronomer, whether he be American, Russian, Hebrew, Mohammedan, or Chinese.

The number of days between any two events, even centuries apart, is at once found by merely taking the difference between their Julian-day numbers.

The Nautical Almanacs give the Julian year and the Julian day corresponding to January 1 of each year. Thus

1896 =	Julian	Year 6609.	January	1, 1896 =	Julian	Day	2 413560.
1897 =	"	" 6610.	"	1, 1897 =	"	"	2 413926.
1898 =	"	" 6611.	"	1, 1898 =	"	"	2 414291.
1899 =	"	" 6612.	"	1, 1899 =	"	"	2 414656.

224. Aberration. — Although in strictness the discussion of aberration does not belong to a chapter describing the earth and its motions, yet since it is a phenomenon due to the earth's motion, and affects the right ascension and declination of the stars in much the same way as do precession and nutation, it may properly enough be considered here.

Aberration is the apparent displacement of a star, due to the combination of the motion of light with the motion of the observer.

The direction in which we have to point our telescope in observing a star is not the same that it would be if the earth were at rest. It lies beyond our scope to show that according to the wave theory of light the apparent direction of a ray will be affected by the observer's motion precisely in the same way (within very narrow limits) as it would be if light consisted of corpuscles shot off from a luminous body, as Newton supposed. This is the case, however, as Doppler and others have shown; and assuming it, the explanation of aberration is easy: Suppose an observer standing at

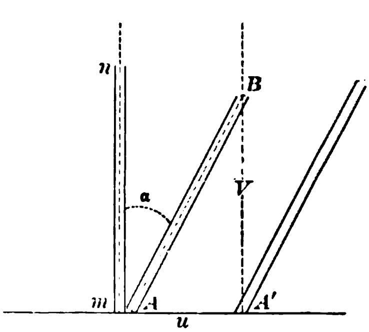

FIG. 70. — Aberration of a Raindrop.

rest with a tube in his hand in a shower of rain where the drops are falling vertically. If he wishes to have the drops descend axially through the tube without touching the sides, he must of course keep it vertical; but if he advances in any direction, he must draw back the bottom of the tube by an amount which equals the advance he makes in the time while the drop is falling through the tube, so that when the drop falling from B reaches A', the bottom of the tube will be there also; *i.e., he must incline the tube forward by an angle a, such that* $\tan a = u \div V$, where V is the velocity of the raindrop and u that of his own motion. In Fig. 70 $BA' = V$ and $AA' = u$.

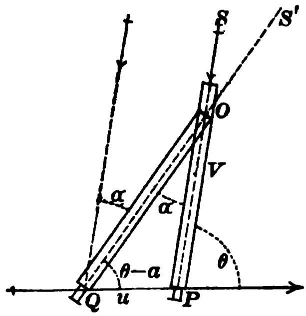

FIG. 71. — Aberration of Light.

225. Now take the more general case: Suppose a star sending us light with a velocity V in the direction SP, Fig. 71, which makes the angle θ with the line of the observer's motion. He himself is carried by the earth's orbital velocity in the direction QP. In pointing the telescope so that the light may pass exactly along its optical axis, he will have to draw back the eye-end by an amount QP, which just equals the distance he is carried by the earth's motion during the time that the light moves from O to P. The star will thus apparently be displaced towards the point towards which he is moving, the angle of displacement POQ, or a, being determined by the relative length and direction of the two sides OP and QP of the triangle OPQ. These sides are respectively proportional to the velocity of light, V, and the orbital velocity of the earth, u.

The angle at P being θ, the angle OQP will be $(\theta - a)$, and we shall have from trigonometry the proportion $\sin a : \sin(\theta - a) = u : V$.

To find a from this, develop the second term of the proportion and divide the first two terms by $\sin a$, which gives us

$$1 : \sin\theta \cot a - \cos\theta = u : V,$$

whence

$$u \sin\theta \cot a = V + u\cos\theta,$$

and

$$\cot a = \frac{V + u\cos\theta}{u \sin\theta}.$$

Taking the reciprocal of this, we have

$$\tan a = \frac{u}{V + u\cos\theta}\sin\theta.$$

The second term in the denominator is insensible, since u is only about one ten-thousandth of V, so that we may neglect it.[1] This gives the formula in the shape in which it ordinarily appears, *viz.*,

$$\tan \alpha = \frac{u}{V} \sin \theta.$$

The value of α (denoted by α_0) which obtains when $\theta = 90°$ and $\sin \theta =$ unity, is called the *Constant of Aberration.*

The value of this constant, adopted by the Astronomical Conference at Paris in 1896, is **20.47**, but is still uncertain to the amount of 0″.01 or 0″.02. Aberration was discovered and explained by Bradley, in 1726.

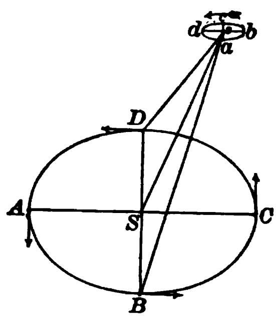

FIG. 72.
Aberrational Orbit of a Star.

226. The Effect of Aberration upon the Apparent Places of the Stars. — As the earth moves in an orbit nearly circular, and with a velocity so nearly uniform that we may for our present purpose disregard its variations, it is clear that a star at the pole of the ecliptic will be always displaced by the same amount of 20″.5, but in a direction continually changing. It must, therefore, appear to describe a little circle 41″ in diameter during the year, as shown in Fig. 72. Now the direction of the earth's orbital motion is always in the plane of the ecliptic, and towards the right hand as we stand facing the sun. At the vernal equinox, therefore, we are moving toward the point of the ecliptic, which is 90° *west of the sun, i.e.*, towards the winter solstitial point, and the star is then displaced in that direction. Three months later the star will be displaced in a line directed towards the vernal equinox, and so on. The earth, therefore, so to speak, *drives the star before it* in the aberrational orbit, keeping it just a quarter of a revolution ahead of itself.

A star on the ecliptic simply appears to oscillate back and forth in a straight line 41″ long.

Generally, in any latitude whatever, the aberrational orbit is an ellipse, having its major axis parallel to the ecliptic, and always 41″

[1] The velocity of light, according to the latest determinations of Newcomb and Michelson, is 299860 kilometers ± 30 kilometers (which equals 186330 miles ± 20 miles). The mean velocity of the earth in its orbit, if we assume the solar parallax to be 8″.8, is 29.77 kilometers, or 18.50 miles; this makes the constant of aberration 20″.478, a little larger than that given in the text.

long, while its minor axis is 41″ × sin β. β being the star's latitude, or distance from the ecliptic.

It is worth noting that since the "hodograph"[1] of the earth's orbital motion, as shown by Hamilton, is an exact circle, the aberrational orbit is ~~an exact ellipse~~, notwithstanding the eccentricity of the earth's orbit, and the variations in its velocity; but the *mean place* of the star (*i.e.*, the place it would occupy if there were no aberration) is not exactly in the centre of this ~~ellipse.~~

226*. Diurnal Aberration. — The motion of an observer due to the earth's *rotation* also produces a slight effect known as the *diurnal aberration.* Its "constant" is only 0″.31 for an observer situated at the equator; anywhere else it is 0″.31 cos ϕ, ϕ being the latitude of the observer.

For any given star it is a maximum when the star is crossing the meridian, and then its whole effect is slightly to *increase the right ascension* by an amount given by the formula, $\Delta\alpha = 0''.31 \cos\phi \sec\delta$, δ being the star's declination.

Exercises on Chapter VI.

1. What is the meridian altitude of the sun at Princeton (Lat. 40° 21′) on the day of the summer solstice?

2. What is the sun's approximate right ascension at that time?

3. On what days during the year will the sun's right ascension be approximately an *even* hour (*i.e.*, 0 h. 2 h. 4 h., etc.)?

4. On what days will it be an *odd* hour?

5. What is the approximate sidereal time at 10 P.M. on May 12?

Ans. 13 h. 26 min.

6. At what time will Arcturus (R. A. = 14 h. 10 min.) come to the meridian on August 1?

Ans. About 5 h. 26 min. P.M.

7. About what time of night is Mizar (R. A. = 13 h. 20 min.) vertically under the pole on October 10?

Ans. Midnight.

8. In what latitude has the sun a meridian altitude of 80° on June 21?

Ans. + 33° 28′.

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[1] The hodograph of an orbit is a curve in which the *direction* of the radius vector at each point is the direction of the orbital motion at the corresponding point of the orbit, and its *length* is proportional to the velocity.

9. What are the longitude and latitude (celestial) of the north celestial pole?

10. What are the right ascension and declination of the north pole of the ecliptic?

11. What are the greatest and least angles made by the ecliptic with the horizon at New York (Lat. 40° 43′)?

Ans. $(90° - 40° 43') \pm 23° 28'$.

12. Does the vernal equinox always occur on the same day of the month? If not, why not? And how much can the date vary?

13. Will the ephemeris of the sun for one year be correct for every other year, and, if not, how much can it be in error?

Ans. A difference of one and three-quarters days motion of the sun is possible; as, for instance, between 1897 and 1903, the leap-year being omitted in 1900.

14. When the sun is in the sign of Cancer in what constellation is he?

15. What obliquity of the ecliptic would reduce the width of the temperate zone to zero?

16. How long before the earth will be in perihelion on July 1, instead of January 1 as at present? (See Arts. 199, 206 and 216.)

17. What will be the Russian date corresponding to February 28, 1900, in our calendar? What corresponding to May 1 of the same year?

18. When the equation of time is minus 16 min., as it is on November 1, how does the forenoon from sunrise till 12 o'clock compare in length with the afternoon from 12 o'clock till sunset?

19. Why do the afternoons begin to lengthen about December 8, a fortnight before the winter solstice?

20. There were five Sundays in February, 1880. The same thing has not occurred since, and will not until — when?

Ans. 1920.

NOTE TO ART. 182.

The difference between the radius-vector of the ellipse, and that of the eccentric circle proposed by Hipparchus for the orbit of the earth, is so small that the method given in the text would not practically suffice to discriminate between them. But the investigation of Newton (Arts. 421 and 1006) shows that the orbit *must* be elliptical like that of the other planets.

CHAPTER VII.

THE MOON: HER ORBITAL MOTION AND VARIOUS KINDS OF MONTH. — DISTANCE AND DIMENSIONS, MASS, DENSITY, AND GRAVITY. — ROTATION AND LIBRATIONS. — PHASES. — LIGHT AND HEAT. — PHYSICAL CONDITION AND INFLUENCES EXERTED ON THE EARTH. — TELESCOPIC ASPECT. — SURFACE AND POSSIBLE CHANGES UPON IT.

227. We pass next to a consideration of our nearest neighbor in the celestial spaces, the moon, which is a satellite of the earth and accompanies us in our annual motion around the sun. She is much smaller than the earth, and, compared with most of the other heavenly bodies, a very insignificant affair; but her proximity makes her far more important to us than any of them except the sun. The very beginnings of Astronomy seem to have originated in the study of her motions and in the different phenomena which she causes, such as the eclipses and tides; and in the development of modern theoretical astronomy the lunar theory with the problems it raises has been perhaps the most fertile field of invention and discovery.

228. Apparent Motion of the Moon.—Even superficial observation shows that the moon moves eastward[1] among the stars every night, completing her revolution from *star to star* again in about 27¼ days. In other words, she revolves around the earth in that time; or, more strictly speaking, they both revolve about their common centre of gravity. But the moon is so much smaller than the earth that this centre of gravity is situated within the ball of the earth on the line joining the centres of the two bodies at a point about 1100 miles below its surface.

As the moon moves eastward so much faster than the sun, which takes a year to complete its circuit, she every now and then, at the time of the new moon, overtakes and passes the sun; and as the phases of the moon depend upon her position with reference to the sun, this interval from new moon to new moon is what we ordinarily understand as the month.

[1] She moves in declination, also, — alternately north and south (Art. 234).

229. Sidereal and Synodic Revolutions. — *The* Sidereal *revolution of the moon is the time occupied in passing from a star to the same star again,* as the name implies. It averages $27^d\ 7^h\ 43^m\ 11^s.55 \pm 0^s.03$, or $27^d.32166$, but varies as much as 3 hours on account of the eccentricity of its orbit and "perturbations."

The moon's daily motion among the stars equals $360° \div 27.32166$, or $13°\ 11'$ (nearly).

The Synodic *revolution is the interval from new moon to new moon again,* or from full to full. It varies about 13^h on account of the eccentricity of the moon's orbit and of that of the earth around the sun, but its *mean* value is $29^d\ 12^h\ 44^m\ 2^s.86 \pm 0^s.03$, or $29^d.53059$; and *this is the ordinary month.* (The word synodic is derived from the Greek σύν and ὁδός, and has nothing to do with the *nodes* of the moon's orbit. The word is *syn-odic*, not *sy-nodic.*)

A synodical revolution is longer than the sidereal, because during each sidereal month of 27.3 days the sun has advanced among the stars, and must be overtaken.

230. Elongation, Syzygy, etc. — The angular distance of the moon from the sun is called its *Elongation.* At new moon it is zero, and the moon is then said to be in "*Conjunction.*" At full moon it is 180°, and the moon is then in "*Opposition.*" In either case the moon is said to be in "*Syzygy*" (σύν ζυγόν). When the elongation is 90°, as at the half-moon, the moon is in "*Quadrature.*"

231. Determination of the Moon's Sidereal Period. — This is effected directly by observations of the moon's right ascension and declination (with the meridian circle), kept up systematically for a sufficient time.

If it were not for the so-called "secular acceleration" of the moon's motion (Arts. 459–461), an exceedingly accurate determination of the moon's *synodic* period could be obtained by comparing ancient eclipses with modern.

The earliest authentically recorded eclipse is one that was observed at Nineveh in the year 763 B.C. between 9 and 10 o'clock on the morning of June 15th.

By comparing this eclipse with (say) the eclipse of August, 1887, we have an interval of more than 30000 months, and so an error of ten hours even, in the observed time of the Nineveh eclipse, would make only about one second in the length of the month. But the month is a little shorter now than it was 2000 years ago.

232. Relation of Sidereal and Synodic Periods. — The fraction of a revolution described by the moon in one day equals $\frac{1}{M}$, M being the length of the *sidereal* month. In the same way $\frac{1}{E}$ represents the earth's daily motion in its orbit, E being the length of the year. The difference of these two equals the fraction of a revolution which the moon *gains* on the sun during one day. In a synodic month, S, it gains one whole revolution, and therefore must gain each day $\frac{1}{S}$ of a revolution; so that we have the equation

$$\frac{1}{M} - \frac{1}{E} = \frac{1}{S};$$

or, substituting the numerical values of E and S,

$$\frac{1}{M} - \frac{1}{365.25635} = \frac{1}{29.53059},$$

whence we derive the value of M.

Another way of looking at it is this: In a year there must be *exactly one more* sidereal revolution than there are synodic revolutions, because the sun completes one entire circuit in that time. Now the number of synodic revolutions in a year is given by the fraction

$$\frac{365\frac{1}{4}}{S} = 12.369 +.$$

There will therefore be 13.369 sidereal revolutions in the year, and the length of one sidereal revolution equals $365\frac{1}{4}$ days divided by this number 13.369, which will be found to give the length of the sidereal revolution as before.

233. Moon's Path among the Stars. — By observing with the meridian circle the right ascension and declination of the moon daily during the month, just as in the case of the sun, we obtain the position of the moon for each day, and joining the points thus found, we can draw the path of the moon in the sky. It is a great circle, cutting the ecliptic in two points called the *nodes*, at an angle which ranges from 4° 57′ to 5° 20′, the mean being about 5° 08′.

We say the path is found to be a great circle. This must be taken with some reservation, since at the end of the month the moon never returns *precisely* to the position it occupied at the beginning, owing

to the regression of the nodes and other so-called "perturbations," which will be discussed hereafter.

234. Moon's Meridian Altitude. — Since the moon's orbit is inclined to the ecliptic 5° 8′, its inclination to the equator varies from 28° 36′ (23° 28′ + 5° 8′), when the moon's ascending node is the vernal equinox, to 18° 20′, when, 9½ years later, the same node is at the autumnal equinox. In the first case the moon's declination will change during the month by 57° 12′, from − 28° 36′ to + 28° 36′. In the other case it will change only by 36° 40′. Since the range of the moon's meridian altitude is the same as that of its declination, the difference becomes striking.

235. Interval between Moon's Transits. — On the average the moon gains 12° 11′.4 on the sun daily, so that it comes to the meridian about 51 minutes of solar time later each day.

To find the mean interval between the successive transits of the moon we may use the proportion

$$(360° - 12°\ 11'.4) : 360° = 24^h : x;\ \text{whence}\ x = 24^h\ 50^m.6.$$

The variations of the moon's motion in right ascension, which are very considerable (much greater than in the case of the sun), cause this interval to vary from $24^h\ 38^m$ to $25^h\ 06^m$.

236. The Daily Retardation of the Moon's Rising and Setting. — The *average* daily retardation of the moon's rising and setting is, of course, the same as that of her passage across the meridian, viz., 51^m; but the actual retardation of rising is subject to very much greater variations than those of the meridian passage, being affected by the moon's changes in declination as well as by the inequalities of her motion in right ascension. When the moon is very far north, having her maximum declination of 28° 36′, she will rise in our latitudes much earlier than when she is farther south, though having the same right ascension.

In the latitude of New York the least possible daily retardation of moon-rise is 23 minutes, and the greatest is 1 hour and 17 minutes. In higher latitudes the variation is greater yet.

237. Harvest and Hunter's Moons. — The variations in the retardation of the moon's rising attract most attention when they occur at the time of the full moon. When the retardation is at its minimum, the moon rises soon after sunset at nearly the same time for several successive evenings; whereas, when the retardation is greatest, the moon appears to plunge nearly

vertically below the horizon by her daily motion. When the full moon occurs at the time of the autumnal equinox, the moon itself will be near the first of Aries.

Now, as will be seen by reference to Fig. 73, the portion of the ecliptic near the first of Aries makes a much smaller angle with the eastern horizon than the equator.

[The line *HN* is the horizon, *E* being the east point — the figure being drawn to represent a celestial globe, as if the observer were looking at the eastern side of the celestial sphere *from the outside.*]

EQ is the equator. Now, when the autumnal equinoctial point or first of Libra is on the horizon at *E*, the position of the ecliptic will be that represented by *ED*; more steeply inclined to the horizon than *EQ* is, by the angle *QED*, 23½°. But when the first of Aries is at *E*, the ecliptic will be in

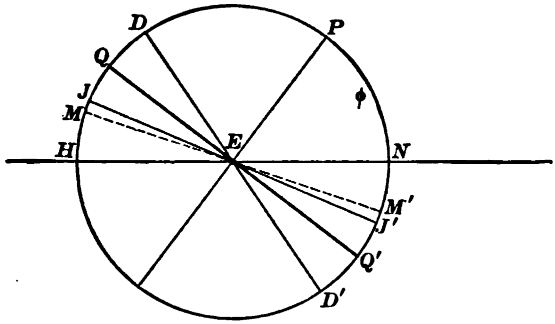

FIG. 73. — Explanation of the Harvest Moon.

the position *JJ'*. And if the ascending node of the moon's orbit happens then to be near the first of Aries, the moon's path will be *MM'*.

Accordingly, when the moon is in Aries, it, so to speak, coasts along the eastern horizon from night to night, its time of rising not varying very much; and this, when it occurs near the full of the moon, gives rise to the phenomenon known as the harvest moon, the *harvest moon being the full moon nearest to the autumnal equinox.* The full moon next following is called the *hunter's* moon.

In Norway and Sweden, under these circumstances, the moon's orbit may actually coincide with the horizon, so that she will rise at absolutely the same time for a considerable number of successive evenings.

238. The Moon's Orbit. — As in the case of the sun, the observation of the moon's path in the sky gives no information as to the real size of its orbit; but its *form* may be found by measuring the apparent diameter of the moon, which ranges from 33′ 30″ to 29′ 21″ at different points. The orbit turns out to be an ellipse like the orbit of the earth, but with an eccentricity more than three times as great

— about $\frac{1}{18}$ on the average, but varying from $\frac{1}{14}$ to $\frac{1}{22}$ on account of perturbations.

The extremities of the major axis of the moon's orbit are called the *perigee* and *apogee* (from περὶ γῇ and ἀπὸ γῇ).

The line of apsides, which passes through these two points, moves around towards the east once in about nine years, also on account of perturbations.

239. Distance and Parallax of the Moon. — These can be found in several ways, of which the simplest is the following: At two observatories B and C (Fig. 74) on, or very nearly on, the same meridian and very far apart (in the northern and southern hemispheres if possible; Greenwich and the Cape of Good Hope, for instance) let the moon's zenith distance ZBM and $Z'CM$ be observed simultaneously with the meridian circle. This gives[1] in the quadrilateral $BOCM$ the two angles at B and C, each of which is the supplement of the geocentric zenith distance. The angle at the centre of the earth, BOC, is the difference of the geocentric latitudes and is known from the geographical positions of the two observatories. Knowing the three angles in the quadrilateral, the fourth at M is of course known. The sides BO and CO are known, being radii of the earth; so that we can solve the whole quadrilateral by a simple trigonometrical process.

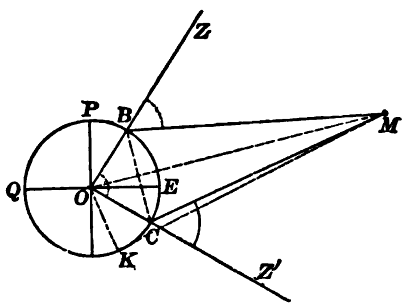

FIG. 74. — Determination of the Moon's Distance.

First find from the triangle BOC the partial angles OCB and OBC, and the side BC. Then in the triangle BCM we have BC and the two angles CBM and MCB, from which we can find the two sides BM and CM. Finally, in the triangle OBM, we now know the sides OB and BM and the included angle OBM, so that the side OM can be computed, which is the distance of the moon from the earth's centre. Knowing this, the horizontal parallax KMO, or the semi-diameter of the earth as seen from the moon, follows at once from the right-anlged triangle OKM.

The moon's parallax can also be deduced from observations at a single station on the earth, but not so simply. If she did not move among the stars, it would be very easy, as all we should have to do would be to compare her apparent right ascension and declination at different points in her diur-

[1] By correcting the observed zenith-distance for the angle of the vertical (Art. 156).

nal circle. Near the eastern horizon the parallax (always depressing an object) increases her right ascension; at setting, *vice versa*. On the meridian the declination only is affected. But the motion of the moon must be allowed for, as the observations to be compared are necessarily separated by considerable intervals of time, and this complicates the calculation.

A third, and a very accurate, method is by means of occultations of stars, observed at widely separated points on the earth. These occultations furnish the moon's place with great accuracy, and so determine the parallax very precisely; but the calculation is not very simple, as the moon's motion in this case also enters into it, since the observations cannot be simultaneous.

240. The Distance of the Moon is continually changing on account of the eccentricity of its orbit, varying all the way, according to Neison, between 252,972 and 221,614 miles; the *mean* distance being 238,840 miles, or 60.27 times the equatorial radius of the earth. The mean parallax of the moon is 57′ 2″, subject to a similar percentage of change. This value of the parallax, it will be noted, indicates that the earth, as seen from the moon, has a diameter of nearly 2°.

Knowing the size of the moon's orbit and the length of the month, the velocity of her motion around the earth is easily calculated. It comes out 2288 miles per hour, or about 3350 feet a second.

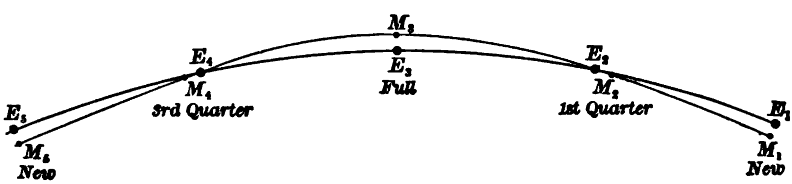

FIG. 75. — Moon's Path with Reference to the Sun.

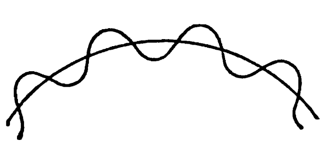

FIG. 76.

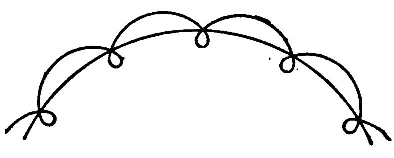

FIG. 77.

False Representations of Moon's Motions.

241. Form of the Moon's Orbit with Reference to the Sun.— While the moon moves in a small elliptical orbit around the earth, it also moves around the sun in company with the earth. This *common*

motion of the moon and earth, of course, does not affect their *relative* motion; but to an observer outside the system the moon's motion around the earth would only be a very small component of the moon's movement as seen by *him*.

The distance of the moon from the earth, 239,000 miles, is very small compared with that of the earth from the sun, 93,000,000 miles — being only about $\frac{1}{400}$ part. The speed of the earth in its orbit around the sun is also more than thirty times faster than that of the moon in its orbit around the earth, so that for the moon the resulting path in space is one which is always *concave towards the sun*, as shown in Fig. 75. It is *not* like Figs. 76 and 77, as often represented. If we represent the orbit of the earth by a circle with a radius of 100 inches (8 feet 4 inches), the moon would only move out and in a quarter of an inch, crossing the circumference twenty-five times in going once around it.

242. Diameter of the Moon. — The mean apparent diameter of the moon is 31′ 7″. This gives it a real diameter of 2163 miles (plus or minus one mile), which equals 0.273 of the earth's diameter. Since the surfaces of globes are as the squares of their diameters, and their volumes as their cubes, this makes the surface of the moon 0.0747 of the earth's (between $\frac{1}{13}$ and $\frac{1}{14}$); and the volume 0.0204 of the earth's volume (almost exactly $\frac{1}{49}$); that is, it would take 49 balls each as large as the moon in bulk to make a ball of the size of the earth.

243. Mass of the Moon. — This is about $\frac{1}{80}$ of the earth's mass, different authorities giving the value from $\frac{1}{78}$ to $\frac{1}{83}$. It is not easy to determine it with accuracy. In fact, though the moon is the nearest of all the heavenly bodies, it is more difficult to "weigh" her than to weigh Neptune, although he is the most remote of the planets.

There are four ways of approaching the problem: (1) (perhaps easiest to understand) *by finding the position of the common centre of gravity of the earth and moon with reference to the centre of the earth.* Since it is this *common centre of gravity* of the two bodies which describes around the sun the ellipse which we have called the earth's orbit, and since the earth and moon revolve around this common centre of gravity once a month, it follows that this monthly motion of the earth causes an alternate eastward and westward displacement of the sun in the sky, which can be measured. At the time of the new and full moon this displacement is zero, the centre of gravity being on the line which joins the earth and sun;

but when the moon is at *quadrature* (that is, 90° from the sun, as at the time of half-moon), the sun is apparently displaced in the sky *towards the moon*, as is evident from Fig. 78. It will be about 6″.4 east of its mean place at the first quarter of the moon, Fig. 78 (*B*), and as much west at the time of the last quarter, Fig. 78 (*A*); (*i.e.*, when the angle *MGS* is 90°, the angle *MCS* is always *less* than 90° by 6″.4, which is therefore the value of the angle *CSG*). Now since the parallax of the sun (which is the earth's semi-diameter seen from the sun — the angle *CSK*) is about 8″.8, it follows that the distance of the centre of gravity of the earth and moon from the centre of the earth is the fraction $\frac{64}{88}$ of the earth radius, or about 2880 miles. This is just about $\frac{1}{82.5}$ of the distance from the earth to the moon; whence we conclude that the mass of the earth is 81.5 times that of the moon.

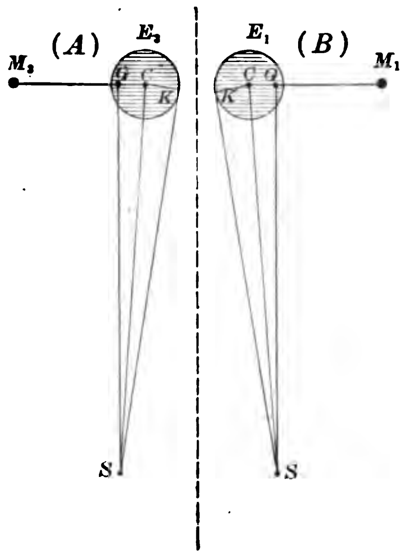

FIG. 78.
Apparent Displacement of Sun at First and Third Quarters of the Month.

244. (2) A second method is by comparing the moon's *actual period* with the *computed period which a single particle at the moon's distance from the earth ought to have*, according to the known force of gravity of the earth, as determined by pendulum experiments. The explanation of this method cannot be given until we have further studied the motion of bodies under the law of gravitation. It is not susceptible of great accuracy.

(3) Still another method is by comparing the *tides produced by the moon with those produced by the sun*. This gives us the mass of the moon as compared with that of the sun; and the mass of the sun compared with that of the earth being known, it gives us ultimately the mass of the moon compared with that of the earth.

(4) The ratio of the moon's mass to the sun's can also be computed from the *nutation* of the earth's axis. (See Chap. XIII.)

245. No other satellite is nearly as large as the moon, in comparison with its primary planet. The earth and moon together, as seen from a dis-

tant star, are really in many respects more like a *double planet* than like a planet and satellite, as ordinarily proportioned to each other. At a time, for instance, when Venus happens to be near the earth, at a distance of about twenty-five millions of miles, the earth to her would appear about twice as bright as Venus at her best does to us; and the moon would be about as bright as Sirius, at a distance of about half a degree from the earth.

246. Density and Superficial Gravity of the Moon. — Since the density of a body is equal to $\frac{\text{mass}}{\text{volume}}$, the density of the moon as compared with that of the earth is found from the fraction

$$\frac{\frac{1}{81}}{\frac{1}{49}}, \text{ or } \frac{0.0124}{0.0204}.$$

This makes the moon's density 0.61 of the earth's density, or about $3\frac{4}{10}$ the density of water — somewhat above the average density of the rocks which compose the crust of the earth.

This small density of the moon is not surprising, nor at all inconsistent with the belief that it once formed part of the same mass with the earth, since if such were the case, the moon was probably formed by the separation of the *outer portions* of that mass, which would be likely to have a smaller specific gravity than the rest.

247. *The superficial gravity*, or the attraction of the moon for bodies at its own surface, may be found by the equation

$$g' = g \times \frac{m}{r^2},$$

in which g' signifies the superficial gravity of the moon, g is the force of gravity of the earth, while m and r are the mass and radius of the moon as compared with those of the earth. This gives us

$$g' = g \times \frac{0.0124}{0.0747},$$

or (very approximately) g' equals *one-sixth* of g; that is, a body which weighs six pounds on the earth's surface would at the surface of the moon weigh only one (in a spring balance). A man on the moon could jump six times as high as he could on the earth and could throw

a stone six times as far. This is a fact to be remembered in connection with the enormous scale of the surface-structure of the moon. Volcanic forces, for instance, upon the moon would throw the ejected materials to a vastly greater distance there than on the earth.

248. Rotation of the Moon.—The moon rotates on its axis once a month, in precisely the same time as that occupied by its revolution around the earth. In the long run it therefore keeps the same side always towards the earth: we see to-day precisely the same face and aspect of the moon as Galileo did when he first looked at it with his telescope, and the same will continue to be the case for thousands of years more, if not forever.

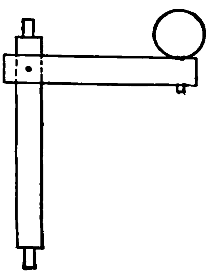

FIG. 79.

It is difficult for some to see why a motion of this sort should be considered a *rotation* of the moon, since it is essentially like the motion of a ball carried on a revolving crank. See Fig. 79. Such a ball, they say, "*revolves* around the shaft, but does not *rotate* on its own axis." It does rotate, however. The shaft being vertical and the crank horizontal, suppose that a compass needle be substituted for the ball, as in Fig. 80. The pivot turns underneath it as the crank whirls, but the compass needle does not rotate, maintaining always its own direction with the marked end north. On the other hand, if we mark one side of the ball (in the preceding figure), we shall find the marked side presented successively to every point of the compass as the crank revolves, so that the ball as really turns on its own axis as if it were whirling upon a pin fastened to a table. The ball has *two* distinct motions by virtue of its connection with the crank: *first*, the motion of translation, which carries its centre of gravity, like that of the compass needle, in a circle around the axis of the shaft; *secondly*, an additional motion of rotation around a line drawn through its centre of gravity parallel to the shaft.

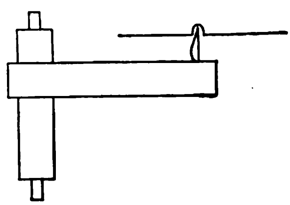

FIG. 80.

248*. Definition of Rotation.—A body "*rotates*" whenever *a line drawn from its centre of gravity outward, through any point selected at random in its mass, describes a circle in the heavens.* In every rotating body, one such line can be so drawn that the circle described by it in the sky becomes infinitely small. This is the *axis* of the body. Another set of points can be found such that lines drawn from the centre of gravity outward through

them describe a great circle in the sky 90° distant from the point pierced by the axis, and these points constitute the *equator* of the body.

249. Librations of the Moon. — 1. *Libration in Latitude.* The axis of revolution of the moon is not perpendicular to its orbit. It makes a constant angle of about 88½° with the ecliptic, and the moon's equator is so placed that it is always edge-wise to the earth when the moon is at her node, being maintained in that position by an action of the earth, which produces a precessional motion of the moon's axis. The angle between the moon's equator and the plane of her orbit, therefore, is 1½° + the inclination of the moon's orbit, which together make up an angle of a little more than 6½°; but, as the inclination of the moon's orbit to the ecliptic is constantly varying slightly, this inclination of the moon's axis to her orbit also changes correspondingly. This inclination of the moon's axis produces changes in the aspect of the moon towards the earth similar to those produced by the inclination of the earth's axis towards the ecliptic. At one time, just as the north pole of the earth is turned towards the sun, so also the north pole of the moon is tipped towards the earth at an angle of 6½°, and in the opposite half of the moon's orbit the south pole is similarly presented to us.

The period of this libration is the time of the moon's revolution from node to node, called a *nodical revolution.* This is 27.21 days — about 2 hours and 38 minutes shorter than the sidereal revolution of the moon, since the nodes always move westward, completing the circuit in about 19 years.

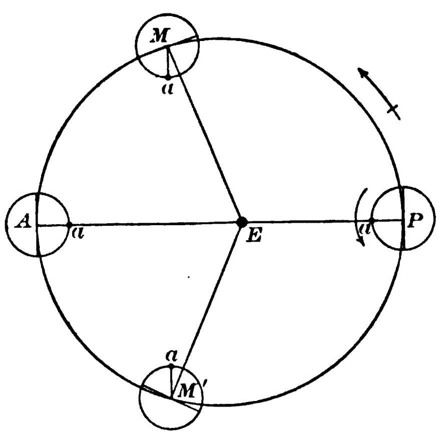

FIG. 81. — The Libration in Longitude.

250. 2. *Libration in Longitude.* The moon's orbit being eccentric, she moves faster when near perigee, and slower when near apogee; half-way between perigee and apogee she is more than 6° ahead of the position she would have if she had moved with the *mean* angular velocity. Now the rotation is *uniform.*[1] A point, therefore, on the moon's

[1] Very nearly; a minute "physical libration" of about 3½′ affects it slightly.

surface which is directed toward the earth at perigee will not have revolved far enough to keep it directed toward the earth when she is half-way (*in time*) between perigee and apogee, as is evident from Fig. 81. For in the quarter-month next following the perigee, the moon will travel to a point *M*, considerably more than half-way to apogee. But the point *a* will have made only one quarter-turn, which is not enough to bring it to the line *ME*. We shall therefore see a little around the *western* edge. Similarly on the other side of the orbit, half-way between apogee and perigee, we shall look around the *eastern* edge to the same extent. At perigee and apogee both, the libration is, of course, zero. The amount of this libration is evidently at any moment just the same as that of the so-called "equation of the centre," which, it will be remembered, is the difference between the *mean* and *true anomalies* of the moon at any moment. Its maximum possible value is 7°45′.

The period of this libration is the time it takes the moon to go around from perigee to perigee — the so-called *anomalistic revolution*, which is 27.555 days, about 5 hours and 36 minutes longer than the sidereal month, and 8 hours 14 minutes longer than the moon's *nodical* revolution, which determines the libration in latitude.

The cause of the increased length of the anomalistic revolution is of course the fact that the line of apsides continually advances *eastward*, making one revolution every nine years. (Art. 238.)

251. 3. *Diurnal Libration.* This is strictly a libration not of the moon, but of the observer; still, as far as the aspect of the moon goes, the effect is precisely the same as if it were a true lunar libration. The moon's motions have reference to the earth's centre. We, on the surface of the earth, look down over the western edge of the moon when it is rising, and over the eastern when it is setting, by an amount which is equal to the semi-diameter of the earth as seen from the moon; that is, about one degree (the moon's parallax).

On the whole, taking all three librations into account, we see considerably more than half the moon, the portion which never disappears being about *forty-one per cent* of the moon's surface, that never visible also *forty-one* per cent, while that which is alternately visible and invisible is *eighteen* per cent.

252. The agreement between the moon's time of rotation and of her orbital revolution cannot be accidental. It is probably due to the action of the earth on some slight protuberance on the moon's surface.

analogous to a tidal wave. If the moon were ever plastic the earth's attraction must necessarily have produced a tidal "bulge" upon her surface, and the effect would ultimately be to force an agreement between the lunar day and the sidereal month. The subject will be resumed later. (See Arts. 483–484.)

253. The Phases of the Moon. — Since the moon is an opaque globe, shining entirely by reflected light, we can see only that hemisphere of her surface which happens to be illuminated, and of course

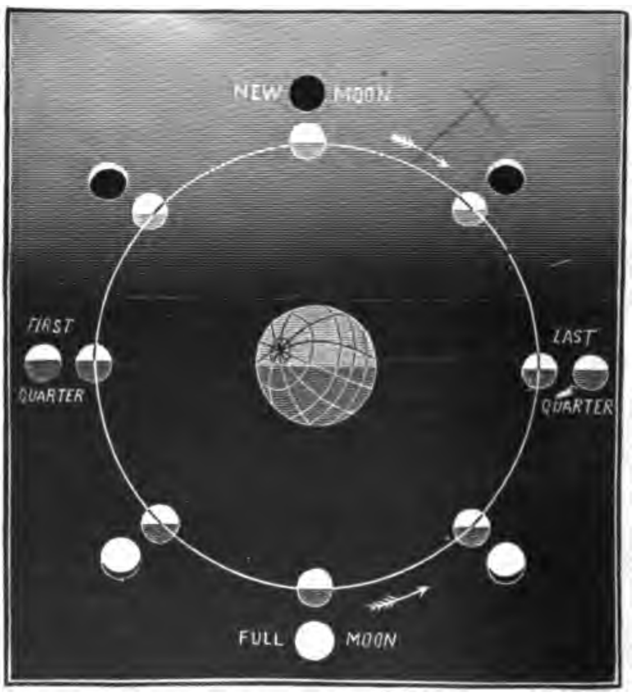

FIG. 82. — Explanation of the Phases of the Moon.

only that part of the illuminated hemisphere which is at the time turned towards the earth. At new moon, when the moon is between the earth and the sun, the dark side is towards us. A week later, at the end of the first quarter, half of the illuminated hemisphere is seen, and we have the half moon, just as we do a week after the full. Between the new moon and the half moon, during the first and last quarters of the lunation, we see less than half of the illuminated portion, and then have the "crescent" phase. See Fig. 82 (in which the

light is supposed to come from a point far above the moon's orbit). Between the half moon and the full, during the second and third quarters of the lunation, we see more than half of the moon's illuminated side, and have what is called the "gibbous" phase.

Since the terminator or line which separates the dark portion of the disc from the bright is always a *semi-ellipse* (being a semi-circle viewed obliquely), the illuminated surface is always a figure made up of a *semi-circle* plus or minus a *semi-ellipse*, as shown in Fig. 83, *A*.

It is sometimes incorrectly attempted to represent the crescent form by a construction like Fig. 83, *B* (where a smaller circle is cut by a larger one). It is to be noticed that *ab*, the line which joints the cusps, is always perpendicular to the line directed to the sun, and *the horns are always turned away from the sun;* so that the precise position in which they will stand at any time is always predictable, and has nothing whatsoever to do with the weather. Artists are sometimes careless in the manner in which they introduce the moon into landscapes. One occasionally sees the moon near the horizon with the *horns turned downwards*, a piece of drawing fit to go with Hogarth's barrel which shows both its heads at once.

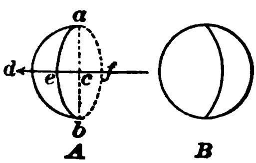

FIG 83.

254. Earth-Shine on the Moon. — Near the time of new moon the whole disc of the satellite is easily visible, the portion on which sunlight does not fall being illuminated by a pale ruddy light. This light is *earth-shine*, the earth as seen from the moon being then nearly full; for seen from the moon the earth shows all the phases that the moon does, the earth's phase in every case being exactly supplementary to that of the moon as seen by us.

As the earth has a diameter nearly four times that of the moon, the earth-shine at any phase would be about thirteen times as strong as moonlight, if the reflective power of the earth's surface were the same. Probably, taking the clouds and snow into account, the earth's surface on the whole is rather more brilliant than the moon's, so that near new moon the earth-shine, by which the dark side of the moon is then illuminated, is from fifteen to twenty times as strong as full moonlight. The ruddy color is due to the fact that light sent to the moon from the earth has twice penetrated our atmosphere and so has acquired the sunset tinge.

255. Physical Characteristics of the Moon. — 1. *Its Atmosphere.* The moon's atmosphere, if it has any at all, is extremely rare, probably not producing a barometric pressure to exceed $\frac{1}{25}$ of an inch of mercury, or $\frac{1}{750}$ of the pressure at the earth's surface. The evidence on this point is twofold.

(*a*) *The telescopic appearance.* The parts of the moon near the edge of the disc, which, if there were any atmosphere, would be seen through its greatest possible depth, are seen without the least distortion: there is no haze, and all shadows are perfectly black. There is no sensible twilight at the cusps of the moon; no evidences of clouds or storms, or anything like atmospheric phenomena.

(*b*) *The absence of refraction* when the moon intervenes between us and any more distant object. For instance, at an eclipse of the sun there is no distortion of the sun's limb where the moon cuts it, nor any ring of light running out on the edge of the moon like that which encircles the disc of Venus at the time of a transit. The most striking evidence of this sort comes, however, from occultations of the stars. When the moon hides a star from sight, the phenomenon, if it occurs at the moon's dark edge, is an exceedingly striking one. The star retains its full brightness in the field of the telescope until all at once, without the least warning, it simply is not there, the disappearance generally being absolutely instantaneous. Its reappearance is of the same sort, and still more startling. Now if the moon had any perceptible atmosphere (or the star any sensible diameter) the disappearance would be gradual. The star would change color, become distorted, and fade away more or less gradually.

The spectroscope adds its evidence in the same direction. There is no modification of the spectrum of the star in any respect at the time of its disappearance; and we may add that the spectrum of moonlight is identical with that of sunlight pure and simple, there being no traces of any effect whatever produced upon the sunlight by its reflection from the moon, nor any signs of its having passed through an atmosphere.

256. The time during which a star would be hidden behind the moon would also be decreased by the refraction of any sensible atmosphere, making the observed duration of an occultation less than that computed from the known diameter of the moon and its rate of motion. Certain Greenwich observations *apparently show a difference*, amounting to about *two seconds* of time. This may possibly be due in some part to the action of a *real*, but exceedingly rare, lunar atmosphere; for if the whole phenomenon were due simply to atmospheric action, it would indicate an atmosphere having a density about $\frac{1}{2000}$ part of our own, — far within the limits which were stated above. But the difference may be, and very probably is, attributable, in part at least, to a slight error in the measured diameter of the moon, due to

irradiation: the diameter of a bright object always appears a little larger than it really is. An error of about 2″ of this sort would explain the whole discrepancy, without any need of help from an atmosphere.

257. What has become of the Moon's Atmosphere. — If the moon ever formed a part of the same mass as the earth, she must once have had an atmosphere. There are a number of possible and more or less probable hypotheses to account for its disappearance. It has been surmised (1) that there may be great cavities left within the moon's mass by volcanic eruptions, and that the rocks themselves have been transformed into a sort of pumice-stone structure, and that the air has retired into these internal cavities.

(2) That the air has been absorbed by the inner lunar rocks in cooling. A heated rock expels any gases that it may have absorbed; but if it afterwards cools slowly, it reabsorbs them, and can take up a very great quantity. The earth's core is supposed to be now too intensely heated to absorb much gas; but if it goes on cooling, it will absorb more and more, and in time it may rob the surface of the earth of all its air. There are still other hypotheses,[1] which we can not take space even to mention.

258. Water on the Moon's Surface. — Of course without an atmosphere there can be no water, since the water would immediately evaporate and form an atmosphere of water vapor if there were no air present. It is not impossible, however, or even improbable, that *solid* water, that is, ice and snow, may exist on the moon's surface at a temperature too low for any sensible evaporation. There are many things in the moon's appearance that seem to indicate the former existence of seas and oceans on her surface, and the same hypotheses have been suggested to account for their disappearance that were suggested in the case of the moon's atmosphere. It may be added also that many kinds of molten rock in crystallizing would take up large quantities of water of crystallization, not merely absorbed as a sponge absorbs water, but chemically united with the other constituents of the rock. In whatever way, however, it may have come about, it is certain that *now* no substances that are gaseous, or that can be evaporated at low temperatures, exist in any quantity on the moon's surface — at least, not *on our side* of the moon.

There have been speculations that on the other side — that celestial country so near us and so absolutely concealed from us — there may be air and water and abundant life; the idea being that our side of the moon is a great table-land many miles in elevation, while the other side is a corresponding

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[1] See also note on page 181.

depression, like the valley of the Caspian Sea, only vastly deeper. An insufficiently grounded conclusion of Hansen's, that the centre of gravity of the moon is some thirty miles farther from us than its centre of figure, for a time gave color to the idea, but it is now practically abandoned, Hansen's conclusion having been shown to be unwarranted by the facts.

259. The Moon's Light. — As to *quality* it is simple sunlight, showing a spectrum which, as has been said, is identical in every detail with that of light coming directly from the sun. Its *brightness* as compared with that of sunlight is difficult to measure accurately, and different experimenters have found results for the ratio between full moonlight and sunlight ranging all the way from $\frac{1}{300000}$ (Bouguer) to $\frac{1}{800000}$ (Wollaston). The value now usually accepted is that determined by Zöllner, viz., $\frac{1}{618000}$. According to this, if the whole visible hemisphere were packed with full moons, we should receive from it about one-eighth part of the light of the sun.

It is found, also, that the half moon does not give even nearly half as much light as the full moon. The law which connects the phase of the moon with the amount of light given at the time, is rather complicated, but the gist of the matter is that at any time, except at the full, the visible surface is more or less darkened by the shadows cast by the irregularities of the surface. Zöllner has calculated that an average angle of 52° for these elevations and depressions would account for the law of illumination actually observed.

The average "*albedo*," or reflecting power of the moon's surface, Zöllner states as 0.174; that is, the moon's surface *reflects a little more than one-sixth* part of the light that falls upon it. This is about the albedo of a rather light-colored sandstone, and agrees well with the estimate of Sir John Herschel, who found the moon to be very exactly of the same brightness as the rock of Table Mountain when it was setting behind it, illuminated as were the rocks themselves by the light of the rising sun. There are, however, great variations in the brightness of different portions of the moon's surface. Some spots are nearly as white as snow or salt, and others as dark as slate.

260. Heat of the Moon. — For a long time it was impossible to detect the moon's heat. It is too feeble to be detected by the most delicate mercurial thermometer even when concentrated by a large lens. The first sensible effect was obtained by Melloni, in 1846,

with the then newly invented thermopile, by a series of observations from the summit of Vesuvius. Since then several physicists have worked upon the subject with more or less success, especially Lord Rosse and Boys in Great Britain, and Langley, Hutchins, and Very in the United States. With modern apparatus there is no difficulty in detecting the lunar heat, but *measurements* are extremely difficult and liable to error. A considerable percentage of the lunar heat seems to be heat simply *reflected* (like light), while the rest, perhaps three-fourths of the whole, is "*obscure heat*"; that is, heat which has been first absorbed by the moon's surface and then *radiated*, like the heat from a brick surface that has been warmed by sunshine. This is shown by the fact that a comparatively thin plate of glass cuts off some 86 per cent of the heat received from the moon in the same way that it does the heat of a stove, while the heat of direct sunlight, or of an electric arc, would pass through the same plate with very little diminution. The same thing appears also from direct measurements upon the heat-*spectrum* of the moon made by Langley with his bolometer, described further on. (Art. 343.)

The amount of heat sent by the full moon to the earth has been estimated by Lord Rosse as $\frac{1}{80000}$ of that sent us by the sun; Hutchins' measures in 1888 make it only $\frac{1}{185000}$.

261. As to the *temperature of the moon's surface*, it is difficult to affirm much with certainty. On one hand, the lunar rocks are exposed to the sun's rays in a cloudless sky for fourteen days at a time, so that if they were blanketed by air like our own rocks they would certainly become intensely heated. Some years ago, Lord Rosse inferred from his observations that the temperature of the lunar surface rose at its maximum (about three days after full moon) far above that of boiling water.[1] But his own later investigations and those of Langley throw great doubt on this conclusion. There is no air-blanket at the moon's surface to prevent it from losing heat;[2] and it now seems rather more probable that the temperature never rises above the freezing-point of water, as is the case on the highest of our mountains, where there is perpetual ice. So far as we can judge, the condition of things on the moon's surface must correspond to an elevation many times higher than any mountain on the earth; for no terrestrial mountain is so high that the density of the air at its summit is even nearly as low as that of the densest supposable lunar atmosphere.

[1] See note on page 183.

[2] See Art. 377 for Rosse's eclipse observations bearing on this point.

This idea that the moon is very cold is borne out, also, by the fact that the bolometer shows the presence, in the lunar radiations, of a considerable quantity of heat having a wave-length greater than that of the heat radiated from a block of ice.

On the dark portion, during the long fourteen-days night the temperature must probably fall at least as far as — 200° F.

262. Lunar Influences on the Earth. — The moon's attraction coöperates with that of the sun in producing tides, of which we shall speak hereafter. There are also certain distinctly ascertained disturbances of terrestrial magnetism connected with the approach and recession of the moon at perigee and apogee; and this ends the chapter of *ascertained* lunar influences.

The multitude of current beliefs as to the controlling influence of the moon's phases and changes over the weather and the various conditions of life are mostly unfounded, and in the strict sense of the word "superstitions," — mere survivals from a past credulity.

It is quite certain that if there is any influence at all of the sort it is extremely slight — so slight that it cannot be demonstrated with certainty, although numerous investigations have been made expressly for the purpose of detecting it. We have never been able to ascertain, for instance, with certainty, whether it is *warmer or not*, or *less cloudy or not*, at the time of the full moon. Different investigations have led to contradictory results.

As to the supposed connection between "change of the moon" and changes of the weather, it should be enough to note that even within the United States the weather changes are not simultaneous (in Kansas and Maine, for instance), as they should be if they were due to the changing phases of the moon. Since, however, a change of the moon occurs every week, every weather change must necessarily occur within about three days and a half of a lunar change, and half of them ought to fall within about forty-five hours, even if perfectly independent.

Now it requires only a very slight prepossession in favor of a belief in the effectiveness of the moon's changes to make one forget a few of the weather changes that occur too far from the proper time. Coincidences enough can easily be found to justify a preëxisting belief.

THE MOON'S SURFACE.

263. Even to the naked eye the moon is a beautiful object, diversified with darker and lighter markings which have given rise to numerous popular superstitions. With a powerful telescope these naked-eye markings mostly vanish, and are replaced by a countless multitude of smaller details, which are interesting in the highest degree. The moon on the whole, on account of this diversity of

detail, is the finest of all telescopic objects; especially to moderate-sized instruments, say from six to ten inches in diameter, which generally give a more pleasing view of our satellite than instruments either larger or smaller.

264. How near the Telescope brings the Moon. — An instrument of this size, with magnifying powers between 250 and 500, brings up the moon virtually to a distance ranging from 1000 miles to 500; and since an object a mile in diameter on the moon subtends an angle of about $0''.86$, with the higher powers of such an instrument objects less than a mile in diameter become visible under favorable atmospheric conditions. A long line or streak, even less than a quarter of a mile across, could probably be seen. With larger telescopes the power can now and then be carried at least twice as high, and correspondingly smaller details made out. When everything is at its best, the great Lick telescope of 36 inches aperture, with a power of 2500 or so, may possibly reduce the virtual distance of our satellite to about 100 miles for visual purposes. It is evident that while with our telescopes we should be able to see such objects as lakes, rivers, forests, and great cities, if they exist on the moon, it will be hopeless to expect to distinguish single buildings, or any of the ordinary operations and indications of life, if such there are.

There are a few mountains on the earth from which a range of 100 miles is obtained in the landscape. Those who have seen such a landscape know how little is to be made out with the naked eye at that distance. Still, the comparison is not quite fair, because in looking at a terrestrial object a hundred miles away the line of vision passes through a dense atmosphere, while in looking upward towards the moon it penetrates a much less thickness of air.

265. The Moon's Surface Structure. — The moon's surface for the most part is extremely uneven and broken, far more so than that of the earth. The structure, however, is not like that of the earth's surface. On the earth the mountains are mostly in long ranges, such as the Alps, the Andes, and Himalayas. On the moon such mountain ranges are few in number, though they exist; but the surface is pitted all over with great craters, resembling very closely the volcanic craters on the earth's surface, though on an immensely greater scale. One of the largest craters upon the earth, if not the largest, is the Aso San in Japan, about seven miles across. Many of those on the moon are fifty and sixty miles in diameter, and some are

over 100 miles across, while smaller ones from a half-mile to eight or ten miles in diameter are counted by the thousand.

The normal lunar crater is nearly circular, surrounded by an elevated ring of mountains which rise anywhere from 1000 to 20000 feet above the surrounding country. Within the floor of the crater the surface may be either above or below the outside level. Some craters are deep, some filled nearly to the brim. In some cases the surrounding mountain ring is entirely absent, and the crater is a mere hole in the plain. In the centre of the crater there usually rises a group of peaks, of about the same elevation as the encircling ring, and these peaks often show holes or craterlets in their summits.

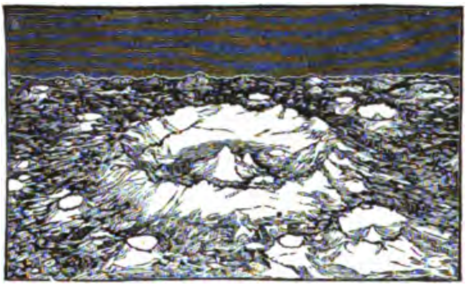

FIG. 84. — A Normal Lunar Crater (Nasmyth).

In most cases the resemblance of these formations to terrestrial volcanic structures, like those exemplified by Vesuvius and others in the surrounding region, makes it natural to assume that they had a similar origin. This, however, is not absolutely certain, for there are considerable difficulties in the way, especially in the case of the great "Bulwark Plains," so-called, which are so extensive that a person standing in the centre could not see the summit of the surrounding ring at any point; and yet no line of demarcation can be drawn between them and the smaller craters. The series is continuous. Moreover, on the earth, volcanoes necessarily require the action of air and water, which do not now exist on the moon. It is obvious, therefore, that if these lunar craters are the result of true volcanic eruptions, they must be 'fossil' formations; for it is quite certain[1] that no evidence of existing volcanic activity has ever been found. The moon's surface appears to be absolutely quiescent — still in death.

On some portions of the moon these craters stand very thickly. Older craters have been encroached upon, or more or less completely obliterated by the newer, and the whole surface is a chaos, of which the counterpart is hardly to be found on the earth, even in the roughest portions of the Alps. This is especially the case near the moon's south pole. It is noticeable that, as on the earth the newest mountains are generally the highest, so on the moon the more newly formed craters are generally deeper and more precipitous than the older ones.

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[1] See note on page 183.

266. Lunar Nomenclature. — The great plains were called by Galileo oceans or seas (*Maria*), and some of the smaller ones marshes (*Paludes*) and lakes, for he supposed that the grayish surfaces visible to the naked eye, and conspicuous in a small telescope, were covered with water. Thus we have the "Oceanus Procellarum," the "Mare Imbrium," and a number of other "seas," of which

Fig. 85. — Map of the Moon. (Reduced from Neison.)

"Mare Fecunditatis," "Mare Serenitatis," and "Mare Tranquillitatis," are the most conspicuous. There are twelve of them in all, and eight or nine Paludes, Lacus, and Sinus.

The ten mountain ranges on the moon are mostly named after terrestrial mountains, as Caucasus, Alps, Apennines, though two or three bear the names of astronomers, like Leibnitz, Dörfel etc.

The conspicuous craters bear the names of the more eminent ancient and mediæval astronomers and philosophers, as Plato, Archimedes, Tycho, Copernicus, Kepler, and Gassendi; while hundreds of smaller and less conspicuous formations bear the names of more modern or less noted astronomers.

The *system* seems to have originated with Riccioli in 1650, but most of the names have been more recently assigned by the later map-makers, the most eminent of whom have been the German astronomers Beer and Maedler (who published their map in 1837), and Schmidt of Athens, whose great map of the moon, on a scale seven feet in diameter, was published by the Prussian government some years ago. It is not at all too much to say that our maps of the earth's surface do not, on the whole, compare in fulness and accuracy with our maps of the moon. Of course this is not true of such countries as France and England, or others that have been trigonometrically surveyed; but there are no such *lacunæ* in our maps of the moon as exist in our maps of Asia and Africa, for instance.

267. Other Lunar Formations. — The craters and mountains are not the only interesting formations on the moon's surface. There are many deep, narrow, crooked valleys that go by the name of "rills" (German *Rillen*), some of which may once have been watercourses. Then there are numerous "clefts," half a mile or so wide and of unknown depth, running in some cases several hundred miles, straight through mountain and valley, without any apparent regard for the accidents of the surface. They seem to be deep cracks in the crust of our satellite. Several of them are shown in Fig. 86. Most curious and interesting of all are the light-colored *streaks* or "*rays*" which radiate from certain of the craters, extending in some cases a distance of several hundred miles. They are usually from five to ten miles wide, and neither elevated nor depressed to any extent with

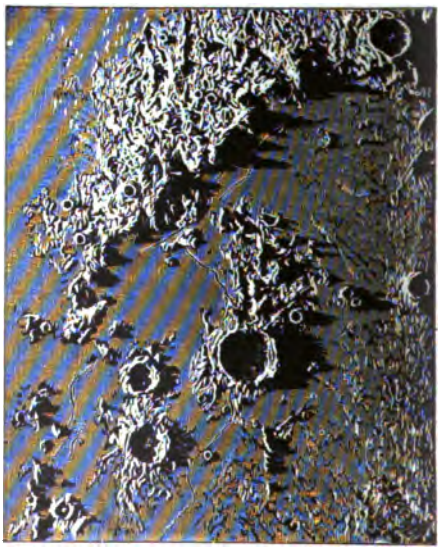

FIG. 86. — Archimedes and the Apennines (Nasmyth).

reference to the general surface. They pass across mountain and valley, and sometimes through craters without any change in width or color. We do not know whether they are like the so-called "trap-dykes" on the earth, — fissures which have been filled up from below with some light-colored material, — or whether they are mere surface markings. No satisfactory explanation has ever been given.

The most remarkable system of "rays" of this kind is the one connected with the great crater Tycho, not very far from the moon's south pole. They are not very conspicuous until within a few days of full moon, but at that time they, and the crater from which they radiate, constitute by far the most striking feature of the whole lunar landscape.

268. Changes on the Moon. — It is certain that there are no *conspicuous* changes. The observer has before him no such ever-varying vision as he would have in looking toward the earth, — no flying clouds, no alternations of seasons with the transformation of the snowy wastes to green fields, nor any considerable apparent movement of objects on the disc. The sun rises on them slowly as they come one after the other to the terminator, and sets as slowly. At the same time it is confidently maintained by many observers that here and there changes are still going on in the details of the surface. Others as stoutly dispute it.

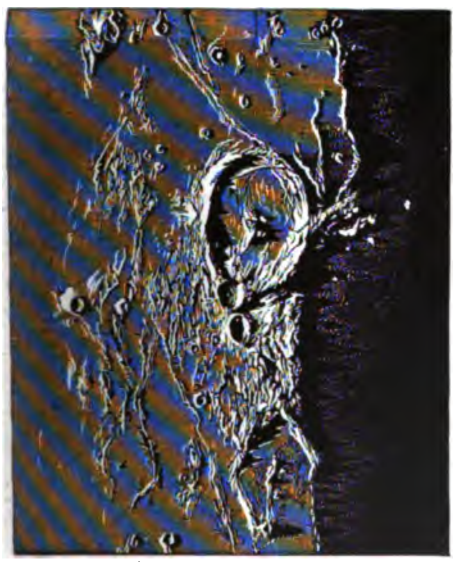

FIG. 87. — Gassendi (Nasmyth).

269. Probably the most notable and best advocated instance of such a change is that of the little crater Linné, in the Mare Serenitatis. It was observed by Schroeter very early in the century, and is figured and described by Beer and Maedler as being about five and a half or six miles in diameter, quite deep and very bright. In 1866 Schmidt, who had several times observed it before, announced that it had disappeared. A few months later it was

visible again, and there were many reported changes in its appearance during the next year or two. There is no question that it does not now at all agree in conspicuousness and size with the representation of Beer and Maedler, for it is at present, and has been for several years, only a minute dark spot, with a whitish spot surrounding it. Astronomers would feel more confident that this was a case of real change were it not that Schroeter's earlier picture much more resembles the present appearance than does that of Beer and Maedler. As the latter observers worked with rather a small telescope, and had no reason for taking any special pains in the delineation of this particular object, the evidence is less conclusive than it might seem at first. The change, however, if real, was certainly as great as in the instance of Krakatoa, the great volcano whose eruption in 1883 filled the earth's atmosphere with smoke and vapor for more than two years, and caused the "twilight conflagrations" of the sky. The phenomenon in the case of Linné, if real, was probably a falling in of the walls of the crater, exposing fresh unweathered surfaces.

FIG. 88.

The difficulty in establishing the reality of such changes lies mainly in the great, but purely apparent, discrepancies due to varying illumination and to the "personal equation" of observers and their telescopes. Comparisons can be safely instituted only between observations made under conditions (lunar, atmospheric, instrumental, and personal) which are sensibly identical, and such identity is of course not easy to secure.

The final appeal will be to photography.[1]

270. Measurement of Lunar Mountains. — The height of a lunar mountain is usually determined by measuring with a micrometer, as accurately as possible, the apparent length of its shadow, and also, a little more roughly, measuring at the same time the distance of the object from the terminator and from what may be called the "equator of illumination," — the line *b* (in Fig. 88) which bisects the phase symmetrically. With these data and those supplied by the almanac, the result is easily calculated by formulæ given in Neison's "Moon." If the mountain is favorably situated, its height can be determined in this way with an error not exceeding five or six hundred feet.

In some cases the height is computed from measurements of the

[1] See note on page 183.

distance between the terminator and the top of the mountain when it first catches the sunlight, and looks like a star outside the terminator, as shown in Figs. 87 and 88. But this is less accurate.

Many of the lunar mountains reach an elevation of 15000 feet and upwards. One of the highest, so situated that its height can be fairly measured, is Mt. Huyghens in the range of the Appenines on the western edge of the Mare Imbrium, — a little over 18000 feet high. Very near the south pole, and only visible in outline under favorable conditions of libration, are the great Leibnitz and Doerfel ranges, which are much higher, — probably between 25000 and 30000 feet.

271. The Best Time to Look at the Moon with a Telescope. — The moon when full is not so satisfactory an object as when near the half, because at the full moon there are no shadows, so that at that time the "relief" of the surface structure is entirely lost. Certain features, however, as has been before mentioned, are then best seen, as, for instance, the streaks or rays. Generally, any particular mountain, crater, rill, or cleft is best studied when it is just on or very near the terminator, that is, at the time when the sun is rising or setting near it, because then the shadows are longest. The best general view of the moon is that obtained a few days after the half moon, when Copernicus and Tycho are both near the terminator, and Plato is still near enough to it to show very well.

272. Photographs of the Moon. — A great deal of attention has been paid to this subject, and some fine results have been reached. The earliest success was that of Bond in 1850, with the old daguerreotype process; then followed the work of De la Rue in England, and of Dr. Henry Draper, and especially of Mr. Rutherfurd in this country. Rutherfurd's pictures have remained absolutely unrivalled until very recently.

Since 1885, however, great progress has been made. In this country the Harvard, the Lick, and the Yerkes observatories have reached admirable results, and in Europe the Paris observatory. From negatives made at the Lick and Paris observatories, and, more recently, by W. H. Pickering in Jamaica, complete atlases of the moon have been made. The Paris atlas is especially fine, showing the features on various scales corresponding to lunar diameters ranging from four to nine feet. But the photograph cannot yet rival the eye in the study of delicate details.

272*. Note to Art. 257. — It is not improbable that the extent and to a certain degree the composition of the atmosphere of a heavenly body may depend directly upon its mass and density. Indeed, if the "Kinetic theory" of gases is true, it must necessarily be so, as was pointed out some years ago by Johnstone Stoney, of Dublin. According to this theory the molecules of a gas are continually flying in all directions with a velocity depend-

ent upon their mass and temperature. Individual molecules move, some faster and some slower, and a certain small percentage may attain a speed six or seven times as great as this mean velocity. At zero (Cent.) the maximum molecular velocity of oxygen is computed as about 1.8 miles a second; that of nitrogen, 2.0; that of water-vapor, 2.5; that of helium, 5.2; and that of hydrogen, 7.4, — values which increase or decrease with the temperature.

Again, at any given distance from a body there is a so-called "critical," or "parabolic," velocity (Arts. 429 and 435) depending on the mass of the body; and if a particle at this distance has a speed greater than this parabolic velocity, it cannot be retained by the body's gravitational attraction, but will fly off into space. At the surface of the sun this critical velocity is about 383 miles a second; at the surface of the earth it is a little less than 7; and on the moon's surface it is only 1.5. It is clear, therefore, that if the sun were cool, not a molecule of any of the gases we have mentioned above would ever escape from its atmosphere. On the earth, however, hydrogen cannot be retained *free*, but only in chemical combination; helium would be likely to go also, since a slight elevation of temperature above the freezing point might increase its molecular velocity beyond the 7-mile limit. Oxygen, nitrogen, and water-vapor, on the other hand, stay by us.

But on the moon the force of gravity is so small that even if she were now by some means once more reheated and reclothed with an atmosphere like our own, its molecules would one after another take flight, and soon leave her airless again.

This, however, all hangs upon the truth of the kinetic theory of gases, which, though very probable, can hardly be considered as yet completely proved.

It is worth noting, also, that on this hypothesis interplanetary space must be populous with wandering molecules of the various gases, which, however, no longer behave like "gas," as we know it on the earth, because they are too far apart and collide with each other too seldom to enable them to manifest the familiar gaseous properties. Now these wandering molecules must be continually entering a planet's atmosphere, and when as many arrive in a day as fly off in the same time, this atmosphere ceases either to grow or to diminish.

Exercises on Chapter VII.

1. If the moon's sidereal period were sixty days, what would be her synodic period? *Ans.* 71.7932 days.

2. In that case, what would be the mean interval between her meridian transits? (See Art. 235.) *Ans.* $24^h\ 20.34^m$.

3. Does the moon rise every day?

4. If the moon rises at $11^h\ 45^m$ P.M. on Wednesday, when (approximately) will she rise next?

5. What is the lowest latitude on the earth where the moon can remain above the horizon for 48 consecutive hours?

Ans. $90° - (23°\ 28' + 5°\ 8') = 61°\ 24'$.

6. At what time of the year does the full moon remain longest above the horizon?

7. How many times does the moon turn on its axis in a year?

8. Does the earth rise and set for an observer on the moon?

9. What determines the direction of the horns of the crescent moon?

10. Can a star ever be seen between the horns of the moon?

11. What point describes "the orbit of the earth" around the sun?

Ans. The centre of gravity of the earth and moon.

12. Does the centre of the sun, as seen from the centre of the earth, follow the ecliptic exactly, and if not, how far can it depart from it on account of the moon's action? (See Arts. 233 and 243.)

Ans. The deviation may be $\frac{64}{88} \times 8''.80 \times \sin 5°\ 20' = 0''.6$.

There are also perturbations of the earth by the planets, producing additional deviations of about $0''.5$, so that at times the *latitude* of the sun's centre may slightly exceed $\pm\ 1''.1$.

NOTE TO ART. 261.

An investigation by Professor Very, published in January, 1899, bears strongly in the same direction as Lord Rosse's result, indicating a maximum temperature on the moon's illuminated surface approaching that of boiling water, but falling immediately on the withdrawal of sunlight.

NOTE TO ARTS. 265 AND 269.

Professor W. H. Pickering considers that his observations and photographs of 1902–1903 show changes upon the moon's surface which indicate the deposition of snow or hoar-frost at certain points during the lunar night, and its subsequent disappearance when the sun's rays reach it: very much as if a subdued volcanic activity still persisted in the moon with vapors issuing from beneath through fissures and fumaroles, as in certain parts of the earth, Iceland for instance. But as yet his views do not seem to have gained general acceptance.

CHAPTER VIII.

THE SUN: DISTANCE AND DIMENSIONS. — MASS AND DENSITY. — ROTATION. — STUDY OF THE SURFACE: GENERAL VIEWS AS TO THE SUN'S CONSTITUTION. — SUN SPOTS: THEIR APPEARANCE, NATURE, DISTRIBUTION, AND PERIODICITY.

273. THE SUN is simply a *star;* a hot, self-luminous globe of enormous magnitude as compared with the earth and the moon, though probably only of medium size among its stellar compeers. But to the earth and the other planets which circle around it, it is the grandest of all physical objects. Its attraction confines its planets to their orbits and controls their motions, and its rays supply the energy which maintains every form of activity upon their surfaces and makes them habitable.

274. Its Distance and Dimensions. — Its distance may be determined from its horizontal parallax, which is the apparent angular semi-diameter of the earth as seen from the sun. The mean value of this parallax is probably very near 8″.8.[1]

We reserve to a separate chapter the discussion of the methods by which this most fundamental and important of all astronomical data has been ascertained, merely remarking here that the problem is one of extreme *practical* difficulty, though the principles involved are simple enough.

Assuming the parallax at 8″.8, the mean distance of the sun (putting r for the earth's radius) equals

$$r \div \sin 8''.8 = 23439 \times r.$$

With Clarke's value of r (Art. 145), this gives 149 500000 kilometers,

[1] In the American Ephemeris the value deduced by Newcomb in 1867 is used, *viz.*, 8″.85. The British "Nautical Almanack" uses the same value, and the French the value deduced by Leverrier a little earlier, 8″.86; but more recent observations show that the number stated, 8″.8, is more nearly correct, and after 1900 it is to be used in all three ephemerides.

or 92 897000 miles; which, however, is uncertain by at least 50000 miles, and is *variable*, also, to the extent of about three million miles on account of the eccentricity of the earth's orbit, the earth being nearer the sun in December than in June.

275. This distance is so much greater than any with which we have to do on the earth that it is possible to reach a conception of it only by illustrations of some sort. Perhaps the simplest is that drawn from the motion of a railway train. Such a train going 1000 miles a day (nearly forty-two miles an hour) would take 254½ years to make the journey.

If sound were transmitted through interplanetary space, and at the same rate as through our own atmosphere, it would make the passage in about fourteen years; *i.e.*, an explosion on the sun would be heard by us fourteen years after it actually occurred. A cannon-ball moving unretarded, at the rate of 1700 feet per second, would travel the distance in nine years. Light does it in 499 seconds.

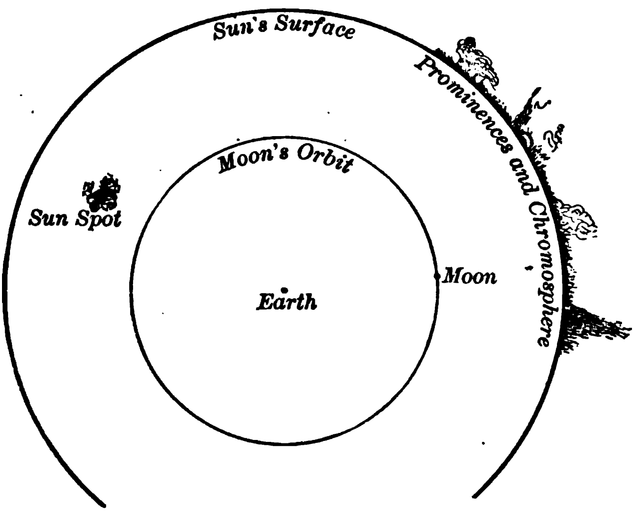

FIG. 90. — Dimensions of the Sun Compared with the Moon's Orbit.

276. Diameter. — The sun's mean apparent diameter is 32′ 04″ ± 2″. Since at the sun one second equals 450.38[1] miles, its diameter equals 866500 miles, or 109½ times the diameter of the earth. It is quite possible that this diameter is variable to the extent of a few

[1] 92 897000 ÷ 206264.8 = 450.38.

hundred miles, since, as will appear hereafter, the sun (at least the surface which we see) is not solid.

Representing the sun by a globe two feet in diameter, the earth would be $\frac{22}{100}$ of an inch in diameter, — the size of a very small pea, or a " 22-calibre" round pellet. Its distance from the sun on that scale would be just about 220 feet, and the *nearest star* (still on the same scale) *would be eight thousand miles away, at the antipodes.*

If we were to place the earth in the centre of the sun, supposing it to be hollowed out, the sun's surface would be 433,000 miles away from us. Since the distance of the moon is only about 239,000 miles, it would be only a little more than half-way out from the earth to the inner surface of the hollow globe, which would thus form a very good background for the study of the lunar motions.

It is perhaps worth noticing, as a help to memory, that the sun's diameter exceeds the earth's just about as many times as it is itself exceeded by the radius of the earth's orbit; or, in other words, the sun's diameter is *nearly* a mean proportional between the earth's distance from the sun and the earth's diameter, 110 being the common ratio.

277. Surface and Volume. — Since the *surfaces* of globes are proportional to the *squares* of their radii, the surface of the sun exceeds that of the earth in the ratio of $(109.5)^2$ to 1; that is, its surface is about 12,000 times the surface of the earth.

The *volumes* of spheres are proportional to the *cubes* of their radii; hence the *sun's volume* is $(109.5)^3$, or 1,300000 times that of the earth.

278. The Sun's Mass. — The mass of the sun is *very nearly three hundred and thirty-two thousand times that of the earth*, subject to a probable error of at least one per cent. There are various ways of getting at this result. For our purpose here, perhaps the most convenient is by comparing the earth's attraction for bodies at her surface (as determined by pendulum experiments) with the attraction of the sun for the earth, — the central force which keeps her in her orbit. Put f for this force (measured, like gravity, by the velocity it generates in one second), g for the force of gravity (32 feet 2 inches per second), r the earth's radius, R the sun's distance, and let E and S be the masses of the earth and sun respectively. Then, by the law of gravitation, we have the proportion

$$f : g :: \frac{S}{R^2} : \frac{E}{r^2}, \text{ or } S = E\left(\frac{f}{g}\right)\left(\frac{R}{r}\right)^2. \qquad (a)$$

Now, $\frac{R}{r} = 23{,}440$ (nearly).

Its square equals 549,433,600. $g = 386$ inches. To find f we have from Mechanics (Physics, pp. 17 and 28),

$$f = \frac{V^2}{R}, \qquad (b)$$

this being the expression for the "central force" in the case of a body revolving in a circle. (We may neglect the eccentricity of the earth's orbit in a merely approximate treatment of the problem.) V is the orbital velocity of the earth, which is found by dividing the circumference of the orbit, $2\pi R$, by T, the number of seconds in a sidereal year. This velocity comes out 18.495 miles per second. Putting this into formula (b), we get $f = 0.2333$ inches,

so that $$\frac{f}{g} = 0.0006044 = \frac{1}{1654} \text{ (nearly)};$$

whence $$S = E \times \frac{1}{1654} \times 549{,}433{,}600; \text{ or } S \text{ equals } 332{,}000.$$

We may note in passing that half of f expresses the distance by which the earth *falls towards the sun* every second, just as half g is the distance a body at the earth's surface falls in a second. This quantity (0.116 inch), a trifle more than a ninth of an inch, is the amount by which the earth's orbit deviates from a straight line in a second. In travelling *eighteen and one-half miles* the deflection is only *one-ninth of an inch.*

278*. By substituting $\frac{2\pi R}{T}$ for V in equation (b), we get

$$f = \frac{4\pi^2 R}{T^2};$$

and putting this value of f into equation (a) and reducing, we obtain

$$S = E\left[\left(\frac{4\pi^2}{T^2}\right)\left(\frac{r}{g}\right)\left(\frac{R}{r}\right)^3\right], \qquad (c)$$

or, since $$\frac{R}{r} = \frac{1}{\sin p}$$

(p being the sun's horizontal parallax), we have finally

$$S = E\left[\left(\frac{4\pi^2}{T^2}\right)\left(\frac{r}{g}\right)\frac{1}{\sin^3 p}\right]. \qquad (d)$$

It will be noticed that in this expression the *cube* of the parallax appears, and this is the reason why an uncertainty of one per cent in p involves an uncertainty of three per cent in S.

In obtaining the mass of the sun it will be seen that we require as data, T, the length of the sidereal year in seconds; the value of gravity, g (which is derived from pendulum experiments); the radius of the earth, r (deduced from geodetic surveys); and finally (and most difficult to get), the sun's parallax, p, or else the sun's distance, R; giving us in either case the ratio $\frac{r}{R}$.

279. The Sun's Density. — This density[1] as compared with that of the earth is found by simply dividing its mass by its volume (both as compared with the earth); that is, it equals the fraction

$$\frac{332000}{1\,300000} = 0.255,$$

a little more than *a quarter* of the earth's density. To get its "*specific gravity*" (*i.e.*, density as compared with *water*), we must multiply this by 5.58, the earth's mean specific gravity. This gives 1.41; that is, *the sun's mean density is not* $1\frac{1}{2}$ *times that of water,* — a most significant result as bearing on its physical condition.

280. Superficial Gravity. — This is found by dividing its mass by the square of its radius; that is,

$$\frac{332000}{(109\frac{1}{2})^2},$$

which equals 27.6. A body weighing one pound on the earth's surface would there weigh 27.6 pounds. A body would fall 444 feet in a second, instead of 16 feet, as here.

281. The Sun's Rotation. — The sun's surface often shows spots upon it, which pass across the disc from east to west. These are evidently attached to its surface, and not bodies circling around the sun at a distance above it, as was imagined by some early astronomers, because, as Galileo early demonstrated, they continue in sight just as long as the time during which they are invisible; which would not be the case if they were at any considerable elevation.

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[1] The determination of the sun's density does not *necessarily* involve its parallax. Put ρ for sun's radius, and Ds for its density; let De be earth's mean density. Substitute in equation (c), and we have $\frac{4}{3}\pi\rho^3 Ds = \frac{4}{3}\pi r^3 De\left[\frac{4\pi^2}{T^2}\left(\frac{r}{g}\right)\left(\frac{R}{r}\right)^3\right]$, whence $Ds = De\left[\frac{4\pi^2}{T^2}\left(\frac{r}{g}\right)\left(\frac{R}{\rho}\right)^3\right]$. But $\left(\frac{\rho}{R}\right) = \sin\Sigma$, Σ being the sun's angular semi-diameter. Hence, finally, $Ds = De\left[\frac{4\pi^2}{T^2}\left(\frac{r}{g}\right)\left(\frac{1}{\sin^3\Sigma}\right)\right]$.

Period of Rotation. — The average time occupied by a spot in passing around the sun and returning to the same position again is 27.25 days, — *average* because different spots show considerable differences in this respect. This interval, however, is not the *true time* of solar rotation, but the *synodic*, since the earth advances in the interval of a revolution so that the sun has to turn on its axis a little farther each time to bring the spot again into conjunction with the earth. The equation by which the true period is deduced from the synodic is the same as in the case of the moon (Art. 232), *viz.:*

$$\frac{1}{T}-\frac{1}{E}=\frac{1}{S},$$

T being the true period of the sun's rotation, E the length of the year, and S the observed *synodic* rotation;

whence, $$\frac{1}{T}=\frac{1}{27.25}+\frac{1}{365.25},$$

which gives $T = 25^d.35$. Different observers get slightly different results. Carrington finds $25^d.38$; Spoerer, $25^d.23$.

282. Position of the Sun's Axis. — On watching the spots with care as they cross the disc, it appears that they usually describe paths more or less oval, showing that the sun's axis is inclined to the ecliptic. Twice a year, however, the paths become straight, at the times when the earth is in the plane of the sun's rotation. These dates are about June 3 and December 5.

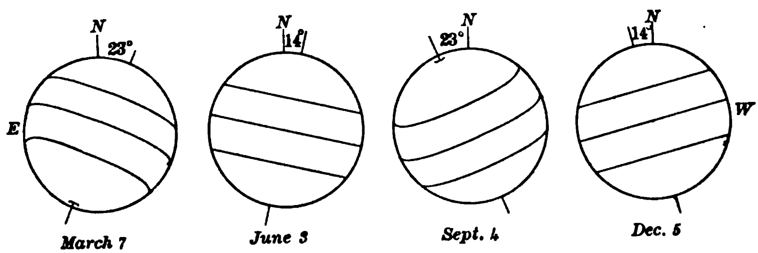

FIG. 90. — Position of the Sun's Axis.

The ascending node of the sun's equator is in celestial longitude 73° 40′ (Carrington), and the inclination of its equator to the plane of the ecliptic is 7° 15′. Its inclination to the plane of the terrestrial equator is 26° 25′. The position of the point in the sky towards which the sun's pole is directed is in right ascension $18^h\ 44^m$, declination + 63° 35′, very nearly half-way between the bright star α Lyræ and the Pole Star.

Fig. 90 shows the position of the sun's axis and equator with reference to the north and south line, and the apparent paths of sun-spots upon the

disc, at the dates indicated. On January 4 and July 6 the axis lies exactly upon the hour-circle, *i.e.*, due north and south in the sky. On April 5 and October 14 the position angle is at its maximum of 26° 25′ west and east, respectively.

283. Peculiar Law of the Sun's Rotation. — *Equatorial Acceleration.* The earth rotates *as a whole*, every point on its surface making its diurnal revolution in the same time; so also with the moon and with the planet Mars. Of course it is necessarily so with any *solid* globe. But this is not the case with the sun. It was noticed quite early that the different spots give different results for the rotation period, but the researches of Carrington between 1853 and 1861 first brought out the fact that the differences follow a regular law, showing that at the solar equator the time of rotation is less than on either side of it. Thus, spots near the sun's equator give $T = 25$ days; at solar latitude 20°, $T = 25.75$ days; at solar latitude 30°, $T = 26.5$ days; at solar latitude 40°, $T = 27$ days. The time of rotation in latitude 40° is fully two days longer than at the solar equator; but we are unable to follow the law further towards the poles, because the spots are rarely found beyond the parallels of 45° on each side of the equator, and there are no well-defined markings between this point and the poles by which we can accurately determine the motion.

284. Various formulæ have been proposed to represent this law of rotation. Carrington gives for the daily motion of a spot $X = 865' - 165' \times \sin^{\frac{7}{4}} l$, l being the solar latitude of the spot. Faye, from the same observations, considering that the exponent $\frac{7}{4}$ could have no physical justification, deduced $X = 862' - 186' \times \sin^2 l$, which agrees almost as well with the observations. Still other formulæ have been deduced by Spoerer, Zöllner, and Tisserand, all giving substantially the same results.

It might be supposed that this apparent equatorial acceleration may be only a motion of the spots over the sun's surface, like that of clouds or railway trains over the earth, and the idea has been tested by observations upon the faculæ (Art. 292), and upon the lower portions of the solar atmosphere where the dark lines of the spectrum originate. The results from the faculæ have been a little discordant among themselves, but a late research of the kind, based upon a series of photographs made at Pulkowa, comes out in substantial agreement with the results obtained from the spots.

The motion of the sun's atmosphere cannot, of course, be studied by direct telescopic or photographic methods, but only *spectroscopically*, as explained hereafter, by making use of the "Doppler-Fizeau Principle" (Art. 321). The earlier observations of this kind were not delicate enough to do much more than to prove that the solar atmosphere actually participates in the general rotation. In 1887 Crew at Baltimore made an elaborate series of observations which indicated for the atmosphere a mean rotation-period

practically the same as that given by the spots, but with a slight (though very doubtful) *retardation* at the equator. The exquisite work of Dunér (in Sweden), however, two years later, demonstrated the equatorial acceleration of the solar atmosphere beyond all question.

The still more recent observations of Jewell at Baltimore in 1897 appear to indicate that the upper portions of the solar atmosphere have a rotation-period several days shorter than the lower; but the matter requires farther investigation.

285. Thus far all the formulæ which attempt to represent the velocity of the sun's surface in different latitudes are simply *empirical;* that is, they are deduced from the observations, without being based upon any satisfactory physical explanation, for no such explanation of this strange equatorial acceleration has yet been found. Probably it has its origin somehow in the effects produced by the outpour of heat from the sun's surface; still, just how such a result should follow in the case of a cooling globe, of which the particles are free to move among each other, is not yet evident.

(See note at end of chapter, Art. 310*.)

It has been suggested (see Art. 306) that the spots may be due to the fall of matter upon the sun's surface, matter which has remained at a great elevation for some time, and acquired a corresponding velocity of rotation. It can be shown that if the matter forming the spots had thus fallen from a height of about 20000 miles, it would account for their apparent acceleration. Matter so falling would have an apparent eastward motion, just as do bodies on the earth when falling from the summit of a tower (Art. 138). From this point of view it is very interesting to inquire whether the minuter markings upon the sun's surface do, or do not, possess the same rate of motion as the spots. At present the evidence is not decisive, but probably they do.

286. The Phenomena of the Sun's Surface. — In order to study the sun with the telescope it is necessary to be provided with some special forms of apparatus. Its heat and light are so intense that it is impossible to look directly at it, as we do at the moon. A very convenient method of exhibiting the sun to a number of persons at once is simply to attach to the telescope a frame carrying a screen of white paper at a distance of a foot or more from the eye-piece, as shown in Fig. 91. On pointing the instrument to the sun

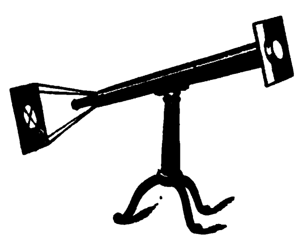

FIG. 91. — Telescope and Screen.

and properly adjusting the focus, a distinct image is formed on the screen, which shows the main features very fairly. It is, however, much more satisfactory to look at it directly, with a proper eye-piece. With a small telescope, not more than two and a half or three inches in diameter, a mere dark glass between the eye-piece and the eye can be used, but this dark glass soon becomes very hot, and is apt to crack. With larger instruments, it is necessary to use eye-pieces especially designed for the purpose and known as *solar eye-pieces* or *helioscopes.*

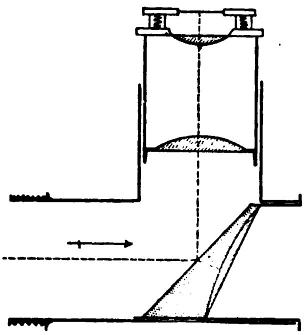

FIG. 92. — Herschel Eye-piece.

The simplest of them, and a very good one for ordinary purposes, is one known as Herschel's, in which the sun's rays are reflected at right angles by a plane of unsilvered glass (Fig. 92). This reflector is made either of a prismatic form or concave, in order that the reflection from the back surface may not interfere with that from the front. About nine-tenths of the light passes through this reflector, and is allowed to pass out uselessly through the open end of the tube. The remaining tenth is sent through the eye-piece, and though still too intense for the eye to endure, it requires only a comparatively thin shade of neutral-tinted glass to reduce it sufficiently, and in this case the shade does not become uncomfortably heated. It is well to have the shade-glass made wedge-shaped, — thinner at one end than at the other, — so that one can choose the particular thickness which is best adapted to the magnifying power employed.

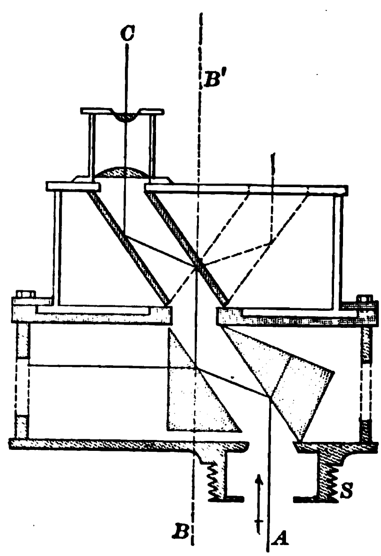

FIG. 93. — Polarizing Helioscope.

287. The polarizing eye-pieces are still better when well made. In these the light is reflected twice at plane surfaces of glass at the "angle of polarization" (Physics, p. 462), and is then received on a second pair of reflectors of black glass. When the upper pair of reflectors is in either of the two positions shown in Fig. 93, a strong beam of light is received at C, — too strong for the eye to

bear, although more than ninety per cent of it has already been rejected; but by simply turning the box which carries the upper reflectors one-quarter of a revolution around the line BB' as an axis, the light may be wholly extinguished; and any desired gradation may be obtained by setting it at the proper angle, without the use of a shade-glass.

288. It may be asked why it will not answer merely to "cap" the object-glass, and so cut off part of the light, instead of rejecting it after it has once been allowed to enter the telescope. It is because of the fact, mentioned in Art. 43, that the smaller the object-lens of the telescope, the larger the image it makes of a luminous point, or the wider its image of a sharp line. To cut down the aperture, therefore, is to sacrifice the definition of delicate details. With a low power there is no objection to reducing the amount of heat admitted into the telescope tube in that way, but with the higher powers the whole aperture should always be used.

289. Photography. — In the study of the sun's surface photography is for some purposes very advantageous and much used. The instrument must have a special object-glass (Art. 42), with an apparatus for the quick exposure of plates. Such instruments are called photo-heliographs, and with them photographs of the sun are made daily at numerous observatories. The necessary exposure varies from $\frac{1}{500}$ to $\frac{1}{10}$ of a second, in different cases. The pictures made by these instruments are usually from two inches up to eight or ten inches in diameter, and some of Janssen's, made at Meudon, bear enlarging up to forty inches in diameter. Photographs have the advantage of freedom from prejudice and prepossession on the part of the observer; but they take no advantage of the instants of fine seeing. They represent the surface as it happened to appear at the moment when the plate was uncovered.

290. The study of the sun has become so important from a scientific point of view that several observatories have recently been established mainly for that purpose, though most of them connect with it that of other topics in astronomical physics. Among the most important of these "astrophysical" observatories may be named those at Potsdam and Meudon, and in this country the Cambridge observatory, and, probably very soon, the Yerkes.

291. General Views. — Before passing to a discussion of the details of the different solar phenomena, it will be well to give a very brief summary of the objects and topics to be considered.

1. *The photosphere; i.e.*, the luminous surface of the sun directly visible to our telescopes. It is probably a sheet of *luminous clouds* formed by condensation into little drops and crystals (like the water-

drops and ice-crystals in our terrestrial clouds) of certain substances which within the central mass of the sun exist in a gaseous form, but are cooled at its surface below the temperature necessary for their condensation ; perhaps such substances as carbon, boron, and silicon. The granules, faculæ, and spots are all phenomena in this photosphere.

2. The so-called *"reversing layer"* is a stratum of unknown thickness, but probably shallow, just above the photosphere, containing the vapors of many of the familiar terrestrial elements; of which the presence, and to some extent their physical condition, can be investigated by means of the spectroscope.

3. Above the photosphere, interpenetrating the atmosphere of vapors just spoken of, and perhaps indistinguishable from it, is an envelope of *permanent* gases; that is, gases which, under the solar conditions, cannot be condensed into clouds of solid or liquid particles. Among them hydrogen is most conspicuous. This envelope is the so-called *Chromosphere;* and from it the *prominences* of various kinds rise, sometimes to the height of hundreds of thousands of miles. These beautiful objects are best seen at total eclipses of the sun, but to a certain extent they can also be studied at any time by the help of a spectroscope.

Fig. 94. — Constitution of the Sun. From "The Sun," by permission of the Publishers.

4. Higher yet rises the mysterious *Corona*, of material still less dense, and so far observable only during total eclipses of the sun.

Fig. 94 shows the relative positions of these different elements of the solar constitution.

5. A fifth subject deals with the *measurement of the sun's light* and the relative brightness of different parts of the solar surface.

6. Another most interesting and important topic relates to the amount of *heat* radiated by the sun, — the sun's *probable temperature* and the mechanism by which its heat-supply is maintained.

292. The Photosphere. — The sun's visible surface is called the *photosphere*, and when studied under favorable atmospheric condi-

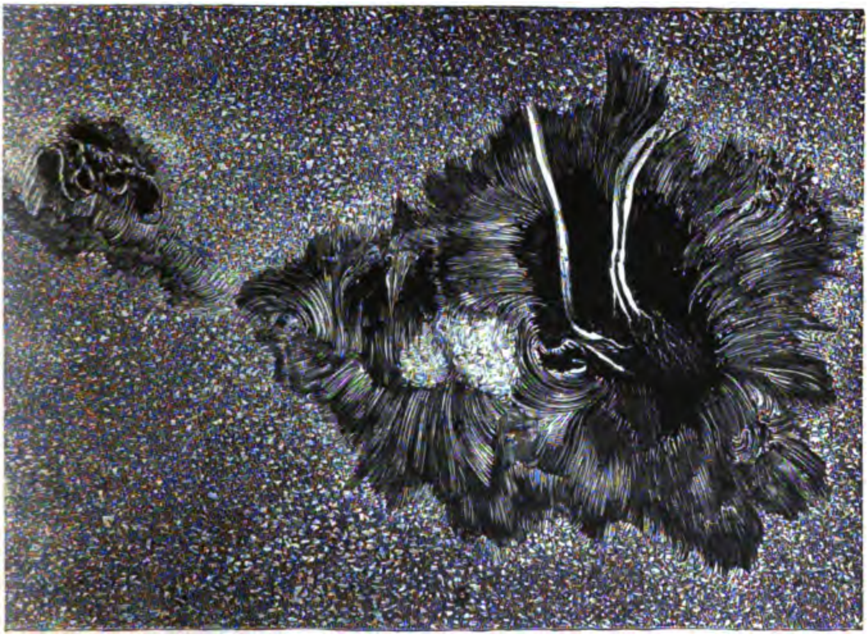

FIG. 95.

The Great Sun Spot of September, 1870, and the Structure of the Photosphere. From a Drawing by Professor Langley. From "The New Astronomy," by permission of the Publishers.

tions, with a rather low magnifying power, it looks like rough drawing-paper. With higher powers it is seen to be, as shown in Fig. 95, made up of a comparatively darkish background sprinkled over with grains, or "nodules," as Herschel called them, of something much more brilliant, — like snowflakes on gray cloth, according to Langley. These are from 400 to 600 miles across, and in the finest seeing are themselves resolved into more minute "granules." For the most

part, these nodules are about as broad as they are long, though of irregular form; but here and there, especially in the neighborhood of the spots, they are drawn out into long streaks. Nasmyth seems first to have observed this structure, and called the filaments "willow leaves." Secchi called them "rice grains." According to Huggins they were "dots"; and there was for a long time a pretty lively controversy as to their true form. Their shape, however, unquestionably varies very much in different parts of the surface and under different circumstances. They are probably luminous clouds[1] floating in a less luminous atmosphere.

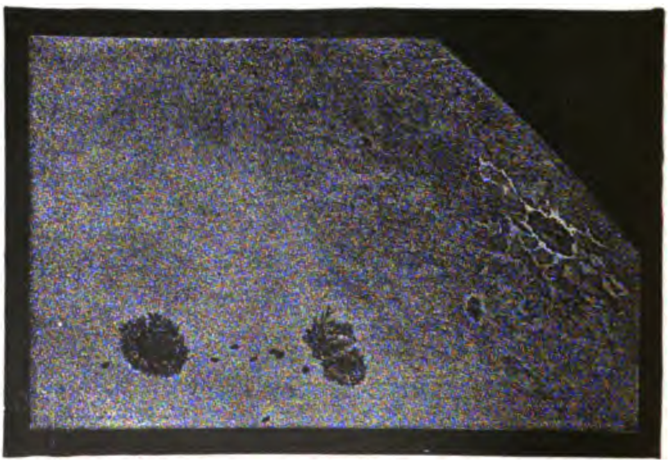

FIG. 96. — Faculæ at Edge of the Sun. (De La Rue.)

Near the edge the photosphere appears generally much less brilliant; but certain bright streaks called "faculæ" (from *fax*, a torch), which, though visible, are not very obvious at points further from the limb, become there conspicuous. These faculæ are elevations, probably of the same material as the rest of the photosphere, but elevated above the general level and intensified in brightness. When one of them passes off the edge of the sun, it is sometimes seen as a little projection. They are most abundant near the sun spots, and they are more conspicuous near the edge of the disc, as shown in Fig. 96, because the sun's surface is overlaid by a gaseous atmosphere which absorbs more of the light there than it does near the centre, and these faculæ push up through it like mountains.

[1] It was many years ago suggested by Stoney that these clouds are probably composed mainly of *carbon*, but this view is not yet by any means universally accepted.

293. The Sun Spots. — The appearance of a normal sun spot, Fig. 97, fully formed, is that of a dark central "*umbra,*" more or less nearly circular, with a fringing "*penumbra,*" composed of filaments directed radially. The umbra itself is not uniformly dark throughout, but is overlaid with filmy clouds which require a good telescope and helioscope to make them visible. Usually, also, in the umbra there are several round and very black spots, which are sometimes called "*nucleoli,*" but are often referred to as "Dawes' holes," after the name of their first discoverer. But while this is the appearance of what may be taken as a normal spot, very few are strictly normal. Most of them are more or less irregular in form. They are often gathered in groups with a common penumbra, and partly covered by brilliant "*bridges*" extending across from the outside photosphere. Often the umbra is out of the centre of the penumbra, or has a penumbra only on one side, and the penumbral

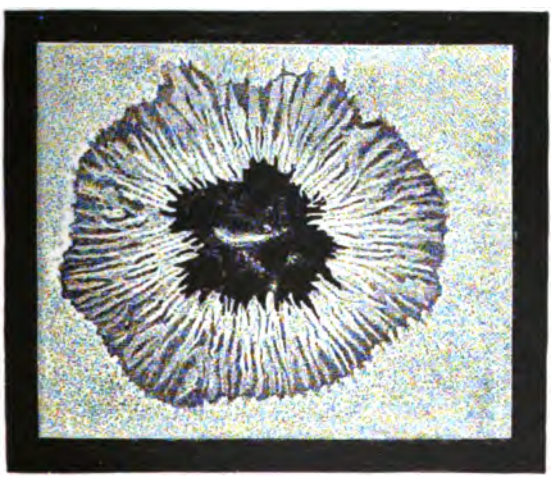

FIG. 97. — A Normal Sun Spot. (Secchi; modified.)

filaments, instead of being strictly radial, are frequently distorted in every conceivable way. In fact, the normal spots form a very small proportion of the whole number.

The darkest portions of the umbra are dark only by contrast. Photometric observations (by Langley) show that even the nucleus gives at least one per cent as much light as a corresponding area of the photosphere; that is to say, as we shall see hereafter, the darkest portion of a sun spot is brighter than a calcium light.

294. The spots are generally believed to be *depressions* in the photosphere, filled with gases and vapors which are cooler than the surrounding portions, and therefore absorb a considerable proportion of light. The evidence that they are "hollows" is the change in the appearance of a spot as it travels across the disc. According to Wilson of Glasgow, who first discovered the fact (if it really is one) more than a century ago, the umbra of a normal spot is central at the centre of the disc, but as the spot approaches the limb the penumbra becomes narrower on the inner edge, and vanishes entirely before the spot disappears around the limb, — the appearance (Fig. 98) being precisely such as would be shown by a saucer-shaped cavity in the surface of a globe if the bottom of the cavity were painted black to represent the umbra, and the sloping sides gray for the penumbra. Evidently observations upon any single spot would be inconclusive, because spots are extremely irregular in form and behavior; but by observing several hundred the truth ought to come

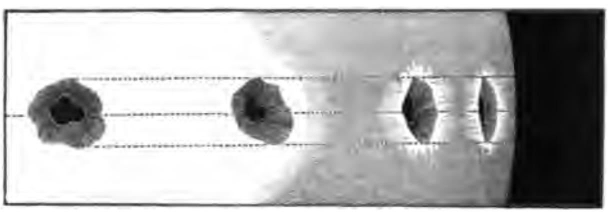

FIG. 98. — Sun Spots as Cavities.

out distinctly, and until recently astronomers were practically unanimous in accepting Wilson's theory. Lately, however, some high authorities have called it in question; partly on the evidence drawn from solar photographs and a very extensive series of sun-spot drawings by Mr. Howlett, an observer of great experience, and partly on account of certain thermal observations referred to in Art. 301*. At present the subject is being vigorously rediscussed, and opinion is much divided as to the real level of a sun-spot nucleus. It is quite clear that in some cases the depression must be very slight, if it exists at all.

295. The *penumbra* is usually composed of "thatch-straws," or long drawn-out granules of photospheric matter, which, as has been said, converge in a general way towards the centre of the spot. At the inner edge the penumbra, from the convergence of these filaments, is usually brighter than the outer. The inner ends of the filaments are generally club-formed; but sometimes they are drawn out into

fine points, which seem to curve downward into the umbra like the rushes over a pool of water. The outer edge of the penumbra is usually pretty definite, and the penumbra there is darker. Around the spot the photosphere is much disturbed and elevated into faculæ, which sometimes radiate outward from the spot like streams of lava from a crater, though, of course, they are really nothing of the sort.

296. Dimensions of Sun Spots. — The diameter of the umbra of a sun spot ranges all the way from 500 to 1000 miles in the case of a very small one, to 50000 or 60000 miles in the case of the larger ones. The penumbra surrounding a group of spots is sometimes 150000 miles across, though that would be rather an exceptional size. Not infrequently sun spots are large enough to be seen by the naked eye, and they have been often so seen at sunset or through a fog. The depth by which the umbra is depressed below the general surface of the photosphere is very difficult to determine, but according to Faye, Carrington, and others, it seldom exceeds 2500 miles, and more often is less than 1000, and sometimes insensible.

297. Development and Changes of Form. — Generally the origin of a sun spot fails to be observed. It begins from an insensible point, and rapidly grows larger, the penumbra usually appearing only *after the nucleus is fairly developed.*

If the disturbance which causes the spot is violent, the spot usually breaks up into several fragments, and these again into others which tend to separate from each other. At each new disturbance the forward portions of the group show a tendency to advance eastward on the sun's surface, leaving behind them a trail of smaller spots.

298. The "segmentation" of a spot, as Faye calls it, is usually effected by the formation of a "bridge," or streak of brilliant light, which projects itself across the penumbra and umbra from the outside photosphere. These bridges are mere extensions of the surrounding faculæ, and are often intensely bright.

Occasionally a spot shows a distinct cyclonic motion, the filaments being drawn inward spirally; and in different members of the same group of spots the cyclonic motions are not seldom in opposite directions.

When a spot at last vanishes it is usually by the rapid encroachment of the photospheric matter, which, as Secchi expresses it, appears to "fall pell-mell into the cavity," completely burying it and leaving its place covered by a group of faculæ. Figs. 99–104 (see page 201) show the changes which took place in the great spot of September, 1870. They are from photographs by Mr. Rutherfurd of New York, and are borrowed from "The New Astronomy" of Professor Langley, through the courtesy of his publishers.

299. Spots within 15° or 20° of the sun's equator generally, on the whole, drift a little *towards* it, while those in higher latitudes drift *away from* it; but the motion is slight, and exceptions are frequent.

In and around the spot itself the motion is usually *inward towards the centre*, and *downward at* the centre. Not infrequently the fragments at the inner end of the penumbral filaments appear to draw off, move towards the centre of the spot, and then descend. Occasionally, though seldom, the motion is vigorous enough to be detected by the displacement of lines in the spectrum.

300. Duration. — The duration of the spots is very various, but, astronomically speaking, they are always short-lived phenomena, sometimes lasting for only a few days, more frequently, perhaps, for a month or two. In a single instance, a spot has been observed through as many as eighteen successive revolutions of the sun.

301. Distribution. — It is a significant fact that the spots are confined mostly to two zones of the sun's surface between 5° and 40° of latitude north and south. A few are found near the equator, none

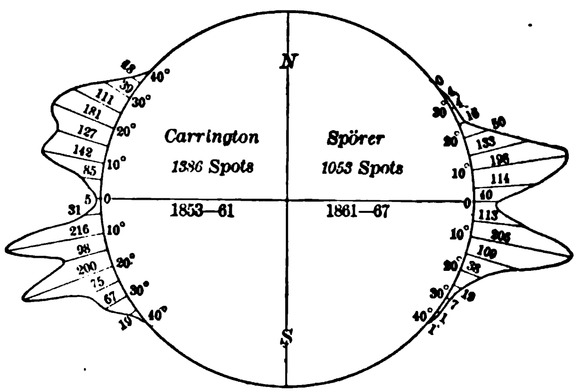

FIG. 105. — Distribution of Sun Spots in Latitude.

beyond the latitude of 45°. Fig. 105 shows the distribution of several thousand spots as observed by Carrington and Spörer.

Occasionally, what Trouvelot calls "veiled spots" are seen beyond the 45° limits — grayish patches surrounded by faculæ, which look as if a dark mass were submerged below the surface and dimly seen through a semi-transparent medium.

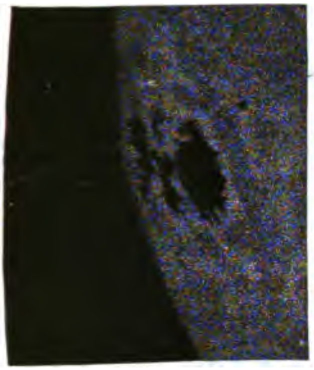

FIG. 99. — Sept. 19.

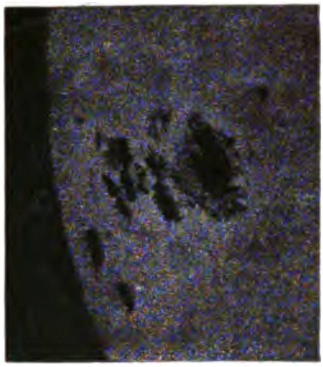

FIG. 100. — Sept. 20.

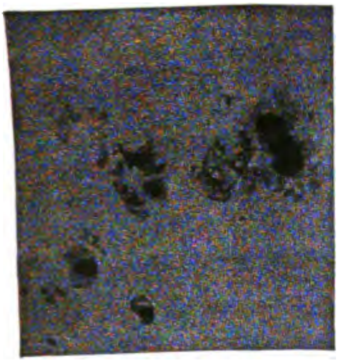

FIG. 101. — Sept. 21.

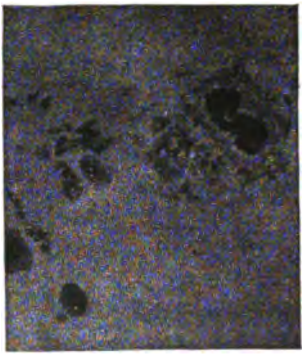

FIG. 102. — Sept. 22.

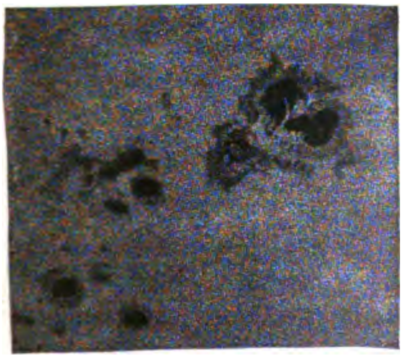

FIG. 103. — Sept 23.

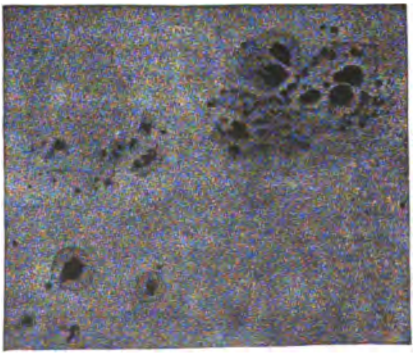

FIG. 104. — Sept. 26.

The Great Sun Spot of 1870.

301*. Radiation and Temperature of Sun Spots. — Thermopile observations upon sun spots, first made in 1845 by Henry, and since then by numerous other observers, show that as the spots are darker, so also they radiate less heat than other portions of the solar surface. But while the umbra of a spot generally emits less than one per cent as much light, it ordinarily radiates nearly *fifty per cent* as much heat as an equal area of the neighboring photosphere, and if the spot is near the edge of the disc the percentage rises much higher; indeed, Langley and Frost have met with cases where the radiation of a spot has appeared actually to exceed that of the brighter regions surrounding it, — an important and rather perplexing observation.

It has generally been inferred hitherto that the lower heat radiation of a sun spot indicates *a lower temperature* than that of the surrounding photosphere, but this does not necessarily follow. Two masses in contact and at the same temperature, *but of different constitution*, may differ widely both in luminosity and in their radiating power for the invisible rays; for instance, the mantle and the gas-flame of a Welsbach burner. At present it is perhaps still uncertain whether the spots are cooler or warmer than the photospheric "mantle."

302. Theories as to the Nature of the Spots. — We first mention (*a*) the theory of Sir William Herschel, because it still finds place in certain text-books, though certainly incorrect. His belief was that the spots were openings through two luminous strata, which he supposed to surround the central globe of the sun. This globe he supposed to be *dark* (*and even habitable!*). The outer stratum, the photosphere, was the brighter of the two, and the opening in it the larger, while the inner shell between it and the solid globe was of less luminous substance, and formed the penumbra. He thought the opening through these might be caused by volcanoes on the globe beneath.

303. (*b*) Another theory, now generally abandoned, but recently endorsed by Proctor in his "Old and New Astronomy," was proposed independently both by Secchi and Faye about 1868. They supposed that the spots were openings in the photosphere caused by the bursting outward of the imprisoned gases underneath it.

They explained the darkness of the centre of the spot by the fact that a heated gas at a given temperature has a lower radiating power and sends out much less light than a *liquid surface*, or than *clouds* formed by the condensation of the same material at even a lower temperature. This is true

of gases at low pressure, but not of gases under great compression, such as must be the case within the body of the sun. Besides, if the gases possessed the small radiating power necessary to this theory, they would also possess small *absorbing* power, and therefore would be transparent; the inner side of the photosphere on the opposite side of the sun would therefore be visible through the opening, so that the centre of such an eruption would not be *dark*, but, if anything, brighter than the general solar surface. Moreover, as we now know from the spectroscopic evidence, the motion at the centre of a spot is usually *inward*, not *outward*.

304. (*c*) Faye more recently has proposed and now maintains a theory which has numerous good points about it, and is accepted by many; *viz.*, that the spots are analogous to storms on the earth, being *cyclones*, due to the fact that the portions of the sun's surface near the equator make their revolution in a shorter time than those in higher latitudes. This causes a relative drift in adjacent portions of the photosphere, and according to him gives rise to *vortices* or *whirlpools* like those in swiftly running water. The theory explains the distribution of the spots (which abound precisely in the regions where this relative drift is at the maximum) and many other facts, such as their "segmentation." According to it, however, *all* spots should be cyclonic, and the spiral motion of all the spots in the southern hemisphere should be *clock-wise*, while in the northern hemisphere they should be *counter-clock-wise*. Now, as a matter of fact, only a very few of the spots show such spiral motions, and there is no such agreement in the general direction of the motion as the theory requires.

Faye attempts to account for this by saying that we do not see the vortex itself, but only the cloud of cooler materials which is drawn together by the down-rushing vortex, itself hidden beneath this cloud. Still, it would seem that in such a case the cloud itself should gyrate. Moreover, the relative drift of the adjacent portions of the photosphere is too small to account for the phenomena satisfactorily. In the solar latitude of 20° two points separated by 1′ of the sun's surface (123 miles) have a relative daily drift of only about four and one-sixth miles, insufficient to produce any sensible whirling.

305. (*d*) Secchi's later theory. He supposed the spots to be due to eruptions from the inner portions of the sun's surface, not *in* the spot, however, but only *near* it; the spot itself being formed by the settling down upon the photosphere of materials thrown out by the eruption and cooled by their expansion and their motion through the upper regions. We have, however, in fact, as a usual thing,

not a single eruption, but a ring of eruptions all around every large spot, all of them converging their bombardment, so to speak, upon the same centre, — a fact very difficult to explain if the spot originates in the eruption, but not difficult to understand if the eruptions are the result of the spot.

Perhaps the true explanation may be that when an eruption occurs at any point, the *photosphere somewhere in the neighborhood settles down in consequence of the diminution of the pressure beneath,* thus forming a "*sink,*" so to speak, which is of course covered by a greater depth of absorbing vapors above, and so looks dark.

306. (*e*) Mr. Lockyer, in his recent work on the "Chemistry of the Sun," revives an old theory, first suggested by Sir John Herschel and accepted by the late Professor Peirce, that the spots are not formed by any action from within, but by *cool matter descending from above,* — matter very likely of meteoric origin; but it is difficult to see how the distribution of the spots with reference to the sun's equator can be accounted for in this way.

Schaeberle, of the Lick Observatory, accounts for them in a somewhat similar manner, but considers that the descending streams consist of matter returning to the sun after having been projected from it by some repulsive force to distances of hundreds of millions of miles. See Art. 331.

306*. (*f*) The newest theory of those that deserve any consideration is one proposed by E. Oppolzer, of Vienna, in 1893, and is based largely on the recent researches of meteorologists upon the thermal effects of vertical currents in our own atmosphere. Such currents are supposed to rise periodically from the polar regions of the sun, to drift slowly toward its equator, and to descend in the spot-zones, *becoming heated and "dried" in their descent,* thus forming in the photosphere hollows which are filled with metallic vapors in a purely gaseous condition. According to this theory, the temperature of a spot is *higher* than that of the surrounding medium. In many ways the theory corresponds admirably with facts, explaining better than any other the peculiar character of the sun-spot spectrum (Art. 321), Spoerer's law of sun-spot latitudes (Art. 307*), and the otherwise puzzling observations of Langley and Frost referred to in Art. 301*. But the "periodical polar streams" remain themselves to be accounted for.

On the whole, it is impossible to say that the problem of the origin of sun spots is yet satisfactorily solved. There is no question that

sun spots are closely associated with eruptions from beneath; but which is cause and which effect, or whether both are due to some external action, remains undetermined.

307. Periodicity of Sun Spots. — In 1843 Schwabe, of Dessau, by the comparison of an extensive series of observations running over nearly thirty years, showed that the sun spots are *periodic*, being at times vastly more numerous than at others, with a roughly regular recurrence every ten or eleven years. This had been surmised by Horrebow more than a century before, though not proved.

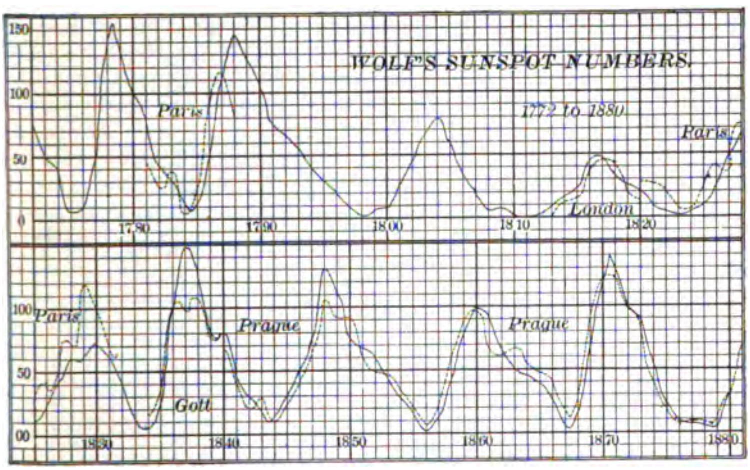

Fig. 106. — Wolf's Sun-spot Numbers.

Subsequent study fully confirms this remarkable result of Schwabe. Wolf of Zürich has collected all the observations discoverable and finds a pretty complete record back to 1610. From these records is constructed the annexed diagram, Fig. 106. The ordinates of the curve represent what Wolf calls his "relative numbers,"[1] which he has adopted as representing the spottedness.

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[1] This "relative number" is formed in rather an arbitrary manner from the observations which Wolf hunted up as the basis of his investigation. The formula is, r (the relative value) $= k\,(10\,g + f)$, in which g is the number of groups and isolated spots observed, f the total number of spots which can be counted in these groups and singly, while k is a coefficient which depends upon the observer and the size of his telescope; it is large for a small telescope

The average period is eleven and one-tenth years, but, as the figure shows, the spot maxima are quite irregular, both in time and extent. The two last maxima occurred in 1883 (two years behind time) and in 1893. Both were feeble; 64 and 70 respectively on the scale of Fig. 106. During a maximum the surface of the sun is never free from spots, from twenty-five to fifty being frequently visible at once. During a minimum, on the other hand, weeks often pass without the appearance of a single one.

307*. Spoerer's Law of Sun-spot Latitudes. — Still another fact, as yet unexplained, and probably of great theoretical importance, has recently been brought out by Spoerer. Speaking broadly, the disturbance which produces the spots of a given sun-spot period first manifests itself in two belts about 30° north and south of the sun's equator. These belts then draw in toward the equator, and the sun-spot maximum occurs when their latitude is about 16°; while the disturbance gradually and finally dies out at a latitude of 8° or 10°, some twelve or fourteen years after its first outbreak. Two or three years before this disappearance, however, two new zones of disturbance show themselves. Thus, at the sun-spot minimum there are four well-marked spot-belts; two near the equator, due to the expiring disturbance, and two in high latitudes, due to the newly beginning outbreak;

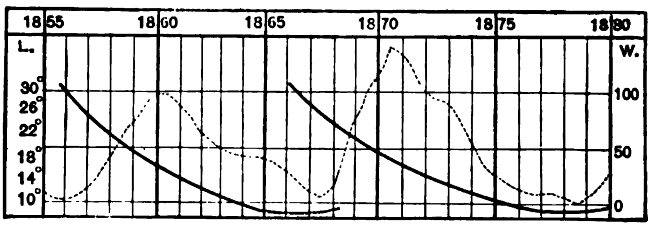

FIG. 106*. — Spoerer's Curves of Sun-spot Latitude.

and it appears that the true sun-spot cycle is from twelve to fourteen years long, each beginning in high latitudes before the preceding one has expired near the equator.

Fig. 106* illustrates this, embodying Spoerer's results from 1855 until 1880. The dotted curves show Wolf's sun-spot curve for that period, the vertical column at the right of the figure, marked W at the top, giving Wolf's "*relative numbers.*" The two continuous curves, on the other hand, give the solar latitudes of the two series of spots that invaded the sun's sur-

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and not very persistent observer, and approaches unity in proportion to the probable ratio between the actual total number of visible spots and the number which the observer has recorded.

face in those years, the scale of *latitudes* being on the left hand. The first series began in 1856 and ended in 1868; the second broke out in 1866 and lasted until 1880.

308. Possible Cause of the Periodicity. — The cause of sun-spot periodicity is not yet known. Attempts have been made to account for it by planetary influences, but with very doubtful success. Sir John Herschel suggested meteoric swarms moving in oval orbits with an eleven-year period, and with a perihelion distance so small that many of the meteors strike the sun's surface when passing perihelion; an idea still favored by Lockyer and some other authorities. Probably, however, the most general impression is that this rather irregular periodicity is more likely to be due not to external causes at all, but to something in the constitution of the photosphere and the rate at which the sun is losing heat: a gathering of deep-lying forces during a period of outward quiescence, followed by an outburst which relieves the internal strain.

309. Terrestrial Influence of the Sun Spots. — One correlation of sun spots with the earth is perfectly demonstrated. When the spots are numerous, magnetic disturbances (the so-called magnetic storms) are most numerous and violent upon the earth, a fact not to be wondered at since violent disturbances upon the sun's surface have been in many individual cases immediately followed by magnetic storms, with a brilliant exhibition of the Aurora Borealis. The nature and mechanism of the connection is as yet unknown, but of the fact there can be no question. The dotted lines in the figure of the sun-spot periodicity (Fig. 106) represent the magnetic storminess of the earth at the indicated dates; and the correspondence between these curves and the curve of spottedness makes it impossible to doubt the connection.

310. It has been attempted, also, to show that greater or less disturbance of the sun's surface, as indicated by the greater frequency of the sun's spots, is accompanied by *effects upon the meteorology of the earth*, upon its temperature, barometric pressure, storminess, and the amount of rainfall. The researches of Mr. Meldrum of Mauritius with respect to the cyclones in the Indian Ocean appear to bear out the conclusion that there may be some such connection in that case, but the general results are by no means decisive. In some parts of the earth the rainfall seems to be greater during a spot maximum; in others, less.

As to the temperature, it is still uncertain whether it is higher or lower at the time of a spot maximum. The *spots usually give less heat*

(as Henry, Secchi, and Langley have shown) than the general surface of the photosphere; but their extent is never sufficient to reduce the amount of heat radiated from the sun by as much as $\frac{1}{1000}$ part. On the other hand, when the spots are most numerous, the generally disturbed condition of the photosphere would, as Langley has shown, necessarily be accompanied by an increased radiation.

Dr. Gould considers that the meteorological records in the Argentine Republic between 1875 and 1885 show an indubitable connection between the *wind currents* and the number of sun spots. But the *demonstration* of such a relation really requires observations running through several spot periods. On the whole, it is now quite certain that whatever influence the sun spots exert upon terrestrial meteorology is very slight, if it exists at all.

310*. Possible Explanation of the Sun's Equatorial Acceleration. — Recent investigations by Wilsing have led him to the conclusion that the peculiar and perplexing equatorial acceleration of the sun depends upon its being in a *transitional* condition between a nebula and a solidified globe. It is tending towards a rotation uniform throughout, and will ultimately reach that state when the relative motions of different portions of its mass have been destroyed, as they will be, by internal friction. Probably they have already practically disappeared in the sun's interior, though still persisting at and near its surface. They die out so slowly, however, that it must require thousands, if not millions, of years to make them vanish entirely, and through any short period of a century or two they appear to us as permanent. Their explanation lies, therefore, not in the *present* constitution of the sun, but in its *past history.*

This view is substantially confirmed by an elaborate mathematical investigation by Professor Sampson of Durham University, published in the last (51st) volume of the Memoirs of the Royal Astronomical Society. Both writers concur in the conclusion that the present laws of surface drift upon the sun, as shown by the observations of Carrington, Dunér, and others, are not only possible, but extremely probable (though only temporary) consequences of the sun's slow condensation from a nebulous mass.

Still more recently (1901), however, Emden of Munich has attempted to show that the acceleration may be mathematically explained as a necessary consequence of physical laws applicable to a gaseous globe rotating and losing heat by radiation from its surface. The question can hardly be considered as finally settled even yet.

CHAPTER IX.

THE SPECTROSCOPE AND THE SOLAR SPECTRUM. — CHEMICAL ELEMENTS PRESENT IN THE SUN. — THE SUN-SPOT SPECTRUM. — DOPPLER'S PRINCIPLE. — THE CHROMOSPHERE. — THE SOLAR PROMINENCES. — THE CORONA.

311. About 1860 the spectroscope appeared in the field as a new and powerful instrument of astronomical research, at once resolving many problems as to the nature and constitution of the heavenly bodies which before had not seemed to be even open to investigation. It is hardly extravagant to say that its invention has done for astronomy almost as much as the invention of the telescope. The latter brings distant objects optically nearer and enables us to determine their positions in the heavens with an accuracy before impossible, and, in the case of the sun, moon, and planets, to study their forms and surface markings. It reveals also millions of stars and nebulæ otherwise invisible.

The spectroscope, on the other hand, enables us to study the light itself, and to read in it a record, more or less complete, of the chemical constitution and physical condition of the body from which the light proceeds, and to measure the rate of its motion towards or from us.

The essential part of the apparatus is either a prism or train of prisms, or else a diffraction grating,[1] which is capable of performing the same office of dispersing — that is, of spreading and sending in different directions — the light rays of different wave-lengths. If, with such a "*dispersion piece*," as it may be called (either prism or grating), one looks at a distant point of light, as a star, he will see instead of a point a long streak of light, red at one end and violet at the other. If the object looked at be not a point, but a *line of light* parallel to the edge of the prism or to the lines of the grating, then, instead of a mere colored streak without width, one gets a

[1] The grating is merely a piece of glass or speculum metal, ruled with many thousand straight, equidistant lines, from 5000 to 20000 in the inch. Usually the surface before ruling is accurately plane, but for some purposes the *concave* gratings, originated by Professor Rowland, are preferable.

spectrum, a colored band of light, which may show markings that will give the observer most valuable information. (Physics, pp. 444, 450–451.) For convenience sake it is usual to form this line of light by admitting the light through a narrow "*slit*," which is at one end of a tube having at the other end an achromatic object-glass at such a distance that the slit is in its principal focus. This tube with slit and lens constitutes the "*collimator*," so called because it is precisely the same as the instrument used in connection with the transit instrument to adjust its line of collimation (Art. 60).

Instead of looking at the spectrum with the naked eye, however, it is better in most cases to use a small telescope; called the "*view-telescope*," to distinguish it from the large telescope, to which the spectroscope is often attached.

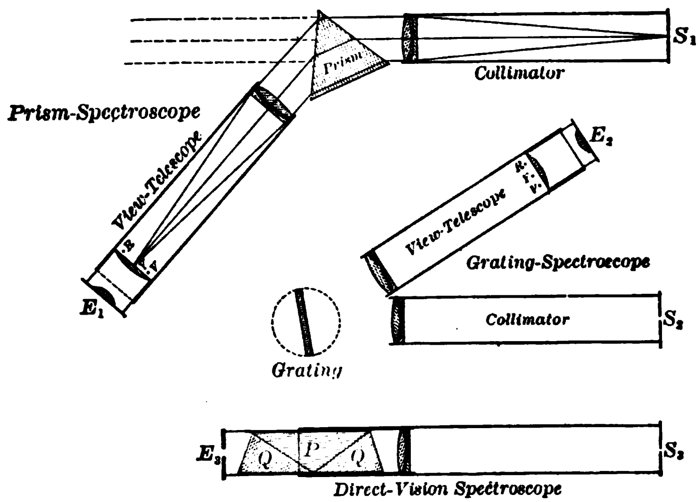


FIG. 107. — Different Forms of Spectroscope.

312. Construction of the Spectroscope. — The instrument, therefore, as usually constructed, and shown in Fig. 107, consists of three parts, — collimator, dispersion-piece, and view-telescope; but in the direct-vision spectroscope, shown in the figure, the view-telescope is omitted. If the slit, *S*, be illuminated by strictly homogeneous light all of one wave-length, say yellow, the "real image" of the slit will be found at *Y*. If at the same time light of a different wave-length be also admitted, say red, a second image will be formed at *R*, and the observer will see a spectrum with two "bright lines,"

the lines being really nothing more than *images of the slit.* If light from a candle be admitted, there will be an infinite number of these slit-images, close packed, like the pickets in a fence, without interval or break, and we then get a "*continuous*" spectrum. If we look at the light emitted by a so-called Geissler-tube, containing rarefied gas made luminous by an electric discharge, or that from an electric spark between two metallic balls, we shall get a "*discontinuous*" spectrum composed of numerous distinct *bright lines* of different color upon a dark background.

The spectrum of sunlight (either direct or reflected) is, on the contrary, characterized by a multitude of *dark lines* crossing a brilliant background of continuous spectrum — as if some of the 'fence pickets' had been destroyed, leaving gaps. These dark lines of the solar spectrum are known as the "Fraunhofer lines," because first studied and mapped by him in 1814, though some of them had been noticed by Wollaston in 1801.

313. Integrating and Analyzing Spectroscope. — If we simply direct the collimator of a spectroscope towards a distant luminous object, every part of the slit receives light from every part of the object, so that in this case every elementary streak of the spectrum is a spectrum of the entire body, without distinction of parts. A spectroscope used in this way is said to be an *integrating* instrument.

If, however, we interpose a lens (the object-glass of a telescope) between the luminous object and the slit, so as to have in the plane of the slit a distinct, real image of the object, then the top of the slit, for instance, will be illuminated wholly by light from one part of the object, the middle of it by light from another point, and the bottom by light from still a third. The spectrum formed by the top of the slit belongs, then, to the light from that particular point of the object whose image falls upon that part of the slit; and so of the rest. We thus separate the spectra of the different parts of the object, and so *optically analyze* it. An instrument thus used is spoken of as an "*analyzing spectroscope.*" The combined instrument formed by attaching a spectroscope to a large telescope for the spectroscopic observation of the heavenly bodies has been called by Mr. Lockyer[1] a "telespectroscope."

For solar purposes a grating spectroscope is generally better than a prismatic, being less complicated and more compact for a given power.

[1] Now Sir Norman Lockyer.

Fig. 108 represents the large grating spectroscope of the Halsted Observatory, as arranged for photography. It can be used visually also by substituting an ordinary view-telescope for the photographic tube.

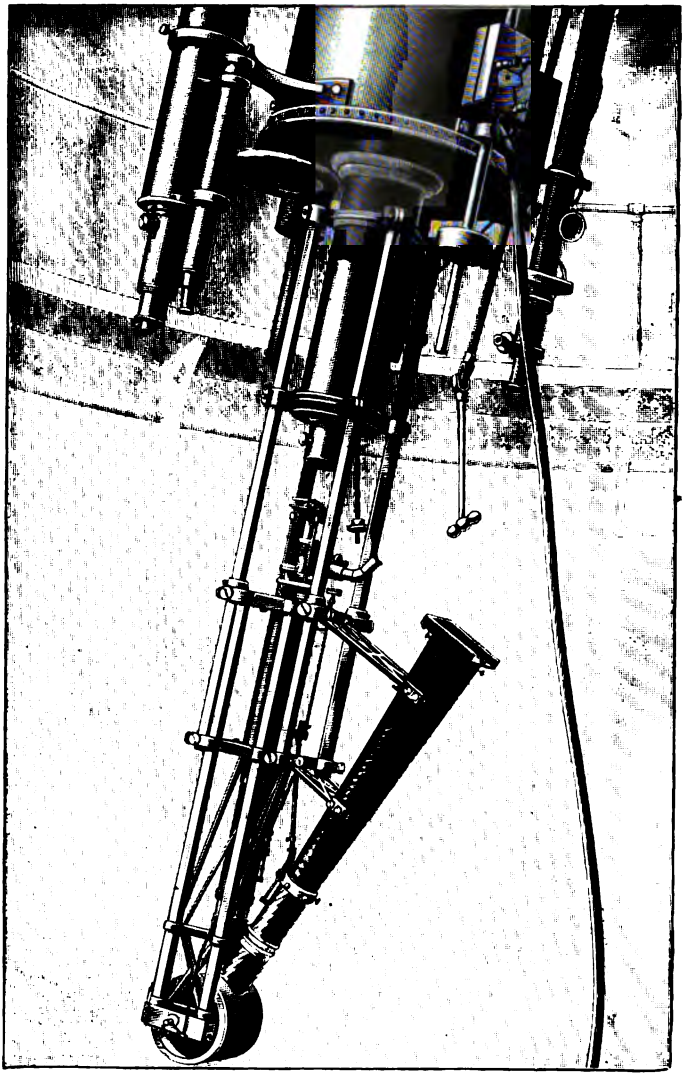


FIG. 108. — Large Spectroscope (fitted for photography). By permission of D. Appleton & Co.

314. Principles upon which Spectrum Analysis Depends.—These, substantially as announced by Kirchhoff in 1858, but with some later modifications, are the three following:—

1. A *continuous spectrum* is given by every incandescent body, the molecules of which so interfere with each other as to prevent their free, independent, luminous vibration; that is, by bodies which are either *solid* or *liquid*, or, if gaseous, are *under high pressure.*

2. The spectrum of a gaseous element, *under low pressure*, is discontinuous, made up of *bright lines*, often hundreds in number, and these lines are characteristic; that is, the same substance under similar conditions always gives the same set of lines, and generally does so even under widely different conditions.

3. A gaseous substance *absorbs* from white light passing through it *precisely those rays of which its own spectrum consists.* The spectrum of white light which has been transmitted through it then exhibits a "*reversed*" spectrum of the gas; that is, one which shows dark lines instead of the characteristic bright lines.

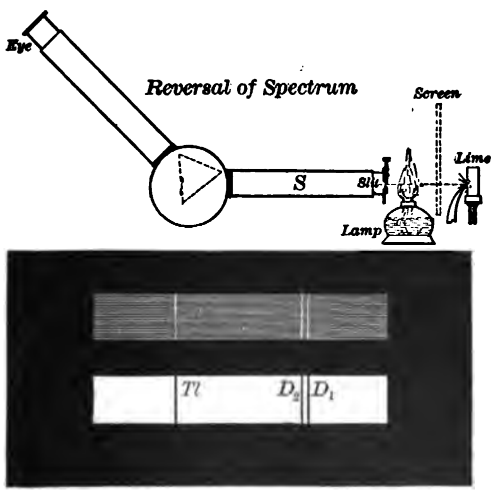


FIG. 109. — Reversal of the Spectrum.

Fig. 109 illustrates this principle. Suppose that in front of the slit of the spectroscope we place a spirit lamp with a little carbonate of soda and some salt of thallium upon the wick. We shall then get a spectrum showing the two yellow lines of sodium and the green line of thallium, *bright.* If now the lime-light be started right behind the lamp flame, we shall at once get the effect shown in the lower

figure, — a continuous spectrum crossed by *black* lines[1] just where the bright lines were before. Insert a screen between the lamp flame and the lime, and the dark lines instantly show bright again.

This experiment at once suggests the explanation of the solar spectrum. Its bright continuous background is due to the photospheric clouds which act the part of the lime-cylinder in the experiment, and the *dark lines* are produced by the absorbing action of gases and vapors which lie between us and the photosphere. Some of the lines, called "*telluric lines*," are produced in our own atmosphere, but most of them originate close to the solar surface, as we shall see. (Art. 319.)

315. Chemical Constituents of the Sun. — By taking advantage of these principles we can detect the presence of a large number of well-known terrestrial elements in the sun. The solar spectrum is crossed by dark lines, which, with an instrument of high dispersion, number several thousand, and by proper arrangements it is possible to identify among these lines many which are due to the presence in the sun's lower atmosphere of known terrestrial elements in the state of vapor. To effect the comparison necessary for this purpose, the spectroscope must be so arranged that the observer can have before him, side by side, the spectrum of sunlight and that of the substance to be tested. In order to do this, half of the slit is fitted with a little "*comparison prism*," so called (Fig. 110), which reflects into it the light from the sun, while the other half of the slit receives directly the light of some flame or electric spark. On looking into the eye-piece of the spectroscope, the observer will then see a spectrum, the *lower* half of which, for instance, is made by sunlight, while the *upper* half is made by light coming from an electric arc or spark containing the vapor of the metal — iron, for instance.


FIG. 110. — The Comparison Prism.

Photography may also be most effectively used in these comparisons instead of the eye. Fig. 111 is a rather unsatisfactory

[1] Their darkness is *relative* only; the "black" lines are actually a little brighter than before, but the background becomes so brilliant that they appear dark by contrast.

reproduction, on a reduced scale, of a negative recently made by Professor Trowbridge at Cambridge. The lower half is the violet portion of the spectrum of the sun, and the upper half that of the vapor of iron in an electric arc. The reader can see for himself with what absolute certainty such a photograph indicates the presence of iron in the solar atmosphere. A few of the lines in the photograph which do not show corresponding lines in the solar spectrum are due to other elements, partly impurities in the carbons of the electric arc.

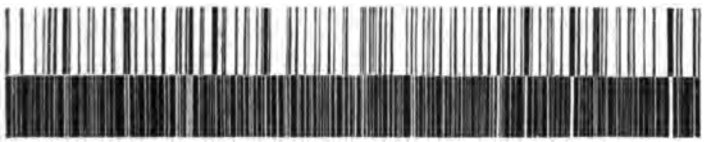


FIG. 111. — Comparison of the Solar Spectrum with that of Iron. From a negative by Professor Trowbridge with a concave-grating spectroscope.

316. Elements thus far Detected in the Sun. — As the result of such comparisons Rowland in 1890 gave the following list of 36 elements whose presence in the sun he regarded as certainly established, and it is probable that a few more will ultimately be added. The elements are arranged according to the intensity of the dark lines by which they are represented in the solar spectrum, while the appended figures denote the rank which each would hold if the arrangement had been based on the *number* instead of the intensity of the lines. In the case of iron the number exceeds 2000.

An asterisk denotes that lines of the element indicated appear often or always as *bright* lines in the spectrum of the chromosphere (Art. 322).

*Calcium, 11.	*Strontium, 23.	Copper, 30.
*Iron, 1.	*Vanadium, 8.	*Zinc, 29.
*Hydrogen, 22.	*Barium, 24.	*Cadmium, 26.
*Sodium, 20.	*Carbon, 7.	*Cerium, 10.
*Nickel, 2.	Scandium, 12.	Glucinum, 33.
*Magnesium, 19.	*Yttrium, 15.	Germanium, 32.
*Cobalt, 6.	Zirconium, 9.	Rhodium, 27.
Silicon, 21.	Molybdenum, 17.	Silver, 31.
Aluminium, 25.	Lanthanum, 14.	Tin, 34.
*Titanium, 3.	Niobium, 16.	Lead, 35.
*Chromium, 5.	Palladium, 18.	Erbium, 28.
*Manganese, 4.	Neodymium, 13.	Potassium, 36.

To these must now be added *helium*, which, however, shows itself only by *bright* lines in the chromosphere spectrum (Art. 323).

317. It will be noticed that all the bodies named in the list, carbon alone excepted, are *metals* (chemically hydrogen is a metal), and that a multitude of the most important terrestrial elements fail to appear; oxygen,[1] nitrogen, chlorine, bromine, iodine, sulphur, phosphorus, arsenic, and boron are all missing. We must be cautious, however, as to *negative* conclusions. It is quite conceivable that the spectra of these bodies under solar conditions may be so different from their spectra as presented in our laboratories that we cannot recognize them; for it is now quite certain that some substances, nitrogen, for instance, under different conditions, give two or more widely different spectra.

318. Mr. Lockyer's Views. — Mr. Lockyer thinks it more probable that the missing substances are not truly elementary, but are decomposed or "dissociated" on the sun by the intense heat, and so do not exist there, but are replaced by their components; he believes, in fact, that none of our so-called elements are really elementary, but that all are decomposable, and, to some extent, actually decomposed in the sun and stars, and some of them by the electric spark in our own laboratories. Granting this, a crowd of interesting and remarkable spectroscopic facts find easy explanation. At the same time the hypothesis is encumbered with great difficulties and has not yet been finally accepted by physicists and chemists. For a full statement of his views the reader is referred to his "Chemistry of the Sun."

319. The Reversing Layer. — According to Kirchhoff's theory the dark lines are formed by the passing of light from the minute solid and liquid particles of which the photospheric clouds are supposed to be formed, through vapors containing the substances which we recognize in the solar spectrum. If this be so, the spectrum of the gaseous envelope, which by its absorption forms the *dark* lines, should by itself show a spectrum of corresponding bright lines. The opportunities are of course rare when it is possible to obtain the spectrum of this gas-stratum alone by itself; but at the time of a total eclipse, at the moment when the sun's disc has just been obscured by the moon, and the sun's atmosphere is still visible beyond the moon's limb, if the slit of the spectroscope be carefully adjusted to the proper point, the observer ought to see this bright-line spectrum. The author succeeded in making this very observation at the Spanish eclipse of 1870. The lines of the solar spectrum, which up to the final obscuration of the sun had remained dark as usual (with the exception of a few belonging to the spectrum of the chromo-

[1] Runge (1896) found evidence, since confirmed, of the presence of oxygen.

sphere), were suddenly "reversed," and the whole length of the spectrum was filled with brilliant-colored lines, which flashed out quickly and then gradually faded away, disappearing in about two seconds, — a most beautiful thing to see. Substantially the same thing has since then been several times observed.[1]

320. The natural interpretation of this phenomenon is, that *the formation of the dark lines in the solar spectrum is mainly, at least, produced by a very thin layer close down to the photosphere,* since the moon's motion in two seconds would cover a thickness of only about 500 miles. It was not possible, however, to be certain that *all* the dark lines were reversed, and in this uncertainty lies the possibility of a different interpretation. Mr. Lockyer doubts the existence of any such *thin stratum.*[1] According to his views the solar atmosphere is very extensive, and those lines of iron, which correspond to the more complex combinations of its constituents, are formed only in the regions of lower temperature, *high up* in the sun's atmosphere. They should appear *early* at the time of an eclipse and *last long,* but not be very bright. Those due to the constituents of iron which are found only close down to the solar surface should be short and bright; and he thinks that the numerous bright lines observed under the conditions stated are due to such substances only.

321. Sun-spot Spectrum. — This is like the general solar spectrum, except that certain lines are much widened, while certain others are thinned, and sometimes the lines of hydrogen and a few other substances are reversed into bright lines; this is almost always the case with the *H* and *K* lines of calcium.

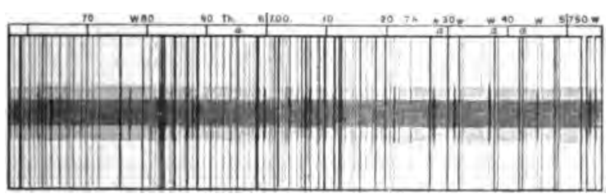


FIG. 111*. — Portion of Sun-spot Spectrum, from Photograph of 1893.

Fig. 111* is from a photograph of the yellow-green portion of a sun-spot spectrum which exhibits the principal peculiarities very

[1] During the eclipse of August, 1896, Mr. Shackleton, in Nova Zembla, obtained with a "prismatic camera" a fine photograph of the spectrum of the solar atmosphere at the instant after totality began. The picture fully confirms the author's visual observations, and appears to establish the reality of the "reversing layer." The bright lines of the chromosphere spectrum (Art. 322) are of course specially conspicuous, but all the prominent Fraunhofer lines are present, "reversed" and bright. (See also note at end of Chapter XI.)

well. The central dark stripe is the spectrum of the nucleus of the spot, the fainter stripe on each side of it being that of the penumbra. Rather more than half the lines are unaltered where they cross the spot-spectrum, about twenty (in this particular spot and in this piece of the spectrum) are more or less widened and darkened, and about half a dozen are thinned or obliterated. Several of the lines most conspicuous in the spot-spectrum are hardly visible at all in the general photospheric spectrum, the two "fish-bellies" at 5728 and 5731 being specially notable. In the case of certain elements. iron, for instance, only a few of their lines are ordinarily affected in the spot-spectrum, while the others remain unchanged, — a fact which Lockyer considers very important and significant.

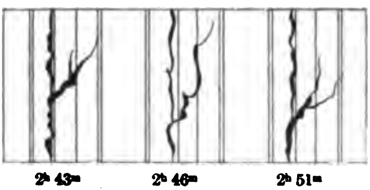


FIG. 112. — The *C* Line in the Spectrum of a Sun Spot, September 22, 1870.

Not infrequently it happens that certain lines of the spectrum are crooked and broken in connection with sun spots, as shown by Fig. 112. Such phenomena are caused, according to *Doppler's Principle*, by the swift motion of matter towards or from the observer. In the particular case shown in the figure, hydrogen is the substance, and the greatest motion indicated was towards the observer at the rate of about 300 miles a second — an unusual velocity. These effects are most noticeable, not *in* the spots, but near them, usually just at the outer edge of the penumbra.

The dark and apparently continuous spectrum which is due to the nucleus of a sun spot is not truly continuous, but under high dispersion is resolved into a range of extremely fine, close-packed, dark lines, separated by narrow spaces. At least, this is so in the green and blue portions of the spectrum: it is more difficult to make out this structure in the yellow and red. It appears to indicate that the absorbing medium which causes the darkness of a sun spot is *gaseous*, and not composed of precipitated particles like smoke.

321*. The Doppler-Fizeau Principle. — *The importance of this principle in the "New Astronomy" can hardly be exaggerated.* Briefly it is simply this: When the distance is increasing between us and a body which is emitting regular vibrations, the *number of waves* received by us in a second is *diminished*, and their *wave-length* is correspondingly *increased;* and *vice versâ* when the distance is decreasing. When, therefore, a luminous mass (say of hydrogen) is rushing towards us, or we towards it, all the lines in its spectrum

have their wave-lengths *diminished,* and are shifted from their normal positions in the spectrum towards the *blue end,* by an amount depending upon the velocity of the motion.

Doppler first announced the principle in 1843 as affecting *color;* Fizeau in 1848 pointed out its effect in shifting the *lines* of the spectrum. (See also Art. 802,* first fine-print paragraph.)

If V is the velocity of light (186330 miles a second), r the speed with which the observer is moving away from the luminous object, and s the speed with which the object is moving from the observer, then, letting λ represent the normal wave-length of a given line in the spectrum, and λ' its apparent wave-length as affected by the motions, we have $\lambda' = \lambda \frac{V+s}{V-r}$. If r and s are both small as compared with V, as of course is usually the case, this gives very approximately $\lambda' - \lambda = \lambda \frac{r+s}{V}$, or $\frac{\lambda' - \lambda}{\lambda} = \frac{v}{V}$, where v is simply the rate at which the distance between the observer and the object is *increasing* (to be taken *minus,* if *decreasing*). With our present spectroscopes a motion of less than a mile a second can be detected in this way.

322. The Chromosphere. — The *chromosphere* is a region of the sun's gaseous envelope which lies close above the photosphere, the "*reversing layer,*" if it exists at all, being only the most dense and hottest part of it. The chromosphere is so called, because, as seen for an instant, during a total solar eclipse, it is of a bright scarlet color, the color being due to the hydrogen which is its main constituent. It is from 5000 to 10000 miles in thickness, and in structure is very like a sheet of scarlet flame, not being composed of horizontal sheets, but of (approximately) upright filaments. Its appearance has been compared very accurately to that of "a prairie on fire"; but the student must carefully guard against the idea that there is any real "burning" in the case; *i.e.,* any *process of combination* between hydrogen and some other substance. Its spectrum is one of *bright* lines containing all that are ever observed in the prominences, besides many others. The lines of hydrogen and helium, and the *H* and *K* lines of calcium are the most conspicuous.

323. The Prominences. — At a total eclipse, after the totality has fairly set in, there are usually to be seen at the edge of the moon's disc a number of scarlet, star-like objects, which in the telescope appear as beautiful, fiery clouds of various form and size. These are the so-called "*prominences,*" which very non-committal name was given while it was still doubtful whether they were solar or lunar.

Photography, in 1860, proved that they really belong to the sun, for the photographs taken during the totality showed that the moon obviously moves over them, covering those upon the eastern limb, and uncovering those upon the western.

Their spectrum, first observed in 1868, is *gaseous*,[1] *i.e.*, *bright-lined*, the lines of hydrogen being especially conspicuous. There are, however, a number of other bright lines, — among them the violet H and K lines ascribed to calcium, and a yellow line known as D_3 because it is near the two D lines of sodium. This is due to "helium," first identified as a terrestrial element in 1895, in the gas liberated from clèveite (and certain other rare minerals).

In connection with this eclipse, Janssen, who observed it in India, found that the lines of the prominence spectrum were so bright that he was able to observe them the next day after the eclipse in full sunlight; and he also found that by a proper management of his instrument he could study the form and behavior of the prominences nearly as well without an eclipse as during one. Lockyer, in England, some time earlier had come to similar conclusions from theoretical grounds, and he practically perfected his discovery a few weeks later than Janssen, although without knowledge of what he had done. By a remarkable but accidental coincidence their discoveries were communicated to the French Academy on the same day; and in their honor the French have struck a medal bearing their united effigies.

324. How the Spectroscope Makes the Prominences Visible. — The only reason we cannot see the prominences at any time is on account of the bright illumination of our own atmosphere. We can screen off the direct light of the sun; but we cannot screen off the reflected sunlight coming from the air which is directly between us and the prominences themselves; a light so brilliant that the prominences cannot be seen through it without some kind of aid.

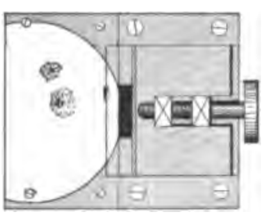


FIG. 113.
Spectroscope Slit adjusted for Observations of the Prominences.

The spectrum of this air-light is, of course, just the same as that of the sun, — a continuous spectrum with the same

[1] Tacchini has reported the existence of *white* prominences (giving a *continuous* spectrum), conspicuous to the eye and on the photographs in the eclipses of 1883 and 1889. The evidence, however, is hardly conclusive.

dark lines upon it. When, therefore, we arrange the apparatus as indicated in Fig. 113, pointing the telescope so that the image of the sun's limb just touches the slit of the spectroscope, then, if there is a prominence at that point, we shall have in our spectroscope two spectra superposed upon each other; namely, the spectrum of the air-illumination and that of the prominence. The latter is a spectrum of *bright lines*, or, if the slit is opened a little, of bright *images* of whatever part of the prominence may fall within the edges of the slit. Now, the brightness of these images is not affected by any increase of dispersion in the spectroscope. Increase[1] of dispersion merely sets these images farther apart, without making them fainter. The spectrum of the aerial illumination, on the other hand, is made very faint by its extension; and, moreover, it *presents dark lines* (or *spaces* when the slit is opened) precisely at the points where the bright images of the prominences fall.

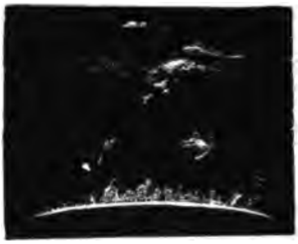
A spectroscope of dispersive power sufficient to divide the two E lines, attached to a telescope of four or five inches aperture, gives a very satisfactory view of these beautiful objects; the *red* image corresponds to the C line, and is by far the best for such observations, though the D_3 line or the F line can also be used. When the instrument is properly adjusted, the slit opened a little, and the image of the sun's limb brought exactly to the edge of the slit, the observer at the eye-piece of the spectroscope will see things about as we have attempted to represent them in Fig. 114, as if he were looking at the clouds in an evening sky through a slightly opened window-blind.[2] (See also Art. 326*.)

FIG. 114.
The Chromosphere and Prominences seen in the Spectroscope.

325. Different Kinds of Prominences; Their Forms and Motions. — The prominences may be broadly divided into two classes, — the

[1] Too high dispersion injures the definition, however, because the lines in the spectrum of hydrogen are rather broad and hazy.

[2] The observation of prominences in this manner was first effected by Huggins (now Sir William Huggins) in 1868.



Clouds.

Diffuse.

Filamentary.

Stemmed.

Plumes.

Horns.

Fig. 115.

Quiescent Prominences. Scale 75,000 Miles to the Inch. From "The Sun," by Permission of D. Appleton & Co.

Vertical Filaments.

Prominences Sept. 7, 1871, 12.30 P.M.

Cyclone.

Same at 1.15 P.M.

Flame.

Jets near Sun's Limb, Oct. 5, 1871.

FIG. 116.

Eruptive Prominences. From "The Sun." By Permission of D. Appleton & Co.

quiescent or diffused, and the *eruptive* or "metallic," as Secchi calls them, because they show in their spectrum the lines of many metals besides hydrogen. The former, illustrated by Fig. 115 (see p. 222), are immense clouds, often 60000 miles in height, and of corresponding horizontal dimensions, either resting upon the chromosphere or connected with it by slender stems like great banyan-trees. They are not very brilliant, and are composed almost entirely of hydrogen and "helium." They often remain nearly unchanged for days together as they pass over the sun's limb. They are found on all portions of the disc, at the poles and equator as well as in the spot zones. Some of them are cloud-like forms floating entirely detached from the sun's surface.

Usually these clouds are simply the remnants of prominences which appear to have been thrown up from below, but in some cases they actually form and grow larger without any visible connection with the chromosphere — a fact of considerable importance, as showing in those regions the presence of hydrogen, invisible to our spectroscopes until somehow or other it is made to give out the rays of its familiar spectrum. All the forms and motions of the prominences, it may be said further, seem to indicate the same thing — that they exist and move, not *in a vacuum,* but in a medium of density comparable with their own, as clouds do in our own atmosphere.

326. The *eruptive* prominences, on the other hand, are brilliant and active, not *usually* so large as the quiescent, but at times enormous, reaching elevations of 100000, 200000, or even 400000 miles. They are illustrated by Fig. 116. Most frequently they are in the form of spikes or flames; but they present also a great variety of other fantastic shapes, and are sometimes so brilliant as to be visible with the spectroscope on the surface of the sun itself, and not merely at the limb. Generally prominences of this class are associated with active sun spots, while both classes appear to be connected with the faculæ. The figures given are from drawings of individual prominences that have been observed by the author at different times.

These solar clouds are most fascinating objects to watch, on account of the beauty of their forms, and the rapidity of their changes. In the case of the eruptive prominences the swiftness of the changes is sometimes wonderful — portions can be actually seen to move, and this implies a real velocity of at least 250 miles a second, so that it is no exaggeration to speak of such phenomena as veritable "explosions"; of course, in such cases the lines in the spectrum are

greatly broken and distorted, and frequently a "magnetic storm" follows upon the earth, with a brilliant Aurora Borealis.

The number visible at a single time is variable, but it is not very unusual to find as many as twenty on the sun's limb at once.

326*. Photography of the Prominences and Chromosphere.—With our present sensitive dry plates, and by utilizing the *H* and *K* lines, which, like the hydrogen lines, are always reversed in the spectrum of the chromosphere and prominences, it has become perfectly easy to photograph these objects with a spectroscope arranged like Fig. 108. Professor George E. Hale, of Chicago, and Deslandres, of Paris, first and almost simultaneously, took up the subject in 1890, and have been especially successful. Both have devised ingenious arrangements (called *spectro-heliographs*),[1] by which they are able to obtain pictures of the chromosphere and prominences around the whole circumference of the sun at once.

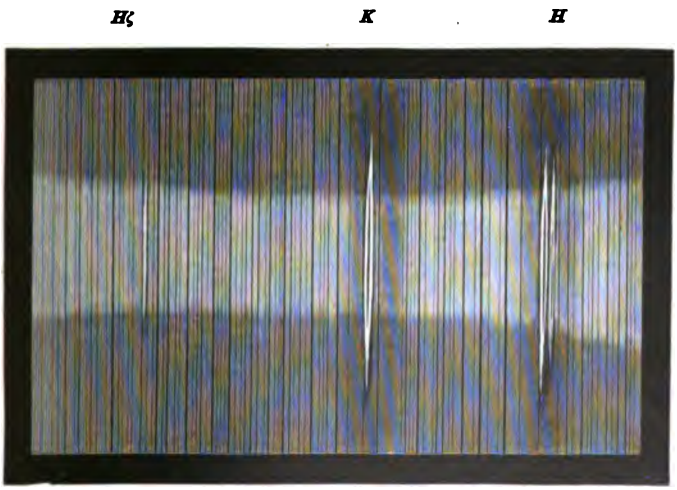


FIG. 116*. — *H* and *K* in Chromosphere Spectrum. By permission of D. Appleton & Co.

It should be noted that, while in the ordinary solar spectrum *H* and *K* are broad, hazy, dark *bands* or *shades*, in the spectrum of a prominence they are thin, bright lines just in the middle of the dark bands. Moreover, *H* is *double*, *i.e.* there is a strong, bright line of hydrogen close to the still brighter calcium line (which occupies the middle of the band) and on its red-ward side.

Fig. 116* is from a photograph of the *chromosphere* spectrum, made by setting the slit of the spectroscope tangential to the edge of the sun. The hydrogen line below *H* is shown and also a second hydrogen line above *K*,

[1] See Addendum A, following page 580.

marked $H\zeta$. But the most notable feature is the *double reversal* of K, H, and its hydrogen companion (which is known as $H\epsilon$). In the spectrum of *faculæ*, H and K are usually bright and doubly reversed in the same manner as in the chromosphere, while over the spots, though usually reversed, they are seldom, if ever, doubled.

327. The Corona. — This is a halo, or "glory," of light which surrounds the sun at the time of the total eclipse. From the remotest times it has been well known, and described with enthusiasm, as being certainly one of the most beautiful of natural phenomena.

FIG. 117. — Corona of the Eclipse of 1871. By permission of D. Appleton & Co.

The portion of the corona nearest the sun is almost dazzlingly bright, with a greenish, pearly tinge which contrasts finely with the scarlet blaze of the prominences. It is made up of streaks and filaments which on the whole radiate outwards from the sun's disc, though they are in many places strangely curved and intertwined. Usually these filaments are longest in the sun-spot zones, thus giv-

ing the corona a more or less quadrangular figure. At the very poles of the sun, however, there are often tufts of sharply defined threads.

For the most part the streamers have a length not much exceeding the sun's radius, but some of them at almost every eclipse go far beyond this limit. In the clear air of Colorado during the eclipse of 1878, two of them could be traced for five or six degrees, — a distance of at least 9 000000 miles from the sun. A most striking feature of the corona usually consists of certain dark rifts which reach straight out from the moon's limb, clear to the extremest limit of the corona.

The corona varies much in brightness at different eclipses, and of course the details are never twice the same. Its total light under ordinary circumstances is at least two or three times as great as that of the full moon.

328. Photographs of the Corona. — While the eye can perhaps grasp some of its details more satisfactorily than the photographic plate can do, it is found that drawings of the corona are hardly to be trusted. At any rate, it seldom happens that the representations of two artists agree sufficiently to justify any confidence in their scientific accuracy. Photographs, on the other hand, may be trusted as far as they go, though they may fail to bring out some things which are conspicuous to the eye. Fig. 117 is from a photograph of the Indian eclipse of 1871, and 117* is from the photograph of the Egyptian eclipse of 1882, when a little comet was found close to the sun.

FIG. 117*. — Corona of the Egyptian Eclipse, 1882.

Of course, as in the case of the prominences, the only reason we cannot see the corona without an eclipsed sun is the illumination of the earth's atmosphere. If we could ascend above our atmosphere, and manage to exist

and to observe there, we could see it by simply screening off the sun's disc. So long, however, as the brightness of the illuminated air is more than about sixty times that of the corona, it must remain invisible to the eye. Sir William Huggins has thought that it might be possible by means of photographs to detect differences of illumination less than $\frac{1}{60}$ (the limit of the eye's perception), and so to obtain pictures of the corona at any time; especially as it appears that the coronal light is far richer in ultra-violet rays (the photographic rays) than the general sunlight with which the air is illuminated. Thus far, however, no success has been obtained either by himself or by others who have made the attempt.

329. Spectrum of the Corona. — This was first definitely observed in 1869 during the eclipse which passed over the western part of the United States in that year. It was then found that its most remarkable characteristic is a bright line in the green, which the writer incorrectly identified as coinciding with the dark line at 1474 on the scale of Kirchhoff's map ($\lambda = 5317$). This line was also seen by two or three other observers, but either not recognized as belonging to the corona, or differently identified.

This result was very puzzling, since the line, also conspicuous in the chromosphere spectrum, is ascribed to *iron* by Ångström and other authorities. The mystery was cleared up by the eclipse-photographs[1] obtained since 1896, which show that the real corona line is not "1474" at all, but is slightly more refrangible with a wave-length of 5304, and has no corresponding line either in the ordinary solar spectrum or in that of the chromosphere. It is now generally referred to an hypothetical element provisionally named *coronium*, lighter than hydrogen, existing upon the earth, like helium, only sparingly if at all, and as yet unrecognized in our laboratories.

Besides this conspicuous green line, the hydrogen lines are also faintly visible in the spectrum of the corona; and by means of a photographic camera used during the Egyptian eclipse of 1882, it was found that the upper or violet portion of the spectrum is very rich in lines, among which H and K were specially conspicuous. Later observations, however, in 1893 and 1896, have made it nearly certain that these lines were not really coronal, but only due to reflection in our own atmosphere of light from the chromosphere and prominences. There is also, through the whole spectrum, a faint continuous background. In it some observers have reported the presence of a few of the more conspicuous dark lines of the ordinary solar spectrum, but the evidence on this point is rather conflicting.

When the corona is photographed with a "prismatic camera," which has a prism or prisms in front of its lens, the picture is com-

[1] See notes on pages 231 and 267.

posed of several rings (seven in 1893), all of which, except the green one, are very faint and lie in the violet portion of the spectrum; the brightest of them falls just below the *H* line. They are all probably due to "coronium." The plate at the same time also shows numerous other partial rings which are quite different in appearance, and are clearly due to the hydrogen, helium, and calcium of the prominences. As the evidence now stands, it is not probable that either of these elements exists in the corona.

330. Nature of the Corona. — It is evident that the corona is a truly solar and not merely an optical or atmospheric phenomenon, from two facts: first, *the identity of detail in photographs made at widely separate stations.* In 1871, for instance, photographs were obtained at the Indian station of Bekul, in Ceylon, and in Java, three stations separated by many hundreds of miles; but, excepting minute differences of detail, such as might be expected to have resulted from the changes that would naturally go on in the corona during the half-hour while the moon's shadow was travelling from Bekul to Java, all the photographs agree exactly, which of course would not be the case if the corona depended in any way upon the atmospheric conditions at the observer's station.

Second (but *first* historically), *the presence of bright lines in the spectrum of the corona* proves that it cannot be a terrestrial or lunar phenomenon, by demonstrating the presence in the corona of a *self-luminous gas*, which observation fails to find either near to the moon or in our own atmosphere. It must, therefore, be at the sun.

But while it is thus certain that the corona contains luminous gas, it also is very likely that finely divided solid or liquid matter may be present in the corona; that is, fog or dust of some kind, as is indicated by the partial polarization of its light.

331. The corona cannot be a true "solar *atmosphere*" in any strict sense of the word. No gaseous envelope in any way analogous to the earth's atmosphere could possibly exist there in gravitational equilibrium under the solar conditions of pressure and temperature. The corona is probably a phenomenon due somehow to the intense activity of the forces there at work; meteoric matter, cometic matter, matter ejected from within the sun, are all concerned.

That this matter is inconceivably rare is evident from the fact that in several cases comets have passed directly through the corona without experiencing the least perceptible disturbance of their mo-

tions. It is altogether probable that at a very few thousand miles above the sun's surface its density becomes far less than that of the best vacuum we can make in an electric lamp.

No wholly satisfactory theory of the corona has yet been found. Before 1869 it was very generally regarded as a purely optical phenomenon, either due to diffraction at the limb of the moon, or, like rainbows and halos, produced in our own atmosphere. Later, some sought an explanation in meteoric matter descending upon the sun from interplanetary space. Many, recognizing the striking resemblance between the appearance of the corona and that of the Aurora Borealis, have inferred a similarity of nature; that the corona, in short, is a "permanent solar aurora," consisting of streams of electrical discharge, directed and arranged by solar magnetism. A "magnetic theory" based on this general idea has been elaborately developed by Professor Bigelow, of the U. S. Weather Bureau.

Professor Schaeberle, of the Lick Observatory, on the other hand, contends for a purely "mechanical" theory, regarding the coronal streamers as jets of rare material ejected from the solar surface (chiefly from the sun-spot zones) to planetary distances, from which it falls back in a state of diffusion. To this returning matter he attributes many important solar phenomena.

None of the theories, however, seem to allow sufficient weight to the fundamental spectroscopic fact that "coronium," the characteristic substance of the corona, appears thus far to be absolutely unique in nature, — utterly distinct from any other known form of matter, terrestrial, solar, or cosmical.

Exercises on Chapters VIII and IX.

1. Assuming Faye's equation for the solar rotation (Art. 284), what are the rotation-periods at the sun's equator, in latitude 30°, in latitude 45°, and at the pole?

Ans. At equator, 25.06 days.
Lat. 30°, 26.49 "
Lat. 45°, 28.09 "
At pole, 31.95 "

2. What would be the synodic or apparent time of rotation for a spot in latitude 45°?

Ans. 30.43 days.

3. If the diameter of the sun were doubled, its density remaining unchanged, what would be the force of gravity at its surface?

4. If the sun were expanded into a homogeneous sphere, with a radius equal to the distance from the earth to the sun, its mass remaining unchanged, what would be the force of gravity at its surface?

5. In this case, what change, if any, would result in the orbit of the earth?

6. In the neighborhood of a sun spot a point is found in its spectrum where a portion of the C line ($\lambda = 6563.0$) is deflected to 6566.0. What is the velocity (in the line of sight) of the hydrogen at that point? (See Art. 321*.)

Ans. 85.17 miles, receding.

7. How great is the displacement of the hydrogen line F ($\lambda = 4861.5$) at that point?

Ans. 2.22 units (of wave-length).

8. How great a displacement is produced in the line D ($\lambda = 5896.16$) by a velocity of 100 miles a second?

Ans. 3.16 units.

9. If a luminous body were moving towards us with a velocity one quarter that of light, what would be the effect upon the apparent length of the portion of the spectrum included between two lines, — say C and F?

10. What if it were moving towards us with the speed of light?

11. What if it were receding at that rate?
Ans. The wave-length of every ray would be apparently doubled.

NOTE TO ART. 329.

THE CORONA LINE. The photographs of spectra obtained in India by Lockyer and Campbell during the eclipse of January, 1899, made it very probable that the true wave-length of the corona line is 5304, instead of 5316. The "1474" line ($\lambda = 5316$) is by far the most conspicuous of the chromosphere lines in that region of the spectrum, persisting some time after the others vanish, when the total phase of the eclipse begins, and it would be easy to make an erroneous identification of the corona line with it. It now seems certain that this mistake was actually made in 1869. The results obtained in the eclipses of May, 1900 and 1901, have confirmed those of the Indian eclipse.

CHAPTER X.

THE SUN'S LIGHT AND HEAT: COMPARISON OF SUNLIGHT WITH ARTIFICIAL LIGHTS. — MEASUREMENT OF THE SUN'S HEAT, AND DETERMINATION OF THE "SOLAR CONSTANT." — PYRHELIOMETER, ACTINOMETER, AND BOLOMETER. — THE SUN'S TEMPERATURE. — THEORIES AS TO THE MAINTENANCE OF THE SUN'S RADIATION, AND CONCLUSIONS AS TO THE SUN'S POSSIBLE AGE AND FUTURE DURATION.

332. The Sun's Light. — *The Quantity of Sunlight.* It is very easy to compare (approximately) sunlight with the light of a standard[1] candle; and the result is, that when the sun is in the zenith, it illuminates a white surface about 60,000 times as strongly as a standard candle at a distance of one metre. If we allow for the atmospheric absorption, the number would be fully 70,000. If we then multiply 70,000 by the square of 150,000 million (roughly the number of metres in the sun's distance from the earth), we shall get what a gas engineer would call the sun's "*candle power.*" The number comes out 1575 billions of billions (English); *i.e.*, 1575 with twenty-four ciphers following.

333. One way of making the comparison is the following: Arrange matters as in Fig. 118. The sunlight is brought into a darkened room by a mirror *M*, which reflects the rays through a lens *L* of perhaps half an inch in diameter. After the rays pass the focus they diverge and form on the screen *S* a disc of light, the size of which may be varied by changing the distance of the screen. Suppose it so placed that the illuminated circle is just ten feet in diameter; that is, 240 times the diameter of the lens. The illumination of the disc will then be less than that of direct sunlight in the ratio of 240^2 (or 57,600) to 1 (neglecting the loss of light produced by

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[1] A standard candle is a sperm candle weighing one-sixth of a pound and burning 120 grains an hour. The "decimal candle" proposed by the Paris Congress in 1890 is about one per cent less. It is one-twentieth of the light emitted by a square centimetre of melting platinum. An ordinary gas-burner consuming five feet of gas hourly gives a light equivalent to from twelve to fifteen standard candles.

the mirror and the lens, a loss which of course must be allowed for). Now place a little rod like a pencil near the screen, as at *P*, light a standard candle, and move the candle back and forth until the two shadows of the pencil, one formed by the candle, and the other by the light from the lens,

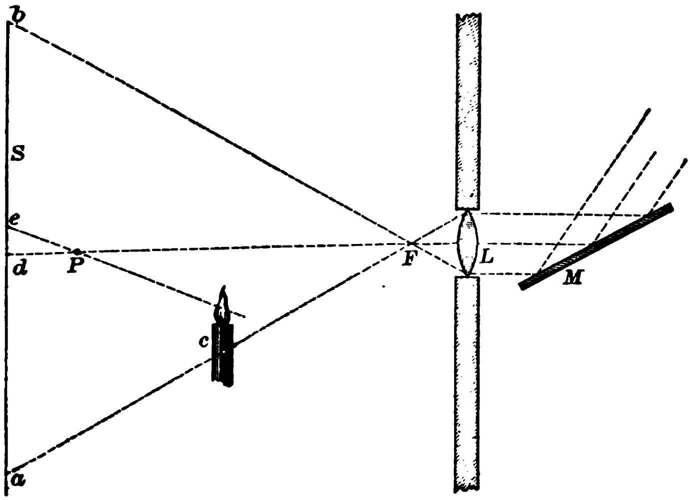

FIG. 118. — Comparison of Sunlight with a Standard Candle.

are equally dark. It will be found that the candle has to be put at a distance of about one metre from the screen ; though the results would vary a good deal from day to day with the clearness of the air.

334. When the sun's light is compared with that of the full moon and of various stars, we find, as stated (Art. 259), that it is about 600000 times that of the full moon. It is 7000 000000 times as great as the light received from Sirius, and about 40000 000000 times that from Vega or Arcturus.

335. The Intensity of the Sun's Luminosity. — This is a very different thing from the total quantity of its light, as expressed by its "candle power" (a surface of comparatively feeble luminosity can give a great quantity of light if large enough). It is the *amount of light per square inch of luminous surface* which determines the intensity. Making the necessary computations from the best data obtainable (only roughish approximations being possible), it appears that the sun's surface is about 190000 times as bright as that of a

candle flame, and about 150 times as bright as the lime of the calcium light. *Even the darkest part of a solar spot outshines the lime.* The intensely brilliant spot in the so-called "crater" of an electric arc comes nearer sunlight than anything else known, being from one-half to one-fourth as bright as the surface of the sun itself. But either the electric arc or the calcium light, when interposed between the eye and the sun looks like a dark spot on the disc.

336. Comparative Brightness of Different Portions of the Sun's Surface. — By forming a large image of the sun, say a foot in diameter, upon a screen, we can compare with each other the rays coming from different parts of the sun's disc. It thus appears that there is a great diminution of light at the edge, the light there, according to Professor Pickering's experiments, being just about one-third as strong as at the centre. There is also an obvious difference of color, the light from the edge of the disc being brownish red as compared with that from the centre. The reason is, that the red and yellow rays of the spectrum lose much less of their brightness at the limb than do the blue and violet. According to Vogel, the latter rays are affected nearly twice as much as the former. For this reason, photographs of the sun exhibit the darkening of the limb much more strongly than one usually sees it in the telescope.

337. Cause of the Darkening of the Limb. — It is due unquestionably to the general absorption of the sun's rays by the lower portion of the overlying atmosphere. The reason is obvious from the figure (Fig. 119). The *thinner* this atmosphere, other things being equal, the *greater the* ratio *between the percentage of absorption at the centre and edge of the disc, and the more obvious the darkening of the limb.*

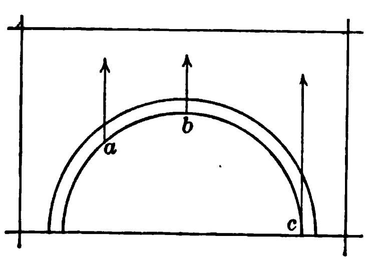

FIG. 119.

Cause of the Darkening of the Sun's Limb.

Attempts have been made to determine from the observed differences between the brightness of centre and limb the total percentage of the sun's light thus absorbed. Unfortunately we have to supplement the observed data with some very uncertain assumptions in order to solve the problem; and it can only be said that it is *probable* that the amount of light absorbed by the sun's atmos-

phere lies between fifty and eighty per cent; *i.e.*, the sun deprived of its gaseous envelope would probably shine from two to five times as brightly as now. It is noticeable, also, as Langley long ago pointed out, that thus stripped, the "complexion" of the sun would be markedly changed from yellowish white to a good full *blue*, since the blue and violet rays are much more powerfully absorbed than those at the lower end of the spectrum.

THE SUN'S HEAT.

338. Its Quantity; the "Solar Constant." — By the "*quantity of heat*" received by the earth from the sun we mean the number of heat-units received in each unit of time by a square unit of surface when the sun is in the zenith. The heat-unit most employed by engineers is the *calorie*, which is the quantity of heat required to raise the temperature of one kilogram of water one degree centigrade. It is found by observation that each square metre of surface exposed perpendicularly to the sun's rays receives from the sun each minute very approximately thirty of these calories; or rather it *would do so* if a considerable portion of the sun's heat were not stopped by the earth's atmosphere, which absorbs some thirty per cent of the whole, even when the sun is vertical, and a much larger proportion when the sun is near the horizon. This quantity, *thirty calories*[1] *per square metre per minute*, is known as the "*Solar Constant.*"

339. Method of Determining the "Solar Constant." — The method by which the solar constant is determined is simple enough in principle, though complicated with serious practical difficulties which affect its accuracy. It is done by allowing *a beam of sunlight of known cross-section to shine upon a known weight of water* (*or other substance of known specific heat*) *for a known length of time, and*

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[1] For many scientific purposes the engineering calorie is inconveniently large, and a smaller one is employed, which replaces the kilogram of water by the *gram* heated one degree — the smaller calorie being thus only $\frac{1}{1000}$ of the engineering unit. As stated by many writers (Langley, for instance), the solar constant is the number of these *small* calories received per square *centimetre* of surface in a minute. This would make the number 3.0 instead of 30. It would perhaps be better to bring the whole down to the "c.g.s. system" by substituting the *second* for the minute; and this would give us for the solar constant exactly *one-twentieth of a* (*small*) *calorie per square centimetre* per second.

measuring the rise of temperature. It is necessary, however, to determine and allow for the heat received from other sources during the experiment, and for that lost by radiation. Above all, the absorbing effect of our own atmosphere is to be taken into account, and this is the most difficult and uncertain part of the work, since the atmospheric absorption is continually changing with every change of the transparency of the air, or of the sun's altitude.

340. Pyrheliometers and Actinometers. — The instruments with which these measurements are made, are known as "pyrheliometers" and "actinometers." Fig. 120 represents the pyrheliometer of Pouillet, with which in 1838 he made his determination of the solar constant, at the same time that Sir John Herschel was experimenting at the Cape of Good Hope in practically the same way. They were the first apparently to understand and attack the problem in a reasonable manner. The pyrheliometer consists essentially of a little cylindrical box *ab*, like a snuff-box, made of thin silver plate, with a diameter of one decimetre and such a thickness that it holds 100 grams of water. The upper surface is carefully blackened, while the rest is polished as brilliantly as possible. In the water is inserted the bulb of a delicate thermometer, and the whole is so mounted that it can be turned in any direction so as to point it directly towards the sun. It is used by first holding a screen between it and the sun for (say) five minutes, and watching the rise or fall of the mercury in the thermometer at *m*. There will usually be some slight change due to the radiation of surrounding bodies. The screen is then removed, and the sun is allowed to shine upon the blackened surface for five minutes, the instrument being continually turned upon the thermometer as an axis, in order to keep the water in the calorimeter box well stirred. At the end of the five minutes the screen is replaced and the rise of the temperature noted. The difference between this and the change of the thermometer during the first five minutes will give us the amount by which a beam of sunlight one decimetre in diameter has raised the temperature of 100 grams of water in five minutes, and were it not for the troublesome corrections which must be made, would furnish directly the value of the solar constant.

FIG. 120. — Pouillet's Pyrheliometer.

341. The second apparatus, Fig. 121, is the actinometer of Violle, which consists of two concentric metal spheres, the inner of which is blackened on the inside, while the outer one is brightly polished, the space between the two being filled with water at a known temperature, kept circulating by a pump of some kind. The thermoscopic body in this case, instead of being a box filled with water, is the blackened bulb of the thermometer T; and the observations may be made either in the same way as with the pyrheliometer, or simply by noting the difference between the temperature finally attained by the thermometer T after it has ceased to rise in the sun's rays, and the temperature of the water circulating in the shell.

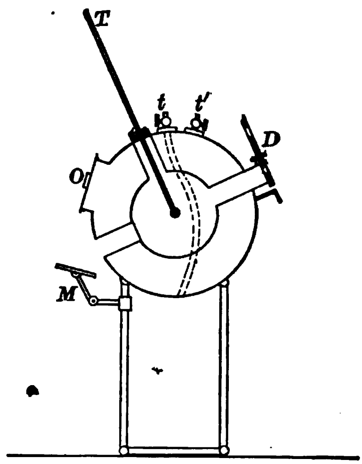

FIG. 121. — Violle's Actinometer.

342. Correction for Atmospheric Absorption. — The correction for atmospheric absorption is determined by making observations at various altitudes of the sun between zenith and horizon. If the rays were *homogeneous* (that is, all of one wave-length), it would be comparatively easy to deduce the true correction and the true value of the solar constant. In fact, however, the *visible* solar spectrum is but a small portion of the whole spectrum of the sun's radiance, and, as Langley has shown, it is necessary to determine the coefficient of absorption separately for all the rays of different wave-length.

343. The Bolometer. — This he has done by means of his "Bolometer," an instrument which is capable of indicating exceedingly minute changes in the amount of radiation received by an extremely thin strip of metal. This strip is so arranged that the least change in its electrical resistance due to any change of temperature will disturb a delicate galvanometer. The instrument is far more sensitive than any thermometer or even thermopile, and has the especial advantage of being extremely quick in its response to any change of radiation. Fig. 122 shows it so connected with a spectroscope that the observer can bring to the bolometer, B, rays of any wave-length he chooses. The rays enter through the collimator lens L, and are then refracted by the rock-salt prism P (or diffracted by a grating) to the reflector M, whence they are sent back to B, and thus produce their electrical effect, which is transmitted to the galvanometer. As the galvanometer needle swings one way or the other, a pencil of light reflected from it falls upon a sensitive photographic plate which is moved by the same clockwork (not shown in the figure) which moves the prism;

and as a consequence the spot of light traces out upon the plate an irregular curve, in which the "hills" correspond to rise of temperature, and the "valleys" to cooler places in the spectrum. The curve may then be transformed into a spectrum-map of the usual form, — a "bolograph," as it is called. Fig. 122* is from Langley's paper of 1894.

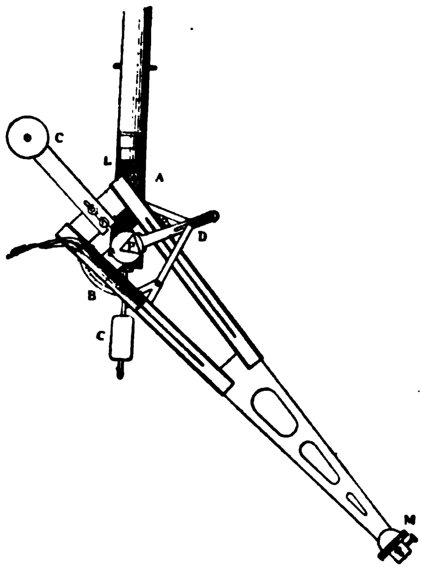

FIG. 122.
Langley's Spectro-Bolometer, as used for Mapping the Energy of the Prismatic Spectrum.

Langley showed that the atmospheric corrections applied by earlier observers were much too low, and has raised the formerly accepted value from 20 or 25 to 30, which we have adopted. Scheiner (1899) considers 40 as more probable, but on the other hand the Smithsonian observations of 1902–1903 give hardly 20, and strongly suggest that the supposed "constant" may be widely variable.

344. A less technical statement of the solar radiation may be made in terms of thickness of the quantity of *ice* which would be melted by it in a given time. Since it requires 79¼ calories to melt a kilogram of ice with a density of 0.92, it follows that 30 calories a minute would melt in an *hour* a sheet of ice one metre square and 24.7 millimetres (0.97 inch) thick. According to this, the sun's heat would melt about 226 feet of ice annually on the earth's equator, or 177 feet yearly over the earth's entire surface if the heat were equally distributed in all latitudes.

(See note at the end of the chapter, page 247.)

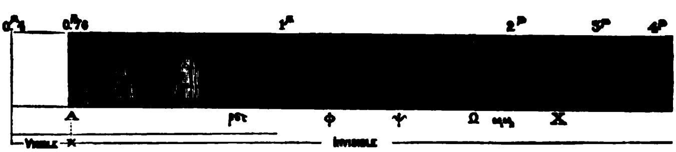

FIG. 122*. — Bolograph of the Infra-red Spectrum, Langley. From "The Sun," by permission of D. Appleton & Co.

345. Solar Heat Expressed as Energy. — Since, according to the known value of the "mechanical equivalent of heat" (Physics, p. 199)

a horse-power corresponds to about $10\frac{7}{10}$ calories per minute, it follows that *each square metre of surface* (neglecting the air-absorption) *would receive, when the sun is overhead,* about *two and three-quarters horse-power continuously.* Atmospheric absorption cuts this down to about one and three-quarters horse-power, of which about one-eighth can be actually utilized by properly constructed machinery, as, for instance, the solar engines of Ericsson and Mouchot (see Langley's "New Astronomy"). In Ericsson's apparatus the reflector, about 11 feet by 16 feet, collected heat enough to work a three-horse-power engine very well. Taking the earth's surface as a whole, the energy received during a year aggregates about a hundred mile-tons for every square foot. That is to say, the *heat annually received on each square foot of the earth's surface, if employed in a perfect heat engine, would be able to hoist about a hundred tons to the height of a mile.*

346. Solar Radiation at the Sun's Surface. — If, now, we estimate the amount of radiation at the sun's surface itself, we come to results which are simply amazing and beyond comprehension. It is necessary to multiply the solar constant observed at the earth (which is at a distance of 93 000000 miles from the sun) by the square of the ratio between 93 000000 and 433250, the radius of the sun. This square is about 46000; in other words, the amount of heat emitted in a minute by a square metre of the sun's surface is about 46000 times as great as that received by a square metre at the earth. Carrying out the calculations, we find that this heat radiation at the surface of the sun amounts to *nearly* 1 400000 *calories per square metre per minute;* that it is nearly 130000 horse-power per square metre continuously acting; that *if the sun were frozen over completely to a depth of sixty-four feet, the heat emitted is sufficient to melt this whole shell in one minute of time;* that if an ice bridge could be formed from the earth to the sun by a column of ice two and one-half miles square at the base and extending across the whole 93 000000 of miles, and if by some means the whole of the solar radiation could be concentrated upon this column, it would be melted in one second of time, and in between seven and eight seconds more would be dissipated in vapor. To maintain such a development of heat *by combustion* would require the *hourly burning of a layer of the best anthracite coal from nineteen to twenty-four feet thick* over the sun's entire surface, — over a ton for every square foot, — at least ten times as much as the consumption of the most powerful blast furnace in existence. At that rate the sun, if made of solid coal, would not last 5000 years.

347. Waste of Solar Heat. — These estimates are of course based on the assumption that the sun radiates heat equally in all directions, and there is no assignable reason why it should not do so. On this assumption, however, *so far as we can see*, only a minute fraction of the whole radiation ever reaches a resting-place. The earth receives about $\frac{1}{2200\,000000}$ of the whole, and the other planets of the solar system, with the comets and the meteors, get also their shares; all of them together, perhaps ten or twenty times as much as the earth. Something like $\frac{1}{100\,000000}$ of the whole seems to be utilized within the limits of the solar system. As for the rest, science cannot yet tell what becomes of it. A part, of course, reaches distant stars and other objects in interstellar space; but by far the larger portion seems to be "wasted," according to our human ideas of waste.

348. Experiments with the thermopile, first conducted by Henry at Princeton in 1845, show that the heat from the edges of the sun's disc, like the light, is less than that from the centre — according to Langley's measurements about half as much. The explanation evidently lies in its absorption by the solar atmosphere.

349. The Sun's Temperature. — While we can measure with some accuracy the *quantity* of heat sent us by the sun, it is different with its *temperature*, in respect to which we can only say that it must be very high — much higher than any temperature attainable by known methods on the surface of the earth.

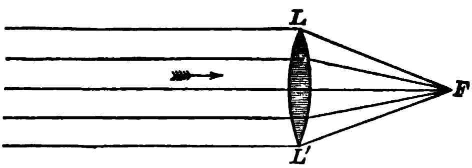

FIG. 123.

This is shown by a number of facts, for instance, by the great *abundance of the violet and ultra-violet rays* in the sunlight.

Again, by the *penetrating* power of sunlight; a large percentage of the heat from a common fire, for instance, being stopped by a plate of glass, while nearly the whole of the solar radiation passes through.

The most impressive demonstration, however, follows from this fact; viz., that at the focus of a powerful burning-lens all known substances melt and vaporize, as in an electric arc. Now at the focus of the lens the *limit* of the temperature is that which would be produced by the sun's direct radiation at a point where the sun's angular diameter equals that of the burning-lens itself seen from the focus, as represented in Fig. 123. An object at *F* would theoretically (that is, if there was no loss of heat conducted away by

surrounding bodies and by the atmosphere) reach the same temperature as if carried to a point where the sun's angular diameter equals the angle *LFL'*. In the most powerful burning-lenses yet constructed a body at the focus is thus virtually carried up to within about 240000 miles of the sun's surface, where its apparent diameter would be about 80°. Here, as has been said, the most refractory substances are immediately subdued. If the earth were to approach the sun as near as the moon is to us, she would melt and be vaporized.

350. Ericsson in 1872 made an exceedingly ingenious and interesting experiment illustrating the intensity of the solar heat. He floated a calorimeter, containing about ten pounds of water, upon the surface of a large mass of molten iron by means of a raft of fire-brick, and found that the radiation of the metal was a trifle over 250 calories per minute for each square foot of surface; which is only $\frac{1}{350}$ part of the amount emitted by the same area of the sun's surface. He estimated the temperature of the metal at 3000° F. or 1649° C.

351. Effective Temperature. — The question of the sun's temperature is embarrassed by the fact that it has no *one* temperature; the temperature at different parts of the solar photosphere and chromosphere must be very different. We evade this difficulty to some extent by substituting for the *actual* temperature, as the object of inquiry, what has been called the sun's "*effective temperature*"; that is, the temperature which a sheet of *lampblack* must have in order to radiate the amount of heat actually thrown off by the sun. (Physicists have taken the radiating power of lampblack as *unity*.) If we could depend upon the laws [1] deduced from laboratory experiments, by which it has been sought to connect the temperature of the body with its rate of radiation, the matter would then be comparatively simple: from the known radiated *quantity of heat* (in calories) we could compute the *effective temperature* in degrees. But at present it is only by a very unsatisfactory process of extrapolation that we can reach conclusions. The sun's temperature is so much higher than any which we can manage in our laboratories, that there is not yet much certainty to be obtained in the matter.

[1] A number of such laws have been formulated; for instance, the well-known law of Dulong and Petit. Pouillet and Vicaire, using this formula, have deduced values for the sun's effective temperature ranging from 1500° and 2500° C. Ericsson and Secchi, using Newton's law of radiation (which, however, is certainly inapplicable under the circumstances), put the figure among the millions. Wilson and Gray's results agree nearly with Stefan's "fourth power" law; viz., $t = kt^4$, t being the "absolute" temperature, and k a constant, depending on the substance which radiates.

Wilson and Gray, the most recent and reliable investigators, from their work in 1894–95, get 8000° C. or 14440° F. for the effective temperature. Almost certainly it lies somewhere between 10000° and 20000° F.

352. Constancy of the Sun's Heat. — It is an interesting and thus far unsolved problem, whether the total amount of the sun's radiation varies perceptibly at different times. It is only certain that the variations, if real, are too small to be detected by our present means of observation. Possibly, at some time in the future, observations on a mountain summit above the main body of our atmosphere may decide the question.

It is not unlikely that changes in the earth's climate such as have given rise to glacial and carboniferous periods may ultimately be traced to the condition of the sun itself, especially to changes in the thickness of the absorbing atmosphere, which, as Langley has pointed out, must have a great influence in the matter. Since the Christian era, however, it is certain that the amount of heat annually received from the sun has remained practically unchanged. This is inferred from the distribution of plants and animals, which is still substantially the same as in the days of Pliny.

353. Maintenance of the Solar Heat. — The question at once arises, if the sun is sending off such an enormous quantity of heat annually, how is it that it does not grow cold?

(*a*) The sun's heat cannot be kept up by *combustion*. As has been said before, it would have burned out long ago, even if made of solid coal burning in oxygen.

(*b*) Nor can it be simply a *heated body cooling down*. Huge as it is, an easy calculation shows that its temperature must have fallen greatly within the last 2000 years by such a loss of heat, even if it had a specific heat higher than that of any known substance.

As matters stand at present, the available theories seem to be reduced to two, — that of Mayer, which ascribes the solar heat to the energy of meteoric matter falling on the sun; and that of Helmholtz, who finds the cause in a slow contraction of the sun's diameter.

354. Meteoric Theory of Sun's Heat. — The first is based on the fact that when a moving body is stopped, its mass-energy becomes molecular energy, and appears mainly as heat. The amount of heat developed in such a case is given by the formula

$$Q = \frac{MV^2}{8339},$$

in which Q is the number of calories of heat produced, M the mass of the moving body in kilograms, and V its velocity in metres per second; the denominator is the "mechanical equivalent of heat" (Physics, p. 159) multiplied by $2g$ expressed in metres; *i.e.*, $425 \times 2 \times 9.81$.

Now, the velocity of a body coming from any considerable distance and falling into the sun can be shown to be about 380 miles per second, or more than 610 kilometres. A body weighing one kilogram would therefore, on striking the sun with this velocity, produce about 45 000000 calories of heat,

$$\left[\frac{(610000)^2}{8339}\right].$$

This is 6000 times more than could be produced by *burning* it, even if it were coal or solidified hydrogen burning in pure oxygen.

Now, as meteoric matter is continually falling upon the earth, it must be also falling upon the sun, and in vastly greater quantities, and an easy calculation shows that a quantity of meteoric matter equal to $\frac{1}{74}$ of the earth's mass striking the sun's surface annually with the velocity of 600 kilometres per second would account for its whole radiation.

355. Objections to Meteoric Theory of Sun's Heat. — There can be no question that a certain fraction of the sun's heat is obtained in this way, but it is very improbable that this fraction is a large one; indeed, it is hardly possible that it can be as much as *one per cent* of the whole.

(1) The annual fall on the sun's surface of such a quantity of meteoric matter implies the presence *near* the sun of a vastly greater mass; for, as we shall see hereafter, only a few of the meteors that approach the sun from outer space would strike the surface; most of them would act like the comets and swing around it without touching. Now, if there were any considerable quantity of such matter near the sun, there would result disturbances in the motions of the planets Mercury and Venus, such as observation does not reveal

(2) Professor Peirce has shown further that if the heat of the sun were produced in this way, the earth ought to receive from the meteors that strike her surface about half as much heat as she gets from the sun. Now the quantity of meteoric matter which would have to fall upon the earth to furnish us daily half as much heat as we receive from the sun would amount to nearly fifty tons for each square mile. It is not likely that we actually get $\frac{1}{1000000}$ of that amount. It is difficult to determine the amount of heat which the earth actually does receive from meteors, but all observations indicate that the quantity is extremely small. The writer has

estimated it, from the best data attainable, as less in a *year* than we get from the sun in a *second*.

356. Helmholtz's Theory of Solar Contraction. — We seem to be shut up to the theory of Helmholtz, now almost universally accepted: namely, that the heat necessary to maintain the sun's radiation is principally supplied *by the slow contraction of its bulk*, aided, however, by the accompanying liquefaction and solidification of portions of its gaseous mass. When a body falls through a certain distance, *gradually*, against resistance, and then comes to rest, the same total amount of heat is produced as if it had fallen *freely, and been stopped instantly*. If, then, the sun does contract, heat is necessarily produced by the process, and that in enormous quantity, since the attracting force at the solar surface is more than twenty-seven times as great as terrestrial gravity, and the contracting mass is immense. In this process of contraction each particle at the surface moves inward by an amount equal to the diminution of the sun's radius: a particle below the surface moves less and under a diminished gravitating force; but every particle in the whole mass, excepting only that at the exact centre of the globe, contributes something to the evolution of heat. In order to calculate the precise amount of heat evolved by a given shrinkage, it would be necessary to know the law of increase of the sun's density from the surface to the centre; but Helmholtz has shown that under the most unfavorable conditions *a contraction of the sun's diameter of about* 90 *metres or* 300 *feet a year* (150 feet in the sun's *radius*) *would account for the whole annual output of heat.* This contraction is so slow that it would be quite imperceptible to observation. It would require very nearly 8000 years to reduce the sun's diameter by a single second of arc; and nothing much less would be certainly detectable by our measurements. *If the contraction is more rapid than this*, the mean temperature of the sun must be actually *rising*, notwithstanding the amount of heat it is losing. Long observation alone can determine whether this is really the case or not.

357. Lane's Law. — It is a remarkable fact, first demonstrated by Lane of Washington, in 1870, that a gaseous sphere, losing heat by radiation and contracting under its own gravity, *must rise in temperature and actually grow hotter*, until it ceases to be a "perfect gas," either by beginning to liquefy, or by reaching a density at which the laws of perfect gases no longer hold. The kinetic energy developed by the shrinkage of a gaseous mass is more than sufficient to replace the loss of heat which caused the shrinkage. In the case of a *solid or liquid* mass this is not so. The shrinkage of such a mass contracting under its own gravity on account of the loss of heat is

never sufficient to make good the loss; but the temperature falls and the body cools. At present it appears that in the sun the relative proportions of true gases and liquids are such as to keep the temperature nearly stationary, the liquid portions of the sun being of course the little drops which are supposed to constitute the clouds of the photosphere.

358. Future Duration of the Sun. — If this shrinkage theory of the solar heat is correct (and there is every reason to accept it), it follows that in time the sun's heat must come to an end, and, looking backwards, we see that there must have been a beginning.

We have not sufficient data to enable us to calculate the future duration of the sun with exactness, though an approximate estimate can be made. According to Newcomb, if the sun maintains its present radiation, it will have shrunk to half its present diameter in about 5 000000 years at the longest. Since, when reduced to this size, it must be about eight times as dense as now, it can hardly then continue to be mainly gaseous, and its temperature must begin to fall. Newcomb's conclusion, therefore, is that it is not likely that the sun can continue to give sufficient heat to support such life on the earth as we are now acquainted with, for 10 000000 years from the present time.

359. Age of the Sun. — As to the past of the solar history on this hypothesis, we can be a little more definite. It is only necessary to know the present amount of radiation, and the mass of the sun, to compute how long the solar fire can have been maintained at its present intensity by the processes of condensation. No conclusion of geometry is more certain than this, — that the contraction of the sun to its present size, from a diameter even many times greater than Neptune's orbit, would have furnished about 18 000000 times as much heat as the sun now supplies in a year, and therefore that the sun cannot have been emitting heat *at the present rate* for more than 18 000000 years, *if its heat has really been generated in this manner.*

But this conclusion rests upon the assumption that the sun has derived its heat solely in this way, and the recent discoveries with respect to radium and radio-activity strongly suggest other causes which may have added large contributions and may still be operative in maintaining the solar radiation.

360. Constitution of the Sun. — (*a*) As to the nature of the main body or nucleus of the sun, we cannot be said to have certain knowledge. It is probably *gaseous*, this being indicated by its low mean density and its high temperature — enormously high even at the

surface, where it is coolest. At the same time the gaseous matter at the nucleus must be in a very different state from gases as we commonly know them in our laboratories, on account of the intense heat and the extreme condensation by the enormous force of solar gravity. The central mass, while still strictly gaseous, because observing the three physical laws of Boyle, Dalton, and Gay Lussac, which characterize gases, would be denser than water, and viscous; probably something like tar or pitch in consistency.[1]

While this doctrine of the gaseous constitution of the sun is generally assented to, there are still some who are disposed to consider the great mass of the sun as liquid.

361. (*b*) The *photosphere* is probably a shell of *incandescent clouds,* formed by the condensation of the vapors which are exposed to the cold of space.

The minute particles of which the photosphere is composed being liquid, or possibly some of them solid, have a radiating power enormously greater than that of the gases in which they float, though the temperature is practically the same. As a source of light and heat the photosphere acts in the same way as the "*mantle*" *of a Welsbach burner.*

362. (*c*) The photospheric clouds float in an atmosphere containing, still uncondensed, a considerable quantity *of the same vapors out of which they themselves have been formed,* just as in our own atmosphere the air around a cloud is still saturated with water vapor. This vapor-laden atmosphere, probably comparatively shallow, constitutes the *reversing layer,* and by its selective absorption produces the dark lines of the solar spectrum, while by its general absorption it probably produces the darkening at the limb of the sun.

But it will be remembered that Mr. Lockyer and others are disposed to question the existence of any such shallow absorbing stratum, considering that the absorption takes place in all regions of the solar atmosphere even to a great elevation.

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[1] The law of Dalton (Physics, p. 218) is, that any number of different gases and vapors tend to *distribute themselves throughout the space which they occupy in common, each as if the others were absent.* The law of Boyle or Mariotte (Physics, p. 118) is, *that at any given temperature the volume of any given amount of gas varies inversely with the pressure; i.e.,* $pv = p'v'$. The law of Gay Lussac (Physics, p. 222) is, that a gas *under constant pressure expands in volume uniformly under uniform increment of temperature,* so that $V_t = V_0(1 + at)$. This is not true of *vapors* in presence of the liquids from which they have been evaporated; for instance, of steam in a boiler.

363. (*d*) The *chromosphere and prominences* are composed of the *permanent gases*, mainly hydrogen and helium, which are mingled with the vapors of the reversing stratum in the region near the photosphere, but usually rise to far greater elevations than do the vapors. The appearances are for the most part as if the chromosphere was formed of jets of heated hydrogen ascending through the interspaces between the photospheric clouds, like flames playing over a coal fire.

364. (*e*) The *corona* also rests on the photosphere, and the peculiar green line of its spectrum (Art. 329) is brightest just at the surface of the photosphere, in the reversing stratum and in the chromosphere itself; but the corona extends to a far greater elevation than even the prominences ever reach, and seems to be not wholly gaseous, but to contain, besides the hydrogen and the mysterious "coronium," dust and fog of some sort, perhaps meteoric. Many of its phenomena are as yet unexplained, and since it can only be observed during the brief moments of total solar eclipses, progress in its study is necessarily slow.

364*. (Note to Art. 344.) Taking the solar constant at 30 cal. per square metre per minute, the amount of heat falling upon a square metre in an hour would raise 1800 kilograms (or cubic decimetres) of water 1° C. in temperature. Since the "heat of fusion" of ice is 79.25, this would melt $\frac{1800}{79.25}$, or 22.7, kgms. of ice; and the specific gravity of ice being 0.92, this would correspond to $\frac{22.7}{0.92}$, or 24.7, cubic decimetres, which spread over a square metre would make a thickness of 24.7^{mm} or 0.97^{in}.

The total heat received by the earth is that intercepted by its diametrical section, or the area of one of its great circles.

The thickness of the sheet of ice melted annually upon this circular plane would be $24.7^{mm} \times 24 \times 365\frac{1}{4} = 216.5$ metres, or 710 feet. On a narrow equatorial belt the thickness melted would be $\frac{216.5^m}{\pi} = 68.9$ metres, or 226 feet, since such a belt intercepts the rays that otherwise would fall upon a diametrical strip of the same width on the circular plane. If the sun's heat were uniformly distributed over the whole surface of the earth, the area of which equals four great circles ($4\pi r^2$), it could melt an ice sheet having a thickness of $\frac{216.5^m}{4}$, or 54.1^m (177.4 feet). It must be remembered, however, that the value of the solar constant is likely to be in error, perhaps as much as ten per cent; and all the numbers above given are affected by the same uncertainty.

It is true that at the sea-level the solar constant is much diminished by atmospheric absorption; and probably does not exceed eighteen calories per minute *directly* received from the sun's rays. But a large part of the solar heat absorbed by the atmosphere reaches the earth's surface *indirectly* through the warming of the atmosphere, so that it must not be considered as lost to the earth because not directly measurable by the actinometer.

CHAPTER XI.

ECLIPSES: FORM AND DIMENSIONS OF SHADOWS. — LUNAR ECLIPSES. — SOLAR ECLIPSES. — TOTAL, ANNULAR, AND PARTIAL. — ECLIPTIC LIMITS AND NUMBER OF ECLIPSES IN A YEAR. — THE SAROS. — OCCULTATIONS.

365. The word eclipse (Greek ἔκλειψις) is strictly a medical term, meaning a *faint* or *swoon*. Astronomically it is applied to the darkening of a heavenly body, especially of the sun or moon, though some of the satellites of other planets besides the earth are also "eclipsed" from time to time. An eclipse of the *moon* is caused by its passage through the shadow of the earth; an eclipse of the *sun*, by the interposition of the moon between the sun and the observer, or, what comes to the same thing, by the passage of the moon's shadow over the observer.

366. Shadows. — If interplanetary space were slightly dusty, we should see, accompanying the earth and moon and each of the planets, a long black shadow projecting behind it and travelling with it. Geometrically speaking, this shadow of a body, the earth for instance, is a *solid — not a surface*. It is the *space* from which sunlight is excluded. If we regard the sun and other heavenly bodies as truly spherical, these shadows are *cones* with their axes in the line joining the centres of the sun and the shadow-casting body, the point being always directed away from the sun, because the sun is always the larger of the two.

367. Dimensions of the Earth's Shadow. — The length of the shadow is easily found. In Fig. 124 we have from the similar triangles OED and ECa, $OD : Ea :: OE : EC$ or l. OD is the difference between the radii of the sun and the earth, $= R - r$. $Ea = r$, and OE is the distance of the earth from the sun $= \Delta$.

Hence $$l = \Delta \times \left(\frac{r}{R - r}\right) = \frac{1}{108.5}\Delta.$$

(The fractional factor is constant, since the radii of the sun and

earth are fixed quantities. Substituting the values of the radii, we find it to be $\frac{1}{108.5}$.) This gives 857200 miles for the length of the earth's shadow when Δ has its mean value of 93 000000 miles, regard-

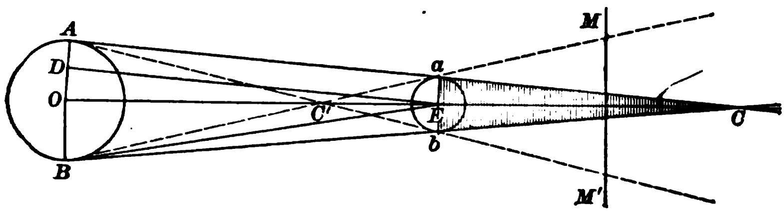

FIG. 124. — Dimensions of the Earth's Shadow.

ing the earth as a perfect sphere and taking its mean radius. This length varies about 14000 miles on each side of the mean as the earth changes its distance from the sun.

The semi-angle of the cone (the angle ECb, or ECB in the figure) is found as follows: Since OEB is exterior to the triangle BEC,

$$OEB = EBC + BCE,$$

or

$$BCE = OEB - EBC.$$

Now, OEB is the *sun's apparent semi-diameter* as seen from the earth, and EBC is the earth's semi-diameter as seen from the sun, which is the same thing as *the sun's horizontal parallax* (Art. 83).

Putting S for the sun's semi-diameter, and p for its parallax, we have —

$$\text{Semi-angle at } C = S - p.^1$$

From the cone aCb all sunlight is excluded, or would be were it not for the fact that the atmosphere of the earth by its refraction bends some of the rays into this shadow. The effect is to make the shadow a little larger in diameter, but less perfectly dark.

368. Penumbra. — If we draw the lines Ba and Ab, crossing at C' between the earth and the sun, they will bound the *penumbra*. Within this space a part, but not the whole, of the sunlight is cut off: an observer outside of the shadow, but within this cone-frustum,

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[1] Also, $l = \dfrac{r}{\sin(S-p)}$, an expression sometimes more convenient than the one given above.

which tapers *towards* the sun, would see the earth as a black body encroaching on the sun's disc. The semi-angle of the penumbra $EC'a$ is easily shown to be $S + p$.

369. Although *geometrically* the boundaries of the shadow and penumbra are perfectly definite, they are not so optically. If a screen were placed at M (Fig. 124) perpendicular to the axis of the shadow, no sharply defined lines would mark the boundaries of either shadow or penumbra; near the edge of the shadow, the penumbra would be very nearly as dark as the shadow itself, only a mere speck of the sun being visible there; and at the outer limit of the penumbra the shading would be still more gradual.

370. Eclipses of the Moon. — The axis of the earth's shadow is always directed to a point exactly opposite the sun. If, then, at the time of full moon, the moon happens to be near the ecliptic (that is, *not far from one of the nodes of her orbit*), she will pass into the shadow and be eclipsed. Since, however, the moon's orbit is inclined about five and one-fourth degrees to the plane of the ecliptic, this does not happen very often (seldom more than twice a year). Ordinarily the moon passes north or south of the shadow without touching it.

Lunar eclipses are of two kinds, — partial and total: total when the moon passes into the shadow completely; partial when she goes so far to the north or south of the centre of the shadow that only a portion of her disc is obscured.

We may also have a "penumbral eclipse" when she passes merely through the penumbra without touching the shadow. In this case, however, the loss of light is so gradual and so slight, unless she almost grazes the shadow, that an observer would notice nothing unusual.

371. Size of the Earth's Shadow at the Point where the Moon crosses it. — Since EC in Fig. 125 is 857,000 miles, and the distance of the moon from the earth is on the average about 239,000 miles, CM must be 618,000 miles, and MN, the semi-diameter of the shadow at this point, will be $\frac{618}{857}$ of the earth's radius. This gives $MN = 2854$ miles, and makes the whole diameter of the shadow a little over 5700 miles, about two and two-thirds times the diameter of the moon. But this quantity varies considerably. The shadow is sometimes more than three times as large as the moon, sometimes hardly more than twice its size.

372. We may reach the same result in another way. Considering the triangle ECN, Fig. 125, we have the angular semi-diameter of the cross-section of the shadow where the moon passes through it, as seen from the earth, represented by MEN.

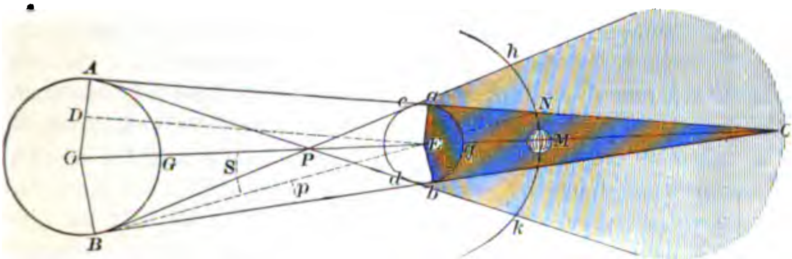
But $$ENa = MEN + ECN;$$

whence $$MEN = ENa - ECN.$$

Now ENa is the semi-diameter of the earth as seen from the moon; that is, it is the moon's *horizontal parallax*, for which write P. Hence, substituting for ECN its value $S - p$, we get

$$MEN = P + p - S.$$

MEN is called "the radius of the shadow." The mean value of P is 57′ 2″; of p, 8″.8; and of S, 16′ 2″, which makes the mean value of $MEN = 41′ 9″$.



E N ..
not
necessarily
a continuation
of B E.

Fig. 125. — Diameter of Earth's Shadow where the Moon crosses it.

The mean value of the moon's apparent semi-diameter is 15′ 40″, the ratio between the semi-diameter of the moon and the radius of the shadow being about $2\frac{2}{3}$, as before.

In computing a lunar eclipse, this angular value for the "radius of the shadow," as it is called, is more convenient than its value in miles. It is customary to increase it by about $\frac{1}{60}$ part in order to allow for the effect of the earth's atmosphere, the value ordinarily used being $\frac{61}{60}(P + p - S)$. Some computers, however, use $\frac{51}{50}$, and others $\frac{76}{75}$. On account of the indistinctness of the edge of the shadow it is not easy to determine what precise value ought to be employed, nor is it important.

373. Duration of a Lunar Eclipse. — When central, a total eclipse of the moon may, all things favoring, continue total for about two hours, the interval from the first contact to the last being about two hours more. This depends upon the fact that the moon's hourly motion is nearly equal to its own diameter. The whole interval from first contact to last is the time occupied by the moon in moving from

a to *d* (Fig. 126). The totality lasts while it moves from *b* to *c*. The duration of a non-central eclipse varies, of course, according to the part of the shadow through which the moon passes.

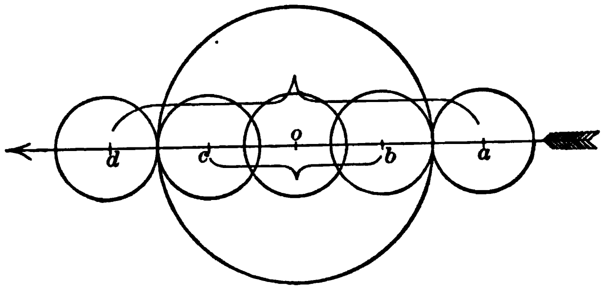


FIG. 126. — Duration of a Lunar Eclipse.

374. Lunar Ecliptic Limit. — The lunar *ecliptic limit* is the greatest distance from the node of the moon's orbit at which the sun can be consistently with having an eclipse. This limit depends upon the inclination of the moon's orbit, which varies a little, and also upon the radius of the shadow at the time of the eclipse and the moon's apparent semi-diameter, which quantities are still more variable. Hence we recognize two limits, the major and minor. If the distance of the sun from the node at the time of full moon exceeds the major limit, an eclipse is impossible; if it is less than the minor, an eclipse is inevitable. The major limit is found to be 12° 15′; the minor, 9° 30′. Since the sun passes over an arc of 12° 15′ in less than thirteen days, it follows that an eclipse of the moon cannot possibly take place more than thirteen days before or after the time when the sun crosses the node.

375. In Fig. 127 let *NE* and *NM* be, respectively, portions of the ecliptic and of the path of the moon, as seen projected upon the celestial

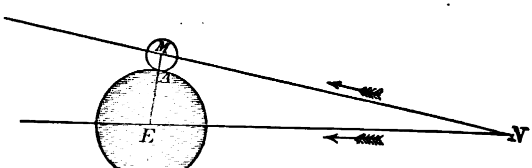


FIG. 127. — Lunar Ecliptic Limit.

sphere. *E* is the centre of the earth's shadow. The sun, of course, is at the point of the celestial sphere directly opposite, and its distance from the

opposite node is equal to *EN*. *M* is the centre of the moon. Call the semi-diameter of the moon S'; then *EM* (the greatest possible distance between *E* and *M* which permits an eclipse) equals the sum of the semi-diameters of the moon and shadow, or $S' + (P + p - S)$, and the corresponding ecliptic limit *EN* is found by solving the spherical triangle *MNE*, having given *ME* and the angle at *N*, which is about $5\frac{1}{4}°$. We must also know one other angle, and with sufficient approximation for such purposes we may regard the angle at *M* as a right angle. The limit is always very nearly *eleven* times *EM*, because the inclination of the moon's orbit is nearly $\frac{1}{11}$ of a "radian."

376. Phenomena of a Total Lunar Eclipse. — Half an hour or so before the moon reaches the shadow its eastern limb begins to be sensibly darkened, and the edge of the shadow itself, when it is first reached, looks nearly black by contrast with the bright parts of the moon's surface. To the naked eye the outline of the shadow appears reasonably sharp; but with even a small telescope it is found to be indefinite and hazy, and with a large instrument and high magnifying power it becomes entirely indistinguishable. It is impossible to determine the exact moment when the edge of the shadow reaches any particular point on the moon within half a minute or so.

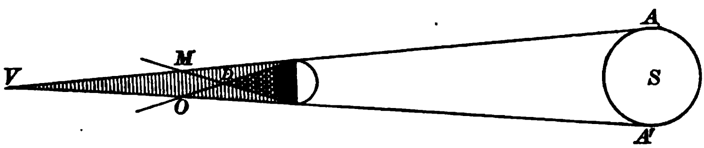


FIG. 128. — Light Bent into Earth's Shadow by Refraction.

After the moon has wholly entered the shadow her disc is usually still distinctly visible, illuminated with a dull, copper-colored light, which is sunlight, deflected around the earth into the shadow by the refraction of our own atmosphere, or rather by that portion of our atmosphere which lies within ten or fifteen miles of the earth's surface. Since the ordinary horizontal refraction is 34′, it follows that light which just grazes the earth's surface will be bent inwards by twice that amount, or 1° 8′. Now, the maximum parallax[1] of the moon is only 1° 2′. In an extreme case, therefore, even when the moon is exactly central in the largest possible shadow, it receives some sunlight coming around the edge of the earth, as shown by Fig. 128. To an observer stationed on the moon, the disc of the earth would appear to be surrounded by a narrow ring of brilliant sunshine, colored with sunset hues by the same vapors which tinge

[1] This is the semi-diameter of the earth as seen from the moon.

terrestrial sunsets, but acting with double power because the light has traversed a double thickness of our air. If the weather happens to be clear at this portion of the earth (upon its *rim* as seen from the moon), the quantity of light transmitted through the atmosphere is very considerable, and the moon is strongly illuminated. If, on the other hand, the weather happens to be stormy in this region, the clouds cut off nearly all the light. In the lunar eclipse of 1884 the moon was absolutely invisible to the naked eye, a very unusual circumstance on such an occasion. At the eclipse of January 28, 1888, Pickering found that the *photographic power* of the centrally eclipsed moon was about $\frac{1}{1400000}$ of that of the moon when uneclipsed.

377. Uses Made of Lunar Eclipses. — In astronomical importance a lunar eclipse cannot be at all compared with a solar eclipse. It has its uses, however. *a.* Many dates in chronology are fixed by reference to certain lunar eclipses. For instance, the date of the Christian era is determined by a lunar eclipse which happened upon the night before Herod died. *b.* Before better methods were devised, lunar eclipses were made use of to some extent in determining the longitude. Unfortunately, as has been said (Art. 119), it is impossible to note the critical instants with any degree of accuracy, on account of the indefiniteness of the earth's shadow. *c.* The study of the spectrum of the eclipsed moon gives us some data as to the constitution of our own atmosphere. We are thus enabled to examine light which has passed through a greater thickness of air than is obtainable in any other way. *d.* The study of the heat radiated by the moon during the different phases of an eclipse gives us some important information as to the absorbing power and temperature of its surface. Observations have been made at Lord Rosse's observatory of all the recent lunar eclipses, with this end in view.[1] *e.* Finally, at the time when the moon is eclipsed, it is possible to observe its passage over small stars which cannot be seen at all when near the moon except at such a time. Observations of these star occultations made at different parts of the earth furnish the best possible data for computing the dimensions of the moon, its parallax, and for determining its precise position in its orbit at the time of observation. The eclipses of the last few years have been very carefully observed in this way by concert between the different leading observatories.

378. Computation of a Lunar Eclipse. — Since all the phases of a lunar eclipse are seen everywhere at the same absolute instant wherever the moon is above the horizon, it follows that a single computation giving the Greenwich times of the different phenomena is all that is needed, and can be made and published once for all. Each observer has merely to correct the predicted time by simply adding

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[1] See Art. 399* at end of chapter.

or subtracting his longitude from Greenwich in order to get the true local time. The computation is very simple.

The method of projecting and calculating a lunar eclipse is given in the Appendix (Art. 1004).

ECLIPSES OF THE SUN.

379. Dimensions of the Moon's Shadow. — By the same method as that used for the shadow of the earth (merely substituting in the formulæ the radius of the *moon* for that of the earth), we find that the length of the moon's shadow at any time is $\frac{1}{399.85}$ of its distance from the sun, and at new moon averages 232,150 miles. It varies not quite 4000 miles each way, and so ranges from 236,050 miles to 228,300. The semi-angle of the moon's shadow is practically equal to the semi-diameter of the sun seen at the earth, or very nearly 16′.

380. The Moon's Shadow on the Earth's Surface. — Since the mean length of the shadow is less than the mean distance of the moon from the earth (which is 238,800 miles), it is obvious that *on the average* it will not reach to the earth. On account of the

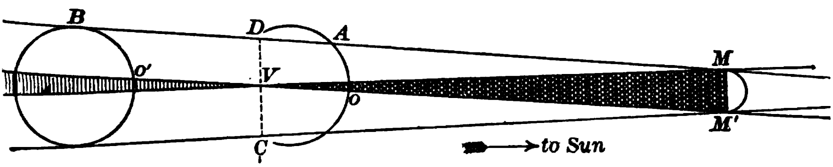

Fig. 129. — The Moon's Shadow on the Earth.

eccentricity of the moon's orbit however, our satellite is much of the time considerably nearer than this mean distance, and may come within 221,600 miles from the earth's centre, or about 217,650 miles from its surface. The shadow, also, under favorable circumstances, may have a length of 236,050 miles. Its point may therefore at times extend nearly 18,400 miles beyond the earth's surface. The cross-section of the shadow where the earth's surface cuts it (at o in Fig. 129) will then be 167 miles. *This is the largest value possible.*

Of course, if the shadow strikes obliquely on the surface of the earth, as it must except when the moon is in the zenith, the shadow spot will be *oval* instead of circular, and the length of the oval along the earth's surface may much exceed the true cross-section of the shadow.

381. The "Negative" Shadow. — Since the distance of the moon may be as great as 252,970 miles from the earth's centre, or nearly

249,000 miles from its surface, while the shadow may be as short as 228,300 miles, we may have the state of things indicated by placing the earth at B in the figure. The vertex of the shadow, V, will then fall 20,700 miles short of the surface, and the cross-section of the "*shadow produced*" will have a diameter of 196 miles where the earth's surface cuts it. When the shadow falls near the edge of the earth, this cross-section may be as great as 230 miles. The shadow-spot which is formed by the intersection of the produced shadow-cone with the earth's surface is sometimes called the *negative shadow*.

382. Total and Annular Eclipses. — To an observer within the true shadow cone, that is, between V and the moon in Fig. 129, the sun will be *totally* eclipsed; but an observer in the produced cone beyond V will see the moon projected on the sun, leaving an uneclipsed ring around it. He will have what is called an *annular* eclipse. These annular eclipses are considerably more frequent than total eclipses — nearly in the ratio of three to two.

383. The Penumbra and Partial Eclipses. — The penumbra can easily be shown to have a diameter on the line CD (Fig. 129) of very nearly twice the moon's diameter.[1] We may take it as having an average diameter at this point of 4400 miles; but as the earth is often be-

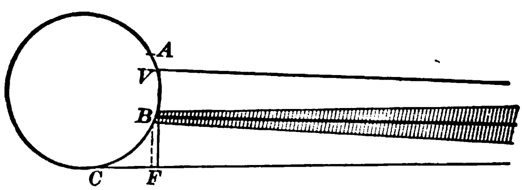

FIG. 130. — Width of the Penumbra of the Moon's Shadow.

yond V, its cross-section *at the earth* is sometimes as much as 4800 miles. An observer situated within the penumbra observes a partial eclipse: if he is near the shadow cone, the sun will be mostly covered by the moon; but if near the outer limit of the penumbra, the moon will only slightly encroach on the sun's disc. While, therefore, total and annular eclipses are visible as such only by an observer within the narrow path traversed by the shadow-spot, the same eclipse will be visible as a partial one everywhere within 2000 miles on

[1] Because the angle DMV (Fig. 129) is the angular diameter of the sun as seen from M, and this is nearly equal to the moon's diameter seen from the earth, *i.e.*, about 31′.

either side of the shadow path; and the 2000 miles is to be reckoned *perpendicularly* to the axis of the shadow. When, for instance, the penumbra falls, as shown in Fig. 130, the distance BC *measured along the earth's surface* will be over 3000 miles, although BF is only 2000.

384. Velocity of the Shadow and Duration of Eclipses. — The moon advances along its orbit very nearly 2100 miles an hour, and were it not for the earth's rotation this is the rate at which the shadow would pass the observer. The earth, however, is rotating towards the east in the same general direction as that in which the shadow moves, and its surface moves at the rate of about 1040 miles

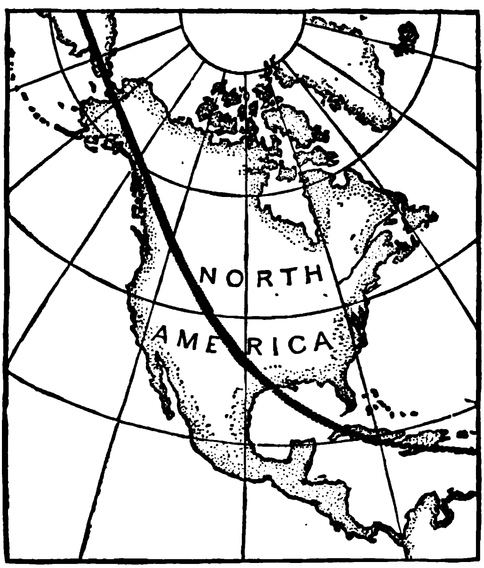

FIG. 131. — Track of the Moon's Shadow, Eclipse of July, 1878.

an hour at the equator. An observer, therefore, on the earth's equator, with the moon near the zenith, would be passed by the shadow with a speed of about 1060 miles per hour (2100 — 1040); and this is its slowest velocity, which is about equal to that of a cannon-ball.

In higher latitudes, where the velocity of the earth's rotation is less, the relative speed of the shadow is higher; and where the shadow falls very obliquely, as it does when an eclipse occurs near sunrise or sun-

set, the advance of the shadow along the earth's surface may become exceedingly swift, — as great as 4000 or 5000 miles per hour. Fig. 131, which we owe to the courtesy of the publishers of Langley's "New Astronomy," shows the track of the moon's shadow during the eclipse of July 29, 1878.

385. Duration of an Eclipse. — A *total* eclipse of the sun observed at a station near the equator under the most favorable conditions possible (the shadow-spot having its maximum diameter of 167 miles), may continue total for *seven minutes and fifty-eight seconds.* In latitude 40° the duration of totality can barely equal six and one quarter minutes. The greatest possible excess of the radius of the moon over that of the sun is only 1′ 19″.

An *annular* eclipse may last for $12^m\ 24^s$ at the equator. The maximum width of the ring of the sun visible around the moon is 1′ 37″.

In the observation of an eclipse four "contacts" are noted: the first, when the edge of the moon first touches the edge of the sun; the second, when the eclipse becomes total or annular; the third, at the cessation of the total or annular phase; and the fourth, when the moon finally leaves the disc of the sun. From the first contact to the fourth the duration may be a little over four hours.

386. The Solar Ecliptic Limits. — It is necessary, in order to have an eclipse of the sun, that the moon should encroach on the cone *ACBD* (Fig. 132), which envelops earth and sun. In this case the "true" angular distance between the centres of the sun and moon

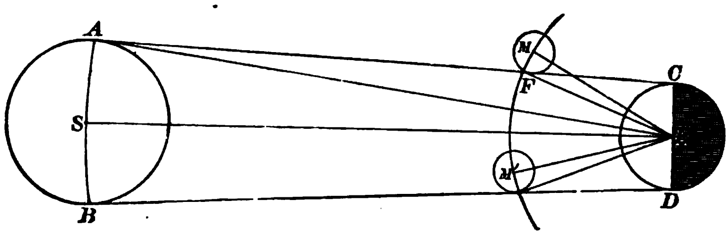

FIG. 132. — Solar Ecliptic Limits.

— that is, their distance as seen from the centre of the earth — would be the angle *MES* in the figure. This is made up of three angles: *MEF*, which equals the moon's semi-diameter, *S′*; *AES*, the sun's semi-diameter, *S*; and *FEA*. This latter angle is equal to the differ-

ence between *CFE* and *FAE*. *CFE* is the moon's horizontal parallax (the semi-diameter of the earth seen from the moon), and *FAE* or *CAE* is the sun's parallax. *FEA*, therefore, equals $P-p$; and the whole angle *MES* equals $S+S'+P-p$. This angle may range from 1° 34′ 13″ to 1° 24′ 19″, according to the changing distances[1] of the sun and moon from the earth.

The corresponding distances of the sun from the node, calculated in the same way as the lunar ecliptic limits (taking the maximum inclination of the moon's orbit as 5° 19′ and the minimum as 4° 57′, according to Neison), give 18° 31′ and 15° 21′ for the major and minor ecliptic limits.

In order that an eclipse may be *central* (total or annular) at any part of the earth, it is necessary that the moon should lie wholly inside the cone *ACBD*, as at *M′*. In this case the angle *M′ES* will be $S-S'+P-p$, and the corresponding major and minor *central* ecliptic limits come out 11° 50′ and 9° 55′.

387. Phenomena of a Solar Eclipse. — There is nothing of special interest until the sun is mostly covered, though before that time the shadows cast by foliage begin to look peculiar. The light shining through every small interstice among the leaves, instead of forming a little *circle* on the earth, makes a little *crescent* — an image of the partly covered sun.

Some ten minutes before totality the darkness begins to be felt, and the remaining light, coming as it does from the edge of the sun only, is much altered in quality, producing an effect very like that of a calcium light rather than sunshine. Animals are perplexed, and birds go to roost. The temperature falls a few degrees, and sometimes dew appears.

[1] We give herewith in a table the different quantities which determine the dimensions of the shadows of the earth and moon, as well as the ecliptic limits and the duration of eclipses.

	Greatest.	Least.	Mean.
Apparent semi-diameter of sun . .	16′ 18″	15′ 46″	16′ 02″
Apparent semi-diameter of moon .	16′ 46″	14′ 42″	15′ 34″
Horizontal parallax of the sun . .	8″.95	8″.65	8″.80
Horizontal parallax of the moon .	61′ 28″	53′ 55″	57′ 02″
Inclination of moon's orbit . . .	5° 19′	4° 57′	5° 8′ 43″

Sun's radius, 433200 miles; earth's (mean), 3956; moon's, 1081.5.

In a few moments, if the observer is so situated that his view commands a distant western horizon, the moon's shadow is seen coming much like a heavy thunder-storm. It advances with almost terrifying swiftness until it envelops him.

For a moment the air appears to quiver, and on every white surface bands or "*fringes*," alternately light and dark, appear. They are a few inches wide and from a foot to three feet apart, and on the whole seem to be parallel to the edge of the shadow. Probably they travel with the *wind;* but observations on this point are as yet hardly decisive. The phenomenon is not fully explained, but is probably due to irregular atmospheric refraction of the light coming from the indefinitely narrow strip of the sun's limb on the point of disappearing.

388. Appearance of the Corona and Prominences. — As soon as the shadow arrives, and sometimes a little before it, the corona and prominences become visible. The stars of the first three magnitudes make their appearance at the same time.

The suddenness with which the darkness descends upon the observer is exceedingly striking; the sun is so brilliant that even the small portion which remains visible up to within a very few seconds of the time of totality so dazzles the eye that it is not prepared for the sudden transition. In a few moments, however, the vision becomes accustomed to the changed circumstances, and it is then found that the darkness is not really very intense. If the totality is of short duration, — that is, if the diameter of the moon exceeds that of the sun by less than a minute of arc, — the lower parts of the corona and chromosphere, which are very brilliant, give a light at least three or four times as great as that of the full moon. Since the shadow also in such a case is of small diameter, a large quantity of light is sent in from the surrounding air, where thirty or forty miles away the sun is still shining; and what may seem remarkable, this intrusion of outside light is greatest when the sky is clouded. In such an eclipse there is not much difficulty in reading an ordinary watch-face. In an eclipse of long duration (say five or six minutes) it is much darker, and lanterns are necessary.

389. Observations of an Eclipse. — A total solar eclipse offers an opportunity of making an immense number of observations of great importance which are possible at no other time, besides certain others which can also be made during a partial eclipse. We mention (*a*) *Times of the four contacts, and direction of the line joining the cusps during the partial phases.*

These observations determine accurately the relative position of the sun and moon at the time, and so furnish the means for correcting the tables of their motion. (*b*) *The search for intra-mercurial planets.* It has been thought likely that there may be one or more planets between the orbit of Mercury and the sun, and during a total eclipse they would become visible, if ever. On the whole, however, the observations, so far made, negative the existence of any body of considerable size in this region, though in 1878, Professor Watson and Mr. Swift, it was thought, had discovered one, if not two, such planets. (*c*) *Observations on the fringes,* which have been described as showing themselves at the commencement of totality. Probably the phenomenon is merely atmospheric and of little importance, but it is not yet sufficiently understood. (*d*) *Photometric measurements* of the intensity of the light at different stages of the eclipse and during totality. (*e*) *Telescopic observations of the details of the prominences and of the corona.* (*f*) *Spectroscopic observations,* both visual and photographic, upon the spectra of the lower atmosphere of the sun, the prominences, and the corona. (*g*) *Observations with the polariscope* upon the polarization of the light of the corona, for the purpose of determining the relation between the reflected and intrinsic light, and perhaps the size of the reflecting particles which are distributed through the corona. (*h*) *Photography,* both of the partial phases and of the corona.

flash spectrum cf §319

390. Calculation of a Solar Eclipse. — The calculation of a solar eclipse cannot be dealt with in any such summary way as that of a lunar eclipse, owing to the moon's parallax, which makes the times of contact and other phenomena different at every different station. The moon's apparent path in the sky, *relative to the centre of the sun*, is not even a portion of a great circle, nor is it described with a uniform velocity. Moreover, since the phenomena of a solar eclipse admit of very accurate observations, it is necessary to take account of numerous little details which are of no importance in a lunar eclipse.

Certain data for each solar eclipse hold good wherever the observer may be. These are calculated beforehand and published in the nautical almanacs; and from them, with the knowledge of his geographical position, the observer can work out the results for his own station. But the calculations are somewhat complicated and lie beyond our scope. The reader is referred to any work on practical astronomy; Chauvenet and Loomis treat the matter very fully. Th. von Oppolzer, lately deceased, published at Vienna in 1887 a most remarkable and monumental work entitled "Canon der Finsternisse" ("Canon of Eclipses"), containing the approximate elements of all eclipses (8000 solar and 5200 lunar) between the years 1207 B.C. and 2162 A.D., with charts showing the approximate track of the moon's shadow for all annular and total eclipses of the sun.

391. Number of Eclipses in a Year. — The least possible number is *two*, both central eclipses of the sun. The largest possible number

is *seven*, *five* of the sun and *two* of the moon. The eclipses each year happen at two seasons (which may be called the "eclipse months"), half a year apart — about the times, of course, at which the sun in its annual path crosses the two nodes of the moon's orbit. If these nodes were stationary, the eclipse months would be always the same; but because the nodes retrograde around the ecliptic once in about nineteen years, the eclipse months are continually changing. The time required by the sun in passing around from a node to the same node again is 346.62 days, which is sometimes called the "eclipse year."

392. Number of Lunar Eclipses. — Representing the ecliptic by a circle (Fig. 133) with the two opposite nodes A and a, it is easy to see *first*, that there can be but two *lunar* eclipses in a year (omitting for a moment one exceptional case). The major lunar ecliptic limit is 12° 15′; hence there is only a space of twice that amount, or 24° 30′, between L and L', at each "node month," within which the occurrence of a full moon might give a lunar eclipse. Now, in a synodic month the sun moves along the ecliptic 29° 6′, while the node moves in the opposite direction 1° 31′, giving the *relative* motion of the sun referred to the node equal to 30° 37′; *i.e.*, *the full-moon points on the circle would fall at a distance of* 30° 37′ *from each other*. Only *one* full moon, therefore, can possibly occur within the lunar ecliptic limits each time that the sun passes the node.

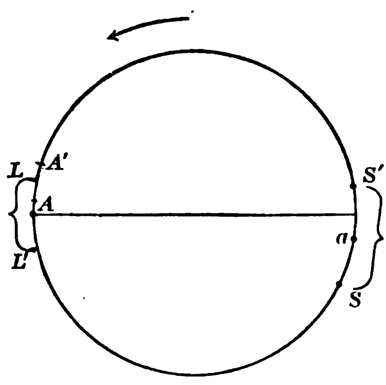

FIG. 133. — Number of Eclipses Annually.

Since the *minor* ecliptic limit for the moon is only 9° 30′, it may easily happen that *neither* of the full moons which occur nearest to the time when the sun is at the node will fall within the limit. There are accordingly many years which have no lunar eclipses.

Three lunar eclipses, however, may possibly happen in one calendar year in the following way. Suppose the first eclipse occurs about Jan. 1, the sun passing the node about that time; the second may then happen about June 25 at the other node, a. The first node, A, will run back during the year, so that the sun will encounter it again about Dec. 13 at A', and thus a third eclipse may occur in December of

the same year. This occurred last in 1852, and will happen again in 1898 and 1917.

393. Number of Solar Eclipses. — Considering now *solar* eclipses, we find that there must inevitably be *two*. Twice the minor limit (Art. 386) of a solar eclipse (15° 21′) is 30° 42′, which is more than the sun's whole motion in a month. One new moon, at least, therefore, *must* fall within the limiting distance of the node, and two *may* do so, since in the figure, SS' is always greater than the distance between the points occupied by two successive new moons.

If the two *new* moons in the two eclipse months happen to fall very near a node, the two *full* moons, a fortnight earlier and later, will both be very likely to fall outside the lunar limit. In that case the year will have only two eclipses, both solar and both central; *i.e.*, either total or annular; as in 1886 and 1904.

Again, if in any year two *full moons* occur when the sun is very near the node, then since the *major* solar limit is 18° 31′, it may happen, and often does, that there will be two partial solar eclipses, one a fortnight before, the other a fortnight after, each of the lunar eclipses, and so the year will have three eclipses in each eclipse month — six eclipses in all, two lunar and four solar. A *fifth* solar eclipse may also come in near the end of the year, if the node was passed about Jan. 15, in the same way that sometimes happens with a lunar eclipse: the year will then have *seven* eclipses. This was the case in 1823, and will next happen in 1935. The most usual number of eclipses is four or five.

394. Relative Frequency of Solar and Lunar Eclipses. — Although, *taking the whole earth into account*, the solar eclipses are the most numerous, about in the proportion of four to three, it is not so with the eclipses *visible at any given place*. A solar eclipse can be seen only from a limited portion of the globe, while a lunar eclipse is visible over considerably more than half the earth, either at the beginning or end, if not throughout its whole duration; and this more than reverses the proportion between lunar and solar eclipses for any given station.

395. Recurrence of Eclipses, and the Saros. — It is not known how early it was discovered that eclipses recur at a regular interval of eighteen years and eleven and one-third days (*ten* and one-third days, if there happen to be five leap years in the interval); but the Chaldeans

knew the period very well, and called it the *Saros* (which means "restitution" or "repetition"), and used it in predicting the recurrence of these phenomena. It is a period of 223 *synodic months*, which is almost exactly equal to nineteen eclipse years. The eclipse year is $346^d.6201$, and nineteen of them equal $6585^d.78$, while 223 months equal $6585^d.32$.

The difference is only $\frac{46}{100}$ of a day (about 11 hours) in which time the sun moves 28′. If, therefore, an eclipse should occur to-day at new moon, with the sun exactly at the node, then after 223 months (18 years 11 days) a new moon will occur again with the sun only 28′ west of the node; so that the circumstances of the first eclipse will be pretty nearly repeated. It would however occur about eight hours of longitude further west on the earth's surface, since the 223 months exceed the even 6585 days by $\frac{32}{100}$ of a day, or $7^h\ 42^m$.

As an example, the four eclipses of 1878 occurred as follows: February 2, solar, annular; February 17, lunar, partial; July 29, solar, total; and August 12, lunar, partial. In 1896 the corresponding eclipses were: February 13, solar, annular; February 28, lunar, partial; August 9, solar, total; and August 23, lunar, partial.

396. Number of Recurrences of a Given Eclipse. — It is usual to speak of eclipses recurring at this regular interval as "repetitions" of one and the same eclipse. Thus, the total solar eclipses of April 1846, May 1864, May 1882, May 1900, June 1918, June 1936, June 1954, July 1972, and August 2008 are for many purposes considered as mere recurrences of one and the same phenomenon. A lunar eclipse is usually thus "repeated" 48 or 49 times. Beginning as a very small partial eclipse, with the sun about 12° *east* of the node, it will be a little larger at its next occurrence eighteen years later; and after 13 or 14 repetitions the sun will have come so near the node that the eclipse will have become total. It will then be repeated as a total eclipse 22 or 23 times, after which it will become partial again with the sun *west* of the node, and after 13 more returns as a partial eclipse will finally dwindle away and disappear, having thus recurred regularly once in every 223 months during an interval of 865½ years.

The same thing happens with the solar eclipses, only since the solar ecliptic limit is larger than the lunar, a solar eclipse has from 68 to 75 returns, occupying some 1260 years. Of these about 25 are only partial eclipses, the sun being so near the ecliptic limit that the *axis* of the shadow does not reach the earth at all. The 45 eclipses in the middle of the period are central somewhere or other on the earth, about 18 of them being total, and about 27 annular. These numbers vary somewhat, however, in different cases.

397. It is to be noticed that the Saros exhibits not only a close commensurability of the *synodic* months with the *eclipse* years, but also with the *nodical*[1] and *anomalistic* months: 242 nodical months equal 6585.357 days; 239 anomalistic months equal 6585.549 days. This last coincidence is important. The moon at the end of the Saros of 223 months not only returns very closely to its original position *with respect to the sun and the node*, but also with respect to *the line of apsides* of its orbit. If it was at perigee originally, it will again be within five hours of perigee at the end of the Saros. If this were not so, the time of the eclipse might be displaced several hours by the perturbations of the moon's motion, to be considered later, in Chap. XII.

398. Number of Eclipses in a Single Saros. — The total number is usually about seventy, varying two or three one way or the other, as new eclipses come in at the eastern limit and go out at the western. Of the 70, 29 are usually lunar and 41 solar; and of the solar, 27 are *central*, 17 being annular and 10 total. (These numbers are necessarily only approximate.) It appears, therefore, that total solar eclipses, *somewhere or other on the earth*, are not very rare, there being about ten in eighteen years. Since, however, the shadow track averages less than 100 miles in width, each total eclipse is visible, *as total*, over only a very small fraction of the earth's whole surface — about $\frac{1}{200}$ in the mean. This gives about one total eclipse in 360 years, in the long run, at any given station.

The total solar eclipses visible in the United States during the nineteenth century have been the following: —

June 16, 1806, in New York and New England, duration $4\frac{1}{2}$ minutes; Nov. 30, 1834, in Arkansas, Missouri, Alabama, and Georgia, duration 2 minutes; July 18, 1860, in Washington Territory and Labrador, 3 minutes; Aug. 7, 1869, in Iowa, Illinois, Kentucky, North Carolina, $2\frac{3}{4}$ minutes; July 29, 1878, in Wyoming, Colorado, Texas, $2\frac{1}{2}$ minutes; Jan. 11, 1880, in California, duration 32 seconds; Jan. 1, 1889, in California and Montana, $2\frac{1}{4}$ minutes; On the morning of May 28, 1900, the moon's shadow crosses the country from Texas to Virginia, the totality lasting in Virginia about two minutes.

Total eclipses visible in the United States occur during the next century in 1918, 1923, 1925, 1945, 1954, 1979, 1984, and 1994, according to Oppolzer's "Canon."

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[1] The *nodical* month is the time of the moon's revolution from one of its *nodes* to the same node again, and is equal to $27^d.21222$; the *anomalistic* month is the time of revolution from *perigee* to perigee again, and equals $27^d.55460$. See Arts. 454, 455. The nodical month is also called the *draconitic* month.

399. Occultations of Stars. — In theory, and in the method of computation, the occultation of a star is precisely like a solar eclipse, except that the shadow of the moon projected by a star is a *cylinder* instead of a *cone*, since, compared with the distance of the sun, that of a star is infinite: moreover, the star is a mere point, so that there is no sensible penumbra. In other words, a star has neither parallax nor semi-diameter, and these circumstances somewhat simplify the formulæ.

As the moon moves always towards the east, the disappearance of the star always takes place at the eastern limb, and the reappearance at the western. In the first half of the lunation the eastern limb is dark and invisible, and the star vanishes without warning. The suddenness with which it vanishes and reappears has already been referred to (Art. 255) as proof of the non-existence of a lunar atmosphere. Observations of this sort determine the moon's place with great accuracy, and when corresponding observations are made at different places, they supply one of the best possible means of determining their difference of longitude.

In some cases observers have reported that a star, instead of disappearing instantaneously when struck by the moon's limb (faintly visible by earthshine), has appeared to cling to the limb for a second or two before vanishing, and in a few instances they have even reported it as having reappeared and disappeared a second time, as if it had been for a moment visible through a rift in the moon's crust. Some of these anomalous phenomena have been explained by the subsequent discovery that the star was double, or had a faint companion, but for the most part are probably due to "bad definition" of air or instrument, or to physiological causes in the observer.

399*. (Supplementary to Art. 377.) During the progress of a lunar eclipse the heat radiated from the moon varies almost exactly with the light, so that when the totality begins the heat has lost full 98 per cent of its original amount, and during the totality falls off about one per cent more. Then, as the light returns, the heat rises almost as rapidly as it fell. This indicates that the lunar surface has almost no power of "*storing*" heat, — a natural consequence of its airlessness.

A very singular fact, moreover, is that *after the eclipse the lunar radiation does not for several hours recover the value it had before the eclipse began.* In

1888, when the moon left the *penumbra*, and was again receiving unobstructed sunshine, the heat had risen to only 80 per cent of the original value, and $1^h\ 40^m$ later had gained only one per cent more. The same thing was observed in 1884. No explanation as yet appears.

Note on the Solar Eclipse of January 22d, 1898.

The shadow of the moon traversed the continent of India in the early afternoon, striking the western coast about 150 miles south of Bombay, and crossing the Himalayas near Mt. Everest. The weather was fine all along its path, and the numerous observers at more than a dozen stations were brilliantly successful, almost without exception. The duration of totality, however, was unfortunately very short, — hardly two minutes.

It is too early as this chapter goes to press to give the final results of the observations, but it is probable that the abundant data collected will be decisive in respect to many important problems of solar physics, — especially those which relate to the origin of the Fraunhofer lines.

The "flash-spectrum" (see note to Art. 319) was successfully photographed at a number of stations, both with spectroscopes of the ordinary form and with prismatic cameras, and with a dispersion fully double that used by Mr. Shackleton in 1896. The comparison of these photographs with those of the ordinary solar spectrum made with similar dispersive power can hardly fail to determine what lines originate low down in the solar atmosphere, and which of them, if any, are produced only in its upper levels.

Fully a hundred photographs of the corona were made with instruments ranging from telescopes of 40-feet focal length down to small cameras. One set of negatives made with a polariscopic apparatus shows distinctly the polarization of one of the longest streamers, indicating the presence of dust or mist. The gaseous elements of the corona were, however, unusually faint, and no advance seems to have been made towards determining more exactly the true position of the violet rings in the corona spectrum, referred to in Art. 329.

The corona on this occasion had the form of an irregular four-rayed star, the principal streamers issuing from the sun-spot zones, while the equatorial extension was short and faint.

The photographs made during the eclipses of 1900 and 1901 fully confirm the results stated above, and those of Mr. Evershed show a number of additional coronium and helium lines in the ultra-violet.

Exercises on Chapters X and XI.

1. If the diameter of the sun is decreasing at the rate of 300 feet a year, how long before its apparent diameter will have decreased by 1″? (See Art. 276.) *Ans.* 7927 years.

2. If the rate of shrinkage be assumed to continue uniform (*i.e.*, 300 feet a year, an improbable assumption), how long will it be before its diameter is diminished by one per cent? *Ans.* Over 150000.

3. How much would its density then be increased?

4. Taking the calorie as equivalent to 428 kilogram-metres of energy, what weight falling 100 metres would at the end of its fall possess an energy equal to that of the solar radiation received in an hour upon ten square metres of the earth's surface, allowing for a loss of fifty per cent absorbed by the air? *Ans.* 38520 kilograms.

5. Assuming (Art. 332) that sunlight at the earth equals 70000 times that of a standard candle at a distance of one metre, at what distance would the light of the sun equal that of a 2000 candle-power electric arc ten metres distant?

Ans. About 59 times the earth's distance from the sun.

6. Can an eclipse of the moon ever occur in the daytime? (Consider the possible effect of refraction.)

7. Why cannot there be an annular eclipse of the moon?

8. Which are most frequent in New York, solar eclipses or lunar?

9. If a lunar eclipse has occurred this year in August, can there be one in June of next year? or in October? If not, why not?

10. Can an occultation of Venus occur during an eclipse of the moon? Is one of Jupiter possible?

11. In a solar eclipse which side of the sun's disc is first touched by the moon, the east or the west?

12. Does the shadow of the moon during a solar eclipse *ever* travel westward over the surface of the earth? (Consider the case of an eclipse within the polar circle occurring near midnight.)

CHAPTER XII.

CENTRAL FORCES: EQUABLE DESCRIPTION OF AREAS.—AREAL, LINEAR, AND ANGULAR VELOCITIES. — KEPLER'S LAWS AND INFERENCES FROM THEM. — GRAVITATION DEMONSTRATED BY THE MOON'S MOTION. — CONIC SECTIONS AS ORBITS. — THE PROBLEM OF TWO BODIES. — THE "VELOCITY FROM INFINITY," AND ITS RELATION TO THE SPECIES OF ORBIT DESCRIBED BY A BODY MOVING UNDER GRAVITATION.—THE INTENSITY OF GRAVITATION.

400. A MOVING body left to itself, according to Newton's first law of motion (Physics, p. 26), moves on forever in a straight line with a uniform velocity. If we find a body so moving, we may, therefore, infer that it is either acted on by *no* force whatever, or, if forces are acting upon it, that they exactly balance each other.

It has been customary with some writers to speak of a body thus moving "uniformly in a straight line" as actuated by a "projectile *force*," a very unfortunate expression, which is a survival of the Aristotelian idea that rest is more "natural" to matter than motion, and that when a body moves, some force must operate to keep it moving. The mere uniform rectilinear motion of a material mass in empty space implies no action of a physical cause, and demands explanation only as mere existence does. *Change* of motion, either in speed or in direction—this alone implies *force in operation.*

401. If a body moves in a straight line, with swiftness either increasing or decreasing, we infer a force acting exactly in the line of motion, and accelerating or retarding it. If it moves in a *curve*, we know that some force is acting *athwart* the line of motion. If the velocity in the curve increases, we know that the direction of the force that acts is *forward*, like *ab* (Fig. 134), making an angle of less than 90° with the "line of motion" *at* (the tangent to the path of the body); and *vice versa*, if the motion of the body is retarded.

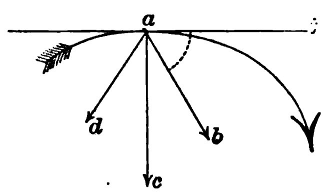

FIG. 134. — Curvature of an Orbit.

If the speed does not alter at all, we know that the force must act along the line *ac*, *exactly perpendicular* to the line of motion.

Here, also, we find many writers, the older ones especially, bringing in the "projectile force," and saying that when a body moves in a curve it does so under the action of *two* forces, one a force that draws it sideways, the other the "projectile force" directed along its path. We repeat; this "projectile force" has no present existence or meaning in the problem. Such a force may have put the body in motion long ago, but its function has ceased, and *now* we have only to do with the action of one single force, — *the deflecting force*, which alters the direction of the body's motion. Of course it is not intended to deny that the deflecting force may itself be the resultant of any number of forces all acting together; but a single force acting athwart a body's line of motion is sufficient to cause it to describe a curvilinear orbit, and from such an orbit we can only infer the *necessary* existence of *one* such force.

402. Description of Areas. — (*a*) If a body is moving uniformly on a straight line, and if we connect the points A, B, C, etc., Fig. 135, which it occupies at the end of successive units of time with any point whatever, as O, we shall have a series of triangles, AOB, etc., *which will all be equal;* since their bases AB, BC, etc., are equal and on the same straight line, and they have a common vertex at O. Calling the line from A to O its radius vector, and O the "centre," we may say, therefore, that when a body is moving undisturbed by any force whatever, *its radius vector, from any centre arbitrarily chosen, describes equal areas in equal times around that centre.* The area enclosed in the triangle described by the radius vector in a unit of time is called the body's "*areal* (or "*areolar*") velocity," and in this case is constant.

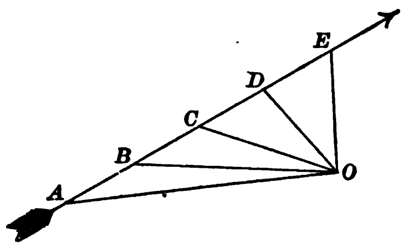

FIG. 135.
Description of Areas in Uniform Motion.

403. (*b*) Moreover, *any impulse in the line of the radius vector, either towards or from the centre, leaves unchanged both the plane of the body's motion and its areal velocity.*

Suppose a body moving uniformly on the line AC (Fig. 136) with such a velocity that it describes AB, BC, etc., in successive units of time; then, by the preceding section, the areal velocity will be con-

stant, and measured by the area of any one of the equal triangles AOB, BOC, or COL. Suppose, now, that at C the body receives an "impulse" directed along the radius vector towards O — a blow, for instance, which by itself would make the body describe CK in a unit of time. The body will now take a new path, which will carry it to the point D, determined by constructing the "parallelogram of

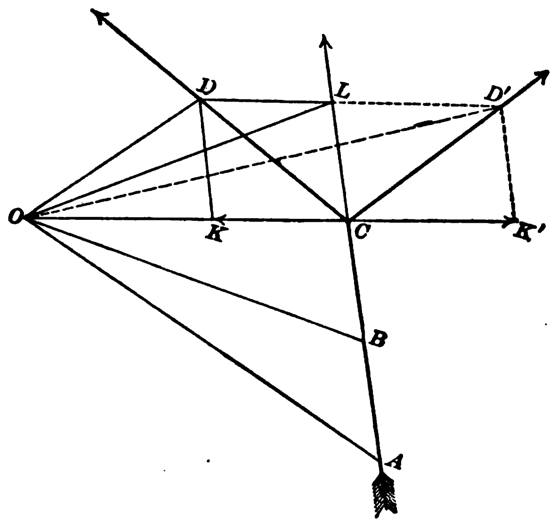

FIG. 136. — Description of Areas under an Impulse directed towards a Centre.

motions" $CKDL$, and thus combining the new motion CK with the former motion CL, according to Newton's second law (Physics, p. 26). The new areal velocity, measured by the triangle OCD, will be the same as before, as is easily shown.

Triangle $BOC =$ triangle COL, because $BC = CL$, and O is their common vertex.

Also, triangle $COL =$ triangle COD, because they have the common base OC, and their summits L and D are on a line which was drawn parallel to this base in constructing the parallelogram of motions. Hence, triangle $BOC =$ triangle COD, and the *areal velocity remains unchanged.*

Also, as may be seen by following out the same reasoning with CK' and CD', the same result would hold true if the impulse had been directed *away from* O instead of towards it.

404. This result depends entirely on the fact that the impulse CK or CK' was *exactly along the radius vector* CO. If it had not been so, then in

constructing the parallelogram of motions to find the points D and D', we should have had to draw LD or LD' *not* parallel to CO, and the two triangles BOC and COD would necessarily have been unequal. COD would be greater than BOC if CK were directed ahead of the radius vector, and less if behind it.

As regards the plane of motion, the point D is on the plane OCL, because LD was drawn through L parallel to OC. OCL is a part of the plane which contains the triangles BOC and AOB, and hence OCD also lies in the same plane.

405. (*c*) From this obviously follows the important general proposition that *when a body is moving under the action of a force always directed towards or from a fixed centre, the radius vector will describe equal areas in equal times; and the path of the body will all lie in one plane.*

Such a force constantly acting is simply equivalent to an indefinite number of separate impulses. Now if no single impulse directed along the radius vector can alter the areal velocity or plane of motion, neither can a succession of them. Hence the proposition follows.

In case of a *continuously* acting force the orbit, however, will become a curve instead of being a broken line.

Observe that this proposition remains true whether the force is attractive or repulsive, and that it is independent of the *law* of the force; that is, the force may vary directly *with the distance*, or *inversely as the square of the distance*, or as *the logarithm* of it, or in any conceivable way; it may even be *discontinuous*, acting only at intervals and ceasing between times: and still the law holds good.

406. Conversely, *if a body moves in this way, describing equal areas in equal times around a point, it is easily shown that all the forces acting upon the body must be directed toward that point.*

We, however, leave the demonstration to the student.

Since the earth moves very nearly in this way in its orbit around the sun, we conclude that the only force of any consequence acting upon the earth is directed towards the sun. We say, "of any consequence," because there are other small forces which do slightly modify the earth's motion, and prevent it from *exactly* fulfilling the law of areas.

As a direct consequence of the law of equal areas we have certain laws with respect to the linear and angular velocities of a body moving under the action of a central force.

407. Law of Linear Velocity.—Suppose a body moving under the action of a force always directed towards S (Fig. 137), and let AB be a portion of its path which it describes in a second. Draw the tangent Bb. Regarding the sector ASB as a triangle (which it will be, sensibly, since the curvature of the path in one second will be very small) the area of this triangle will be $\frac{1}{2}(AB \times Sb)$. Now AB, the distance travelled in a second, is the *linear velocity* of the body (called *linear* because it is measured with the same units as any other *line; i.e.*, in *miles* or in *feet* per second), and Sb is the distance from the centre of force to the "line of motion," as the tangent Bb is called. For Sb, p is usually written; hence in every part of the same orbit, V (the velocity in miles per second) $= \frac{2A}{p}$, and is inversely proportional to p. If p were to become zero, V would become infinite, unless A were zero also.

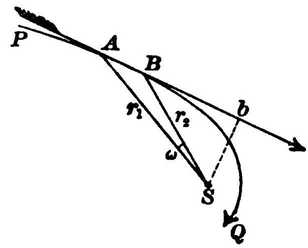

FIG. 137.
Linear and Angular Velocities.

408. Law of Angular Velocity.—Referring again to the same figure, the area of ASB is equal to $\frac{1}{2}$ $(AS \times BS \times \sin ASB)$, or $A = \frac{1}{2} r_1 r_2 \sin \omega$. If we draw r to the middle point of AB, then $r_1 r_2 = r^2$, nearly, since in a second of time the distance would not change perceptibly as compared with its whole length. ω will also be a small angle, so that its sine will equal the angle itself expressed in radians;

hence $$\tfrac{1}{2} r^2 \omega = A, \text{ and } \omega = \frac{2A}{r^2}.$$

Now ω is the *angular velocity* of the body; that is, the number of "radians" which it describes in a second of time, as seen from S, while r is the radius vector.

409. In every case, therefore, of motion under a central force, I. *The Areal velocity* (*acres per second*) *is constant;* II. *The Linear velocity* (*miles per second*) *varies inversely as the distance from the centre of force to the body's line of motion at the moment*, which line of motion is the tangent to the orbit at the point where the body happens to be; III. *The Angular velocity* (*radians, or degrees, per second*) *varies inversely as the square of the distance of the body from the centre of force.*

410. The student will remember that it was found by observation that the sun's angular velocity varies as the square of its apparent diameter, and from this (Art. 186) the law of equal areas was inferred as a fact with respect to the earth's motion. Newton was the first to point out that a body moving under the action of a central force must *necessarily* observe this law of areas, and, conversely, that a body thus observing the law of areas must necessarily be under the control of a central force.

411. Circular Motion. — In the case of a body moving in a *circle* under the action of a central force, the force must be constant, and (Physics, p. 17) is given by the formula

$$f = \frac{V^2}{r}, \qquad (a)$$

in which r is the radius of the circle and V the velocity, while f is the central force measured as an "acceleration," in metres (or feet) per second; that is, by the number of units of velocity which the force would generate in the body in a second of time; just as the force of gravity is expressed by writing, $g = 9.81$ metres.

For many purposes it is desirable to have an expression which shall substitute for V (a quantity not given directly by observation) the time of revolution, t, which is so given. Since V equals the circumference of the circle divided by t, or $\frac{2\pi r}{t}$, we have at once, by substituting this value for V in equation (a),

$$f = 4\pi^2\left(\frac{r}{t^2}\right). \qquad (b)$$

This, of course, is merely the equivalent of equation (a), but is often more convenient.

KEPLER'S LAWS.

412. In 1607–1620 Kepler discovered as facts, without an explanation, three laws which govern the motions of the planets, — laws which still bear his name. He worked them out from a discussion of the observations which Tycho Brahe had made through many preceding years. The three laws are as follows: —

I. *The orbit of each planet is an ellipse, with the sun in one of its foci.*

II. *The radius vector of each planet describes equal areas in equal times.*

III. The "Harmonic law," so-called. *The squares of the periods of the planets are proportional to the cubes of their mean distances from the sun;* i.e., $t_1^2 : t_2^2 = a_1^3 : a_2^3$. (See Art. 423, last sentence.)

413. To make sure that the student apprehends the meaning and scope of this third law, we append a few simple examples of its application.

(1) What would be the *period* of a planet having a mean distance from the sun of 100 astronomical units; *i.e.*, a distance 100 times that of the earth?

$$(\textit{Earth's Dist.})^3 : (\textit{Planet's Dist.})^3 = (\textit{Earth's Period})^2 : (\textit{Planet's Period})^2;$$

i.e., $$1^3 : 100^3 = 1^2 \text{ (year)} : X^2 \text{ (years)},$$

whence, $$X = 100^{\frac{3}{2}} = 1000 \text{ years.}$$

(2) What would be the distance of a planet having a period of 125 years?

$$(1)^2 : 125^2 = 1^3 : X^3,$$

whence, $$X = 125^{\frac{2}{3}} = 25 \text{ (Astron. units).}$$

(3) How long would a planet require to fall to the sun?

If the sun were collected in a single point at its centre, a body starting from a point on the planet's orbit with a slight *side motion*, *i.e.*, motion at right angles to the radius vector, would describe an extremely narrow ellipse around the sun, with its perihelion just *at* the sun, and the aphelion at the starting-point. Practically it would "*fall to the sun*," and return just as if it had *rebounded* from a perfectly elastic surface: the time of "falling" would be just equal to that of returning — the two making up the whole period of the body in the narrow ellipse. Now the semi-major axis of this narrow ellipse is evidently *one-half* the radius of the planet's orbit. Hence, to find the period in this ellipse which is 2τ (τ being taken as the time of "falling"), we have

$$a^3 : (\tfrac{1}{2}a)^3 = t^2 : (2\tau)^2, \text{ or } 1 : \tfrac{1}{8} = t^2 : 4\tau^2;$$

whence, $$\tau = t\sqrt{\tfrac{1}{32}} = 0.1768t, \ t \text{ being the planet's period.}$$

In the case of the earth $\tau = 365\frac{1}{4} \times 0.1768 = 64.56$ days.

(4) What would be the period of a satellite revolving close to the earth's surface?

$$(\textit{Moon's Dist.})^3 : (\textit{Dist. of Satellite})^3 = (27.3 \text{ days})^2 : X^2,$$

or $$60^3 : 1^3 = (27.3)^2 : X^2,$$

whence, $$X = \frac{27.3 \text{ days}}{60^{\frac{3}{2}}} = 1^{\text{h}}\ 24^{\text{m}}.$$

(5) How much would an increase of 10 per cent in the earth's distance from the sun increase the length of the year? *Ans.* 56.13 days.

(6) What is the distance from the sun of an asteroid which has a period of $3\frac{1}{2}$ years? *Ans.* 2.305 Astron. units.

414. Many surmises were made as to the physical meaning of these laws. More than one astronomer *guessed* that a force directed toward the sun, or emanating from it, might be the explanation. Newton proved it. He demonstrated the law of equal areas and its converse as necessary consequences of the laws of motion. He also proved that if a body move, as does the earth, in an ellipse having a centre of force at its focus, then the force at different points in the orbit must vary inversely as the square of the distance from that centre. And, finally, he showed that, granting the harmonic law, the force from planet to planet must also vary according to the same law of inverse squares.

415. The demonstration of this last proposition for circular orbits is so simple that we give it, merely adding (without proof) that the proposition is equally true for elliptical orbits, if for r we put a, the semi-major axis of the orbit.

In a circular orbit, from equation (b), (Art. 411), we have

$$f = 4\pi^2\left(\frac{r}{t^2}\right),$$

where r and t are the distance and period of a planet. In the same way the force acting upon a second planet is found from the equation

$$f_1 = 4\pi^2\left(\frac{r_1}{t_1^2}\right),$$

whence,
$$\frac{f}{f_1} = \frac{r}{t^2} \times \left(\frac{t_1^2}{r_1}\right).$$

But by Kepler's third law $t^2 : t_1^2 :: r^3 : r_1^3$,

whence,
$$t_1^2 = \frac{t^2 r_1^3}{r^3}.$$

Substitute this value of t_1^2 in the preceding equation; we have

$$\frac{f}{f_1} = \frac{r}{t^2} \times \frac{t^2 r_1^3}{r^3 r_1} = \frac{r_1^2}{r^2};$$

i.e.,
$$f : f_1 = r_1^2 : r^2,$$

which is the law of inverse squares.

416. Conversely, the harmonic law is just as easily shown to be a necessary consequence of the law of gravitation in the case of circular orbits.

From Art. 411, Eq. (b), we have

$$f = 4\pi^2 \frac{r}{t^2};$$

also, from the law of gravitation,

$$f = \frac{M}{r^2}, \ M \text{ being the mass of the sun.}$$

Hence, equating the two values of f,

$$\frac{M}{r^2} = 4\pi^2\frac{r}{t^2}, \text{ and } t^2 = \frac{4\pi^2}{M}r^3.$$

Similarly for another planet,

$$t_1^2 = \frac{4\pi^2}{M}r_1^3.$$

Whence, $$t^2 : t_1^2 = r^3 : r_1^3.$$

The demonstration for elliptical orbits is a little more complicated, involving the "law of areas." It is given in all works on Theoretical Astronomy, and may be found in Loomis's "Treatise on Astronomy," p. 134.

417. Correction of Kepler's Third Law. — The "harmonic law" as it stands is not *exactly* true, though the difference is too small to appear in the observations which Kepler made use of in its discovery. It would be exactly true if the planets were mere particles of matter; but as a planet's mass is a sensible, though a very small fraction of the sun's mass, it comes into account. The planet Jupiter, for instance, attracts the sun as well as is attracted by it. If at the distance r Jupiter is drawn towards the sun by a force which would give it in a second an acceleration expressed by $G\frac{M}{r^2}$ (the sun's mass being M), then the sun in the same time is accelerated towards Jupiter by the quantity $G\frac{m}{r^2}$ (m being the mass of Jupiter). The rate at which the two tend to *approach each other* is therefore expressed by $G\frac{M+m}{r^2}$. Hence, in discussing the motions of the planet Jupiter around the *centre of the sun*, instead of writing

$f = G\frac{M}{r^2}$ simply, we must put $f = G\frac{M+m}{r^2}$, G being the "constant of Gravitation" (Art. 161).

But (in the case of circular motion) $f = \frac{4\pi^2 r}{t^2}$.

Hence, we find $$Gt^2(M+m) = 4\pi^2r^3;$$

or, as a proportion, $$t^2(M+m) : t_1^2(M+m_1) = r^3 : r_1^3,$$

which is *strictly* true as long as the planet's motions are undisturbed.

418. Inferences from Kepler's Laws. — From Kepler's laws we are entitled to infer —

First (from the second law), that the *force* which retains the planets in their orbits *is directed towards the sun.*

Second (from the first law), that on any given planet the *force varies inversely as the square of its distance from the sun.*

Third (from the harmonic law), that the force is the same for one planet as it would be for another in the same place; or, in other words, the attracting force *depends only on the mass and distance of the bodies concerned, and is wholly independent of their physical conditions,* such as their temperature, chemical constitution, etc. It makes no difference in the motion of a planet around the sun whether it be made of hydrogen or iron, whether it be hot or cold.

419. Verification of "Gravitation" by Means of the Moon's Motion. — When the idea of gravitation first occurred to Newton he endeavored to verify it by comparing the force which keeps the moon in her orbit with the force of gravity at the earth's surface, reduced in the proper proportion. For lack, however, of an accurate knowledge of the earth's dimensions,[1] he failed at first, there being a discrepancy of about sixteen per cent. He had assumed a degree to be exactly sixty miles in length. Some years afterward, when Picard's measure of the arc of a meridian in Northern France had been made and reported to the Royal Society, making a degree about sixty-nine miles long, he saw at once that the new value would reconcile the discrepancy; and he resumed his unfinished work and completed it.

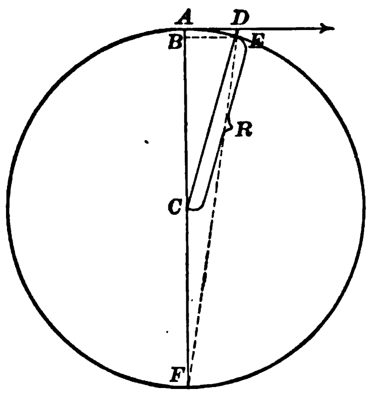

FIG. 138.
Verification of the Hypothesis of Gravitation by Means of the Motion of the Moon.

420. At the earth's surface a body falls about 193 inches in a second. The distance of the moon being very nearly sixty times the earth's radius, if gravity really varies inversely as the square of the

[1] He was long baffled also by the difficulty of proving that the attraction of a globe is the same as if its matter were concentrated at its centre.

distance, a stone at that distance from the earth should fall $\frac{1}{60^2}$ or $\frac{1}{3600}$ as far; that is, it ought to fall $\frac{193 \text{ inches}}{3600} = 0.0535$ inches, — a little more than one-twentieth of an inch. Now the distance which the moon actually *does* fall towards the earth in a second, *i.e.*, the *deflection of its orbit from a straight line in a second of time*, is easily found; and if the force which keeps the moon in its orbit is really the same as that which makes bodies fall towards the centre of the earth, this deflection ought to come out equal to $0^{\text{in.}}.0535$. Let AE (Fig. 138) be the distance the moon travels in a second $= \frac{2\pi r}{t}$, where r is the radius of the moon's orbit, and t the number of seconds in a month. Then, since AEF is a right-angled triangle, we have,

$$AB : AE :: AE : AF \text{ (or } 2r\text{)} ;$$

whence
$$AB = \frac{AE^2}{2r}.$$

The calculation is easy enough, though the numbers are rather large. As a result it gives us $AB = 0.0534$ inches, which is practically equal to the thirty-six hundredth part of 193 inches.

If the quantities did not agree in amount, the discrepancy would disprove the theory, and, as we have said, Newton loyally gave it up until he was able to show that the apparent discordance was the result of a mistake in the original data, and disappeared when the data were corrected. The agreement, however, does not *establish* the theory, but only renders it probable. It does not establish it completely, because it is conceivable that the agreement might be a case of accidental coïncidence, while the forces might really differ as much in their nature as an electrical attraction and a magnetic.

421. Newton was not satisfied with merely showing that the principal motions of the planets and the moon could be explained by the law of gravitation; but he went on to investigate the converse problem, and to determine what must be the motions *necessary* under that law. He found that the orbit of a body moving around a central mass is not of necessity a circle, or even a nearly circular ellipse like the planetary orbits, but that it may be a *conic section* of any eccentricity whatever — a circle, ellipse, parabola, or even an hyperbola; but *it must be a conic.*

422. For the benefit of those of our readers who are not acquainted with conic sections we give the following brief account of them (Fig. 139): —

a. If a cone of any angle be cut *perpendicularly to the axis*, the section will be a *circle* — *MN* in the figure.

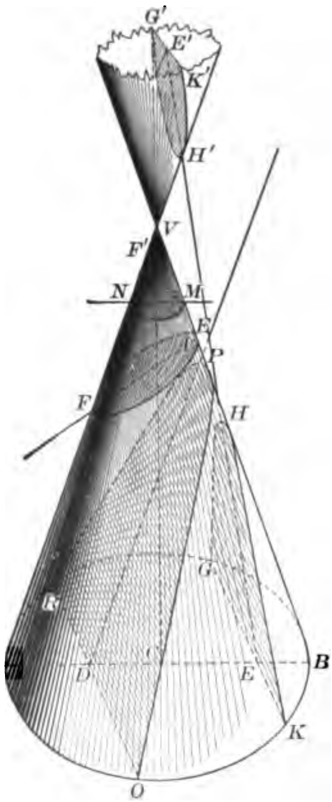

Fig. 139. — The Conics.

b. If it be cut by a plane which makes with the axis an angle *greater* than the semi-angle of the cone, *so that the plane of section cuts completely across the cone* (as *EF*), the section is an *ellipse;* the circle being merely a special case of the ellipse. Ellipses, of course, differ greatly in form, from those which are very narrow to the perfect circle.

c. The parabola is formed by cutting the cone with a plane *parallel to its side; i.e.*, making with the axis an angle *equal to the semi-angle of the cone. RPO* is such a plane. As all circles are alike in form, so are all parabolas, *whatever the angle of the cone at V and wherever the point P is taken.* If the cutting plane is thus situated, then, no matter what is the angle of the cone or the place where the cut is made, the (complete) curve will always be the same *in shape*, though of course *its size* will depend upon a variety of circumstances. The statement seems at first a little surprising; but it is true.

d. If the cutting plane makes an angle with the axis of the cone *less than the semi-angle at V*, so that the cutting plane gets continually deeper and deeper into the cone, then the curve is an *hyperbola;* so called, because the plane in this case "shoots over" (ὑπέρ βάλλειν) and intersects the "cone produced," cutting out of this second cone a curve precisely like the curve cut from the original, as at *H'G'K'* in the figure. The axis of the hyperbola lies outside of the curve itself, being the line *HH'* in the figure, and the "*centre*" of the curve is also outside of the curve at the middle point of this axis.

423. Philosophically speaking there ~~are therefore~~ but *two* species of conic sections, — the ellipse and the hyperbola, with the parabola for a partition between them. (The circle, as has been said before, is merely a special case of the ellipse.) Fig. 140 will give the reader

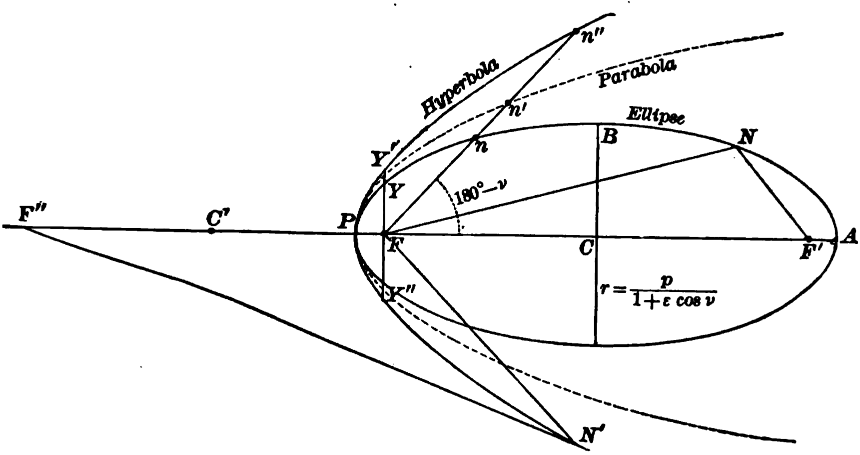

Fig. 140. — The Relation of the Conics to Each Other.

perhaps a better idea of the nature of the curves as drawn on a plane. In the ellipse the sum of the distances from the two foci, $FN + F'N$, equals the major axis of the curve; in the hyperbola it is the difference of these two lines ($F''N' - FN'$) that equals the major axis; in the ellipse the *eccentricity* is *less than unity* (zero in the circle); in the hyperbola it is *greater than unity;* in the parabola *exactly unity.*

The general equation of a conic in polar co-ordinates, applying alike to both the species, is

$$r = \frac{p}{1 + e \cos V},$$

in which r is the distance Fn, or Fn', e is the fraction $\frac{FC}{PC}$ or $\frac{FC'}{PC'}$, the angle V is the angle PFn, PFn', or PFn'', and p is the line FY, FY', or FY'', called the "semi-parameter." The word "*para*meter" means the *cross* measure of a curve, just as "*dia*meter" means the *through* measure of a curve. If e is zero, the curve is a circle, and $r = p$. If $e < 1$, the curve is an ellipse; if $e > 1$, the curve is an hyperbola; if $e = 1$, it is a parabola.

A generalized form of Kepler's third law, applying to hyperbolic and parabolic orbits (which have no periods) as well as elliptical, is this: — *The* ***Areal velocities*** *of bodies revolving around the sun are proportional to the square-roots of the* ***parameters*** *of their orbits.*

424. Problem of Two Bodies. — This problem, proposed and solved by Newton, is the following: —

Given the masses of two spheres and their positions and motions at any moment; given, also, the law of gravitation: required their motion ever afterwards, and the data necessary to compute their place at any future time.

The mathematical methods by which the problem is solved require the use of the calculus, and must be sought in works on analytical mechanics or theoretical astronomy. Some of the results, however, are simple and easily stated.

425. (1) In the first place the motion of the centre of gravity of the two bodies will not be affected by their mutual attraction, *but it* will move on uniformly through space, as if the bodies were united into one at that point, and their motions combined under the same laws which hold good in the case of the collision of inelastic bodies.

The motion of this centre of gravity is most easily worked out graphically as follows: First, in Fig. 141, join the original places of the bodies A and B by a straight line, and mark on it G, the place of the centre of gravity; then take the positions A' and B' they would occupy at the end of a unit of time (if they did not attract each other), and mark the new position of the centre of gravity G' on the line joining them. The line GG' connecting

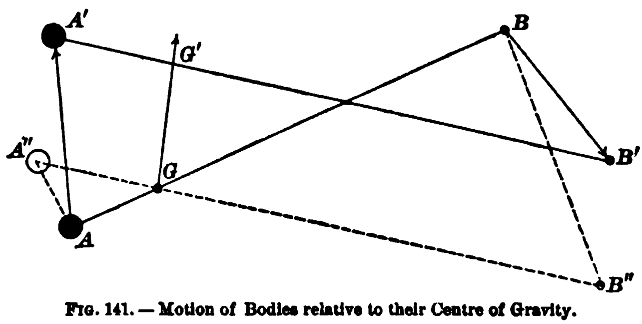

FIG. 141. — Motion of Bodies relative to their Centre of Gravity.

the two positions of the centre of gravity will show the direction and rapidity of its motion; with reference to this point the two bodies will have opposite motions proportional to their distances from it; that is, they will swing around this point as if on a rod pivoted there, and will either both move towards it along the rod, or from it, with speeds inversely proportional to their masses. These relative motions *with respect to the centre of gravity* are easily found by drawing through G a line parallel to $A'B'$, and measuring off on it distances GA'' and GB'' respectively equal to $G'A'$ and $G'B'$. AA'' and BB'' will then be the two motions of A and B *relative to their centre of gravity* G.

426. The Effect of their Mutual Attraction. — This will cause them to describe similar conics around this centre of gravity; the size of their two orbits being inversely proportional to their masses. The form of the orbits and dimensions will be determined by the combined mass of the two bodies, and by their velocities with respect to the common centre of gravity.

427. The Orbit of the Smaller relative to the Centre of the Larger. — It is convenient (though it is not necessary) to drop the consideration of the centre of gravity of the two bodies, and to consider the motion of the smaller one around the centre of the larger one. In reference to that point, it will move precisely as if its mass had been added to that of the larger body, while itself had become a mere particle. This relative orbit will in all respects be like the actual one around the centre of gravity, only magnified in the proportion of $M + m$ to M; *i.e.*, if m is $\frac{1}{10}$ of M, the *actual* orbit around G will be magnified by $\frac{11}{10}$ to produce the *relative* orbit around M.

428. The Orbit determined by Projection. — Suppose that in the figure (Fig. 142) the body P is moving in the direction of the arrow,

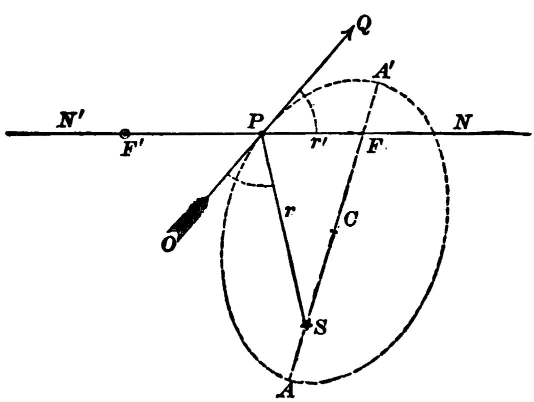

FIG. 142. — Elliptical Orbit determined by Projection.

and is attracted by S, supposed to be at rest. P will thenceforward move in a conic, either in an ellipse or hyperbola, according to its velocity, as we shall see in a moment. S being at one focus of the curve, the other focus will be somewhere on the line PN, which makes the same angle with PQ that r (SP) does (since it is a property of the conics that a tangent-line at any point of the curve makes equal angles with the lines drawn from the two foci to that point).

If we can find the place of the second focus F, or the length of the line PF in the figure, the curve can at once be drawn.

Now, it can be proved, though the demonstration lies beyond our scope, that a, the semi-major axis of the conic, is determined by the equation

$$V^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right), \qquad \text{(Equation 1)}$$

in which r is the distance SP, V is the velocity, and μ is the attracting mass at S expressed in proper units.

(See Watson's "Theoretical Astronomy," p. 49; only for μ he writes $k^2(1+m)$).

V, r, and μ being given, of course a can be found: we get

$$a = \mu \frac{r.}{2\mu - rV^2} \qquad \text{(Equation 2)}$$

Then by subtracting r from $2a$ we shall get r', or the distance PF, if the curve is an ellipse. If it is a hyperbola, a will come out negative; and to find r' we must take $r' = 2a + r$ and measure it off to F', on the other side of the line of motion. In either case, however, we easily find the other focus, and the line drawn through the foci will be the line of apsides; a point half-way between the foci will be the centre of the curve, and any line drawn through this centre will be a diameter. Having the two foci and the major axis $2a$, *i. e.*, AA', the curve can at once be drawn.

429. Expression for a in Terms of the "Velocity from Infinity," or "Parabolic Velocity." — The expression for a admits of a more convenient and very interesting form. It is shown in analytical mechanics that if, under the law of gravitation, a particle falls towards an attracting body whose mass is μ, from one distance s to another distance r, its velocity is given by the simple equation

$$w^2 = 2\mu\left(\frac{1}{r} - \frac{1}{s}\right).[1] \qquad \text{(Equation 3)}$$

---

1 If the difference between s and r is called h, this equation becomes

$$w^2 = 2\mu\left(\frac{1}{r} - \frac{1}{r+h}\right) = 2\mu\left(\frac{h}{r^2 + rh}\right).$$

Now if h is very small as compared with r, this gives

$$w^2 = \left(\frac{2\mu}{r^2}\right)h,$$

which is the same as the usual expression for the velocity of a falling body at the earth's surface, viz., $V^2 = 2gh$, $2g$ being replaced by the fraction $\frac{2\mu}{r^2}$.

If in this equation s be made infinite, w does not also become infinite (that is, a body falling from an infinite distance towards the sun will not acquire an infinite velocity until it actually reaches the centre of the sun, and r becomes zero); but we get in this case

$$w^2 = \frac{2\mu}{r}.$$

This special value of w is usually called "the velocity from infinity for the distance r," or the "*parabolic velocity*" (for a reason which will appear very soon). U is generally used as its symbol; therefore

$$U^2 = \frac{2\mu}{r}, \ U = \sqrt{\frac{2\mu}{r}}, \text{ and } \mu = \tfrac{1}{2} r U^2. \qquad \text{(Equation 4)}$$

The parabolic velocity due to the sun's attraction at any point is therefore inversely proportional to the square-root of the distance from the sun. The sun's mass is such that at the distance unity (the mean distance of the earth from the sun) it is equal to ***26.16*** miles or ***42.10*** kilometres. At the sun's surface it is 383.04 miles or 616.40 kilometres, and at the distance of Neptune it is still 4.77 miles. Again, since $U = \sqrt{\frac{2\mu}{r}}$, *U varies directly as the square-root of the mass* of the attracting centre of force. If the sun's mass were *halved*, the parabolic velocity due to its attraction would everywhere be reduced in the ratio of $\sqrt{\frac{1}{2}}$ to 1, *i.e.*, as 0.7071 to 1. If the mass were doubled, $U =$ would be increased in the ratio of $\sqrt{2}$ to 1, *i.e.*, as 1.4142 to 1.

The square of the parabolic velocity at any point is simply *twice the gravitation potential due to the sun's attraction at that point.* The "potential" may be defined as the *energy* which would be acquired by a mass of one unit, in falling to the point in question from a place where the potential (and attraction) is zero, *i.e.*, from infinity. Now $\frac{1}{2} m V^2$ is the general expression for the kinetic energy of a mass, m, moving with velocity V; if in this expression we make $m = 1$, and $V = U$, we shall have, for the case in hand, Energy $= \frac{1}{2} U^2$, which therefore equals the *Potential at the point.*

430. Relation between the Velocity and the Species of Conic described. In equation 2 substitute for μ its value, $\frac{1}{2} r U^2$ from equation (4), and we get

$$a = \frac{r}{2}\left(\frac{U^2}{U^2 - V^2}\right), \qquad \text{(Equation 5).}$$

From this equation it is clear how the velocity determines whether the orbit will be an ellipse or an hyperbola. If V^2 is *less* than U^2, the denominator of the fraction will be positive, a will also be positive, and the curve will be an *ellipse; i.e.*, if the velocity of the body

P, at the distance r from the central body S, be less than the velocity acquired by the body falling from infinity to that point, the body will move around S permanently in an ellipse.

If, on the other hand, V^2 is *greater* than U^2, the denominator will become negative, a will also come out negative, and the orbit will be an *hyperbola*. In this case P, after once moving past S at the perihelion point, will go off never to return; and it will recede towards a different region of space from that out of which it came, because the two legs of the hyperbola never become parallel. There will in this case be no permanent connection between the two bodies. They simply pass each other, and then part company forever.

If V^2 exactly equals U^2, the denominator of the fraction becomes zero, a comes out infinite, and the curve is a *parabola*. In this case, also, the body will never return; but it will recede from the sun ultimately towards the same point on the celestial sphere as that from which it appeared to come, since the two legs of the parabola tend to parallelism. Obviously, if a body were thus moving in a parabola, the slightest *increase* of its velocity would transform the orbit into an *hyperbola*, and the least *diminution* into an *ellipse;* the bearing of which remark will become evident when we come to deal with comets.

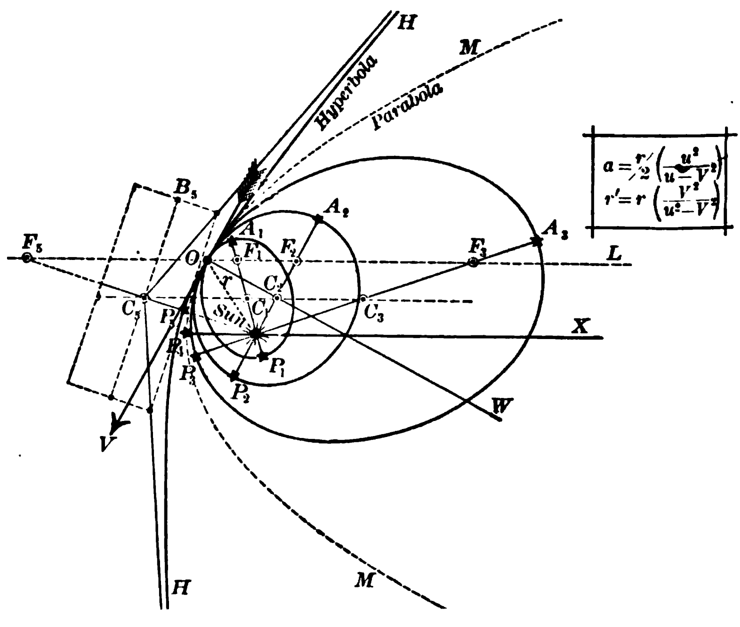

FIG. 143. — Confocal Conics described under Different Velocities of Projection.

431. Again, since $$a = \frac{r}{2}\left(\frac{U^2}{U^2 - V^2}\right),$$

all bodies *having the same velocity V, at the same distance r from the centre of force, will have major axes of the same length for their orbits, no matter what may be the direction of their motion.*

They will have the same *period* also, the expression for the period being

$$t = \frac{2\pi a^{\frac{3}{2}}}{\sqrt{\mu}}. \qquad \text{(Watson, p. 46, Equation 28.)}$$

But observe that when a is negative, *i.e.*, in the hyperbola, the value of t becomes imaginary; there is no periodicity in that case.

If, therefore, a body moving around the sun were to explode at any point, all of its particles which did not receive a velocity greater than the "parabolic velocity" would come around to the same point again, and those which were projected with equal velocities would come around and meet at the same moment, however widely different their paths might be.

432. Fig. 143 represents the orbits which would be described by five bodies projected at O with different velocities along the line OV, the distance OS or r being taken as unity, as well as the parabolic velocity U^2. The squares of the velocities are assumed as given below, with the resulting values of a and r'.

$$V_1^2 = \tfrac{1}{4}; \text{ whence } a_1 = \tfrac{2}{3}; \text{ and } r_1' = \tfrac{1}{3}.$$

This places the empty focus at F_1.

For the next larger ellipse

$$V_2^2 = \tfrac{1}{2};\ a_2 = 1;\ r_2' = 1.$$

In the same way $V_3^2 = \tfrac{3}{4};\ a_3 = 2;\ r_3' = 3.$

$$V_4^2 = 1;\ a_1 = \infty;\ r_4' = \infty. \quad \text{(Parabola.)}$$

$$V_5^2 = 2;\ a_5 = -\tfrac{1}{2};\ r_5' = -2. \quad \text{(Hyperbola.)}$$

433. Fig. 144 shows how three bodies projected at P with *equal velocities, but in different directions*, indicated by the arrows, describe three different ellipses; all, however, having the same period, and the same length of semi-major axis; namely, $a = 2r$; V^2 being taken equal to $\tfrac{3}{4}U^2$.

For a fourth body, V^2 is taken as $= \tfrac{1}{2}U^2$, and with the direction of motion perpendicular to r. This body will move in a perfect circle, a coming out equal to r, when $V^2 = \tfrac{1}{2}U^2$. In order to have *circular* motion, both conditions must be fulfilled; namely, V^2 must equal $\tfrac{1}{2}U^2$, and the direction of motion must be perpendicular to the radius vector.

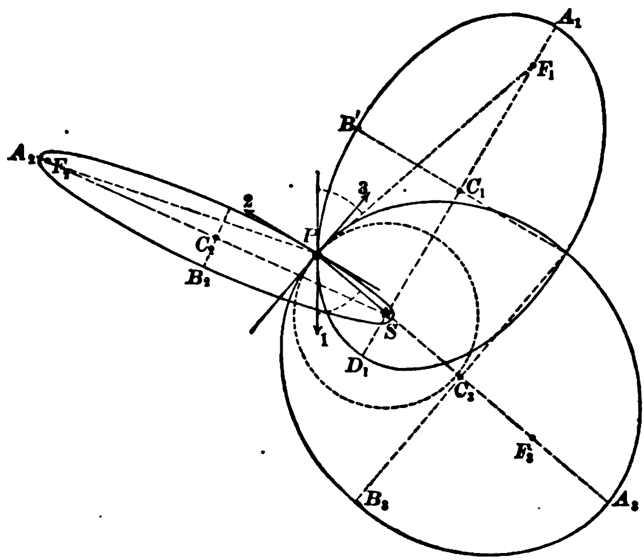

FIG. 144. — Ellipses of the Same Periodic Time.

These conditions are of course fulfilled very nearly in the case of the planets, since they move nearly in circles. Observe also that if the mass of the sun were somehow to be suddenly reduced until the corresponding new value of U^2 were less than V^2, the planets' orbits would at once become parabolas.

434. Velocity of a Planet at Any Point in its Orbit. — If AA' (Fig. 145) be the major axis of a planet's orbit, and KK' the diameter of a circle described around S with AA' as radius, then the velocity of a planet at any point, N, on its orbit is equal to that which it would have acquired by falling to N from the point n on the circumference of the circle. The demonstration is not difficult and may be found in No. 1426 of the "Astronomische Nachrichten."

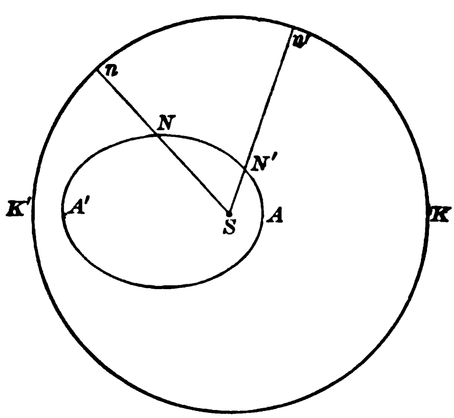

FIG. 145. — Theorem of Whewell and Van der Kolk.

435. Projectiles near the Earth. — A good illustration of the principles stated above is obtained by considering the motion of

bodies projected horizontally from the top of a tower near the earth's surface, supposing the air to be removed so there will be no resistance to the motion.

The "parabolic velocity" due to the earth's attraction equals 6.94 miles per second at the earth's surface; *i.e.*, a body falling from the stars to the surface of the earth, *drawn by the earth's attraction only*, would have acquired this velocity on reaching the earth's surface.

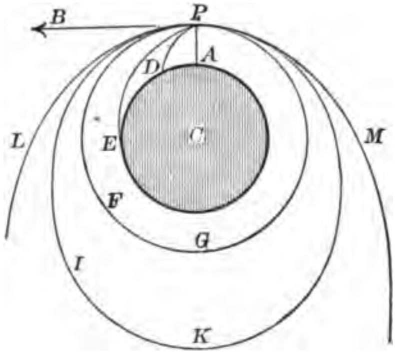

Fig. 146. — Projectiles near the Earth.

First. If a body be projected with a very small velocity, it would fall nearly straight downwards. If the earth were concentrated at the point in its centre so that the body should not strike its surface, it would move in a very long narrow ellipse having the centre of the earth at the further focus, and would return to the original point after an interval of 29.9 minutes.

Second. With a greater velocity the orbit would be a wider ellipse with a longer period, C being still at the remoter focus.

Third. $V = U\sqrt{\frac{1}{2}}$, or about 4.9 miles per second. In this case the orbit of the body would be a *perfect circle*, and the period would be $1^h\,24^m.7$. It will be remembered that we found that if the earth's rotation were 17 times as rapid, thus completing a revolution in $1^h\,24^m.7$, the centrifugal force at the equator would become equal to gravity (Art. 154). Also, Art. 413 (4), this same time, $1^h\,24^m.7$, was found from Kepler's third law as the period of a satellite revolving close to the earth's surface.

Fourth. $V = U = 6.94$ miles. In this case the projectile would go off in a *parabola*, never to return.

Fifth. $V > 6.94$. In this case, also, the body would never return, but would pass off in an *hyperbola*.

At the surface of each of the other planets, the "parabolic velocity" due to its attraction is as follows: Mercury, 2.2 miles per second (probably, but very uncertain); Venus, 6.6; Mars, 1.5; Jupiter, 37; Saturn, 22; Uranus, 13; Neptune, 14.

In the case of the sun and moon, as already stated (Arts. 429, 272*), the parabolic velocities are 383 and 1.5 miles, respectively.

436. Intensity of Solar Attraction. — The attraction between the sun and the earth from some points of view looks like a very feeble action. It is only able, as has been before stated (Art. 278), to bend the earth out of a rectilinear course to the extent of about

one-ninth of an inch in a second, while she is travelling nearly nineteen miles; and yet if it were attempted to replace by bonds of steel the invisible gravitation which holds the earth to the sun, we should find the surprising result that it would be necessary to cover the whole surface of the earth with wires as large as telegraph wires, and only about half an inch apart from each other, in order to get a metallic connection that could stand the strain. This ligament of wires would be stretched almost to the breaking point. The attraction of the sun for the earth expressed as tons of *force* (not tons of *mass*, of course) is 3600000 millions of millions of tons (36 with seventeen ciphers); and similar stresses act through the apparently empty space in all directions between all the different pairs of bodies in the universe.

Exercises on Chapter XII.

1. Given a comet moving in an ellipse with the eccentricity 0.5. Compare the velocities, both linear and angular, at the perihelion and aphelion.

Ans. { Lin. Vel. at perihelion is *three* times that at aphelion.
Ang. Vel. " " *nine* " "

2. At what point in the orbit is the actual linear velocity equal to the *mean* velocity? *Ans.* At the extremity of the minor axis.

3. Is the angular velocity at that point equal to the mean angular velocity; and if not, why not?

4. What would be the periodic time of a small body revolving in a circle around the sun close to its surface? (Apply Kepler's harmonic law.)

Ans. 2 h. 47.4 min.

5. What would be its velocity?

Ans. 191.5 miles a sec. (nearly).

6. If the earth had a satellite with a period of eight months what would its distance be? *Ans.* Four times that of the moon.

7. If Jupiter were reduced to a mere particle how much would its period be lengthened? (Consider its mass to be $\frac{1}{1048}$ of the sun's, and see Art. 417.)

Let x be the new period; then

$$x^2 : t^2 \frac{1049}{1048} = r^3 : r^3 = 1 : 1, \text{ since } r \text{ is not changed. Whence,}$$

$$x = t\sqrt{\frac{1049}{1048}} = t\,(1 + \tfrac{1}{2} \times \tfrac{1}{1048} + \text{etc.}) = t\,(1 + \tfrac{1}{2096}) \text{ very nearly.}$$

$$\text{But } t = 4332.6 \text{ days, and } x - t = \frac{4332.6}{2096} = 2.067 \text{ days.} \quad \textit{Ans.}$$

8. How much longer would the earth's period be if it were a mere particle?

Ans. $\frac{1}{660000}$ of a year, or 47.8 sec.

9. If the sun's mass were a hundred times greater what would be the parabolic velocity at the earth's distance from it? (Art. 429.)

Ans. Ten times its present value, *i.e.*, 261.6 miles a sec.

10. If the sun's mass were reduced 50 per cent what would be the parabolic velocity at the distance of the earth?

Ans. 18.5 miles a sec.

11. If the sun's mass were to be suddenly reduced by 50 per cent or more, what would be the effect upon the now practically circular orbits of the planets? (See Art. 430.)

Ans. They would become parabolas or hyperbolas, and the planets would desert the sun.

12. What would be the effect upon the orbit of the earth if the sun's mass were suddenly doubled?

Ans. It would immediately become an eccentric ellipse, with its aphelion near the point where the earth was when the change occurred.

13. Let V_r be the velocity in an orbit at a point where the radius vector is r, and let U_r and U_{2a} be the *parabolic velocities* at distances r and $2a$ from the sun, a being the semi-major axis of the orbit. Show that $V^2_r = U^2_r \pm U^2_{2a}$. The *plus* sign applies if the orbit is an hyperbola; the *minus*, if it is an ellipse. (See Equations 1 and 4, Arts. 428 and 429.)

CHAPTER XIII.

THE PROBLEM OF THREE BODIES. — DISTURBING FORCES: LUNAR PERTURBATIONS AND THE TIDES.

437. The problem of *two* bodies is completely solved; but if, instead of two spheres attracting each other, we have *three* or more, given completely in respect to their positions, masses, and velocities, the general problem of finding their subsequent motions and predicting their positions at any future date transcends the present power of our mathematics.

This problem of *three* bodies is in itself just as determinate and capable of solution as that of two. Given the initial data, — that is, the *positions, masses, and motions* of the three bodies at a given instant, — then their motions for all the future, and the positions they will occupy at any given date, are absolutely predetermined, provided no forces act upon them except their mutual gravitational attractions. The difficulty of the problem lies simply in the inadequacy of our present mathematical methods, and it is altogether probable that some time in the future this difficulty will be overcome, though at present there is no immediate prospect of success; the problem is one of extreme complexity.

438. But while the *general* problem of *three* bodies is thus intractable, all the special cases of it which arise in the consideration of the moon's motion and in the motions of the planets have been solved by special methods of approximation. Newton himself led the way; and the strongest proof of the truth of his theory of gravitation lies in the fact that it not only accounts for the *regular* elliptic motions of the heavenly bodies, but also for the apparent *irregularities* of these motions.

439. The Disturbing Force. — In the case where two bodies are revolving around their common centre of gravity, and the third body is either *very much smaller* than the central one, or very *remote*, the

motion of the two will be but slightly modified by the action of the third; and in such a case the small differences between the actual motion and the motion as it would be if the third body were not present, are technically called "disturbances" and "perturbations,"[1] and the force which produces them is called the "disturbing force." This disturbing force is not the *attraction* of the disturbing body, but only a *component* of that attraction, and usually only a small fraction of it.

The disturbing force of the attracting body depends upon the difference of its attraction upon the two bodies it disturbs; difference either in amount or in direction, or in both. For instance, if the sun attracted the earth and moon exactly alike (*i.e.*, *equally and along parallel lines*), it would not disturb their relative motions in the least, no matter how powerful its attraction might be. The sun's maximum *disturbing force* on the moon, as we shall see, is only about one eighty-ninth of the earth's attraction; and yet the sun's *attraction* for the moon is actually much greater than that of the earth.

Since the sun's mass is 330,000 times that of the earth, and its distance just about 389 times that of the moon from the earth, its attraction on the moon equals the earth's attraction $\times \frac{330,000}{389^2} = 2.18$; *i.e.*, the *sun's attraction on the moon is more than double that of the earth.*

440. Why the Sun does not take the Moon away from the Earth.—If at the time of new moon, when the moon is between the earth and sun, the sun attracts the moon more than twice as much as the earth does, it is a natural question why the sun does not draw the moon away entirely, and rob us of our satellite. It would do so if it were the case of a "tug of war"; that is, if earth and sun were *fixed in space*, pulling opposite ways upon the moon between them. But it is not so; neither sun nor earth has any *foothold*, so to speak; *but all three bodies are free to move*, like chips floating on water, and the student's difficulty in understanding the action of disturbing forces usually lies in his failure to appreciate the effect of this freedom. The sun attracts the *earth* almost as much as he does the moon, and both earth and moon fall towards him freely; though of course this

[1] The student will bear in mind that these terms ("perturbations" and "disturbances") are mere figures of speech; that philosophically the purely elliptical motion of two mutually attracting bodies alone in space is no more "*regular*" than the (at present) incomputable motion of three or more attracting bodies.

falling motion towards the sun is continually combined with whatever other motion the earth or moon possesses. The only effective disturbance is produced by the fact that, in the case considered, *the new moon, being nearer the sun than the earth is* by about $\frac{1}{389}$ part of the whole distance, falls towards the sun a trifle faster than the earth, and so on that account the curvature of its orbit toward the earth is, for the time being, diminished.

At the half-moon the two bodies are equally attracted towards the sun, but on *converging lines;* and so as they fall towards the sun they *approach* each other slightly; and for this reason, at quadrature, the moon's orbit is a little more curved towards the earth than it would be otherwise.

441. Diagram of the Disturbing Force. — A very simple diagram enables us to find graphically the disturbing force produced by a third body.

(What follows applies *verbatim et literatim* to either of the two diagrams of Fig. 147.)

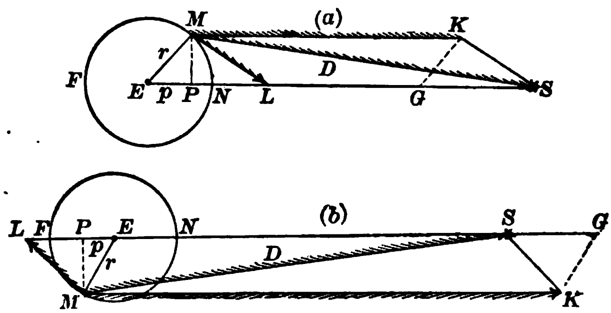

FIG. 147. — Determination of the Disturbing Force by Graphical Construction.

Let E be the earth, M the moon, and S the disturbing body (the sun in this case); and let the sun's attraction on the moon be represented by the line MS. On the same scale the attraction of the sun on the earth will be represented by the line EG, G being a point so taken that $EG : MS = MS^2 : ES^2$; that is, —

The sun's attraction on the earth is to *the sun's attraction on the moon* as *the square of the sun's distance from the moon* is to *the square of the sun's distance from the earth,* according to the law of gravitation. (MS has to do double duty in this proportion: in the first ratio it represents a *force;* in the second, a *distance.*)

From this proportion $\qquad EG = MS \times \frac{MS^2}{ES^2}.$

In figure (*a*) the moon is nearer to the sun than the earth is, and so EG comes out *less* than MS. In figure (*b*) the reverse is the case, and therefore in this case EG is *larger* than MS.

Now if the force represented by the line MS *were parallel and equal to that represented by* EG, there would be no disturbance, as has been said. If, then, we can resolve the force MS into two components, one of which is equal and parallel to EG, this component will be innocent and harmless, and the other one will make all the disturbance.

To effect this resolution, draw through M the line MK parallel and equal to EG. Join KS, and draw ML parallel and equal to it. *ML is then the disturbing force on the same scale as MS; i.e.*, the line ML shows the true *direction* of the disturbing force, and in *amount* the disturbing force is equal to the *sun's attraction for the moon multiplied by the fraction* $\left(\frac{ML}{MS}\right)$. The diagonal of the parallelogram $MLSK$ is MS, which represents the resultant of the two forces MK and ML, that form its sides.

For the sake of clearness the lines which represent forces in the figures are indicated by herring-bone markings.

442. At first it seems a little strange that in figure (*b*) the disturbing force should be directed *away from the sun;* but a little reflection justifies the result. If E and M were connected by a rod, and the E-end of the rod were pulled towards the right more swiftly than the M-end, it is easy to see that the latter would be *relatively* thrown to the left, as the figure shows.

443. The sun is the only body that sensibly disturbs the moon. The planets, of course, act upon the moon to disturb it, but their mass is so small compared with that of the sun, and their distances so great, that in no case is their *direct* action sensible. It is true, however, that some of the lunar perturbations are affected by the existence of one or two of the planets. While they cannot disturb the moon *directly*, they do so *indirectly:* they disturb the earth in her orbit sufficiently to make the sun's action different from what it would be if the planets did not exist, and in this way make themselves felt. There are also a few small disturbances that depend upon the fact that the earth is not a perfect sphere.

444. Since the distance of the sun is nearly four hundred times that of the moon from the earth, and the moon's orbit is very nearly circular, the construction of the disturbing force ML,

Fig. 147, admits of considerable simplification. It is only necessary to drop the perpendicular MP upon the line that joins the earth and the sun, and take the point L upon this line, so that EL equals *three times* EP. The line ML so determined will then *very approximately* (but not exactly) be the true disturbing force.

To prove this relation, let MS, in Fig. 147, be D, $ES = R$, $ME = r$, and $EP = p$, also $R = D + p$, *very nearly*, p being *negative* when $MS > ES$. EG was taken equal to $MS \times \frac{MS^2}{ES^2} = \frac{D^3}{R^2}$.

Now, $EL = GS = (ES - EG) = R - \frac{D^3}{R^2} = \frac{R^3 - D^3}{R^2} = \frac{(D+p)^3 - D^3}{(D+p)^2}$.

Developing this expression, we have

$$EL = \frac{3\,D^2p + 3\,Dp^2 + p^3}{D^2 + 2\,Dp + p^2}.$$

Since p is very small as compared with D, all the terms except the first nearly vanish both in numerator and denominator, and we have

$$EL = \frac{3\,D^2p}{D^2} = 3p \text{ (very nearly).}$$

445. Resolution of the Disturbing Force into Components.— In discussing the effect of the disturbing force it is more convenient to resolve it into three components known as the *radial*, the *tangential*, and the *orthogonal*. The first of these *acts in the direction of the*

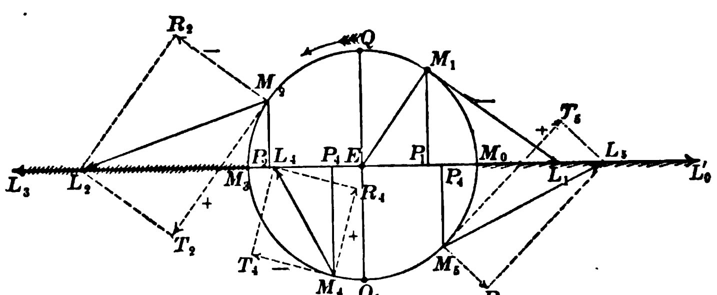

FIG. 148. — Radial and Tangential Components of the Disturbing Force.

radius vector, tending to draw the moon either towards or from the earth. The second, the tangential, *operates to accelerate or retard the moon's orbital velocity*.

Fig. 148 exhibits these two components at different points of the moon's orbit.

The orthogonal component has no existence in cases where the disturbing body lies in the plane of the disturbed orbit; but whenever it lies outside of that plane, the disturbing force ML will generally also lie outside of the orbit-plane, and will have a component tending to *draw the moving body out of the plane of its orbit.* The motion of the moon's node and the changes of the inclination of its orbit are due to this component of the sun's disturbing force, which could not be conveniently represented in the figure.

446. The radial force in the case of the moon's orbit is a maximum at syzygies and quadratures; in fact, at quadratures the whole disturbing force is radial, the tangential and orthogonal components both vanishing. At syzygies (new moon and full moon) the radial force is *negative;* that is, it draws the moon *from* the earth, diminishing the earth's attraction by about *one eighty-ninth* [1] of its whole amount.

At quadrature or half-moon the radial force is *positive;* and since L then falls at E, it is represented by the line QE, and is just half what it is at syzygies; that is, it equals about *one one hundred and seventy-eighth* of the earth's attraction.

It becomes zero at four points 54° 44′ on each side of the line of syzygies.

This angle is found from the condition that the disturbing force M_1L_1, etc., in Fig. 148, must be perpendicular to the radius EM_1 at this point, which gives us $EP_1 : P_1M_1 :: P_1M_1 : P_1L_1$. But $P_1L_1 = 2\,EP_1$; therefore $P_1M_1^2 = 2\,EP_1^2$, and $\frac{P_1M_1}{EP_1} = \tan M_1EL_1 = \sqrt{2}$.

447. The *tangential component* starts at zero at the time of full moon, rises to a maximum at the critical angle of 45° (having at that point a value of $\frac{1}{119}$ of the earth's attraction), and disappears again at quadratures. During the first and third quadrants this force is *negative;* that is, it *retards* the moon's motion; in the second and fourth it is *positive* and accelerates the motion.

[1] At syzygies $ML = NL_0 = 2 \times EN$ (Fig. 147); but $EN = \frac{MS}{389}$. Therefore $ML = \frac{2}{389}$ of the *sun's* attraction on the moon. Now the sun's attraction is 2.18 times the earth's; hence $NL_0 =$ the *earth's* attraction multiplied by $\frac{2 \times 2.18}{389} = \frac{1}{89.2}$.

448. Lunar Perturbations. — So far it has been all plain sailing, for nothing beyond elementary mathematics is required in determining the disturbing force at any point in the moon's orbit; *but to determine what will be the effect of this continually varying force after the lapse of a given time, upon the moon's place in the sky* is a problem of a very different order, and far beyond our scope. The reader who wishes to follow up this subject must take up the more extended works upon theoretical astronomy and the lunar theory. A few points, however, may be noted here.

449. In the first place, it is found most convenient to consider the moon as never deviating from an elliptical orbit, but to consider *the orbit itself as continually changing in place and form*, writhing and squirming, so to speak, under the disturbing forces; just as if the orbit were a material hoop with the moon strung upon it like a bead and unable to get away from it, although she can be set forward and backward in her motion upon it.

450. In the next place, it is found possible to represent nearly all the perturbations by *periodical formulæ* — the same values recurring over and over again indefinitely at regular intervals. This is because the sun, moon, and earth keep coming back into the same, or nearly the same, relative positions, and this leads to recurring values of the disturbing force itself, and also of its effects.

451. Third, the number of these perturbations, each characterized by its own special period, is very large. In the computation of the moon's *longitude* in the American Ephemeris about *seventy* different inequalities are reckoned in, and about half as many in the computation for the *latitude*. Theoretically the number is infinite, but only a certain number produce effects sensible to observation. It is of no use to compute disturbances that do not displace the moon as much as one-tenth of a second of arc; *i.e.*, about 500 feet in her orbit.

452. Fourth, in spite of all that has been done, the lunar theory is still incomplete, or in some way slightly erroneous. The best tables yet made begin to give inaccurate results after fifteen or twenty years, and require correction. The almanac place of the moon at present is not unfrequently "out" as much as 3″ or 4″ of arc; *i.e.*, about three or four miles. Astronomers are continually at

work on the subject, but the computations by our present methods are exceedingly tedious and liable to numerical error.

453. The principal effects of the sun's disturbing action on the moon are the following: —

First: Effect on Length of Month. — Since the radial component of the disturbing force is *negative* more than half the way round (54° 44′ on each side of the line of syzygies) and is twice as great at syzygies as the positive component is at quadrature, the net result is that, taking the whole month through, the earth's attraction for the moon *is lessened by nearly $\frac{1}{350}$ part.* The effect is substantially the same that would follow from a corresponding diminution of the earth's mass, and the moon's period is therefore made about $\frac{1}{710}$ part, or nearly an hour, longer than it otherwise would be at its present distance, since $t = \frac{2\pi a^{\frac{3}{2}}}{\sqrt{\mu}}$. (Art. 431.)

454. *Second: The Revolution of the Line of Apsides.* — This is due mainly to the *radial* component of the disturbing force, though the tangential component assists. When the moon comes to *perigee* or *apogee* at the time of new or full moon (*i.e.*, during those months when the sun is at or near the line of apsides of the lunar orbit), the diminution of the earth's effective attraction for the moon causes it to move on farther than it would otherwise do before turning the corner, so to speak, the consequence being that the line of apsides *advances* in the line of the moon's motion. When perigee or apogee is passed at the time of *quadrature*, the line of apsides is also disturbed and made to *regress*. But at quadrature the radial component is only half as great as at syzygies, and the net result, as has been stated before (Art. 238), is that the line of apsides completes a direct revolution once in about nine years (8.855 years — *Neison*). It does not move forward steadily and uniformly, but its motion is made up of alternate advance and regression. Fig. 149 illustrates this motion of the moon's apsides.

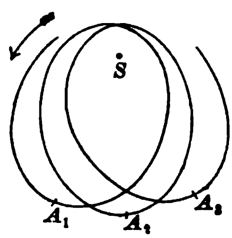

FIG. 149.
Advance of the Apsides of the Moon's Orbit.

For a fuller discussion of the subject, see Herschel's "Outlines of Astronomy," Sections 677–689; or Airy's "Gravitation," pp. 89–100.

455. *Third: The Regression of the Nodes.* — The *orthogonal component generally* (not always) *tends to draw the moon towards the plane*

of the ecliptic. Whenever this is the case at the time when the moon is passing a node, the effect (as is easily seen from Fig. 150) of such a force P_1O_1, acting upon the moon at P_1, is to shift the node backward from N_1 to N_2, the moon taking the new path $P_1b_1N_2$. As the moon is approaching the node, the inclination of its orbit is also increased; but as the moon leaves the node, it is again diminished, the path N_2P_2 being bent at P_2 back to P_2b_2, parallel to P_1a_1: so that while by both operations the node is made to recede from N_1 to N_3, the inclination suffers very little change, if the orthogonal component remains the same on both sides of the node.

Since the orthogonal component vanishes twice a year, — when the sun is at the nodes of the moon's orbit, — and also twice a month, — when she is in quadrature, — the rate at which the nodes regress

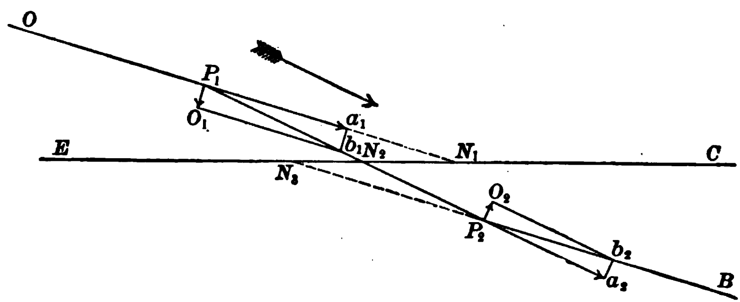

FIG. 150. — Regression of the Nodes of the Moon's Orbit.

is extremely variable. In the long run it makes its backward revolution once in about nineteen years (Arts. 249 and 391). [18.5997 years. — *Neison.*]

See Herschel's "Outlines of Astronomy," section 638 seqq.

456. *Fourth: The Evection.* — This is an irregularity which at the maximum puts the moon forward or backward about $1\frac{1}{4}°$ ($1° 16' 27''.01$ — *Neison*), and has for its period the time which is occupied by the sun in passing from the line of apsides of the moon's orbit to the same line again; *i.e.*, about a year and an eighth. This is the largest of the moon's perturbations, and was earliest discovered, having been detected by Hipparchus about 150 years B.C., and afterwards more fully worked out by Ptolemy, though of course without any understanding of its cause. It was the only lunar perturbation known to the ancients. It depends upon the *alternate increase and decrease of the eccentricity* of the moon's orbit, which is always a maximum when

the sun is passing the moon's line of apsides, and a minimum when the sun is at right angles to it.

This inequality may affect the time of an eclipse by nearly six hours, making it anywhere from three hours early to three hours late, as compared with the time at which it would otherwise occur; it was this circumstance which called the attention of Hipparchus to it.

See Herschel's "Outlines of Astronomy," sections 748 seqq.

457. *Fifth: The Variation.* — This is an inequality due mainly to *the tangential component* of the disturbing force. It has a period of one month, and a maximum amount of 39′ 30″.70, attained when the moon is half-way between the syzygies and quadratures, at the so-called "octants." At the first and third octants the moon is 39½′ *ahead of* her mean place (about an hour and twenty minutes); at the second and fourth she is as much *behind*. This inequality was detected by Tycho Brahe, though there is some reason for believing that it had been previously discovered by the Arabian astronomer, Aboul Wefa, five centuries earlier. This inequality does not *affect the time of an eclipse*, being zero both at the syzygies and quadratures, and therefore was not detected by the Greek astronomers.

See Herschel's "Outlines of Astronomy," sections 705 seqq.

458. *Sixth: The Annual Equation.* — The one remaining inequality which affects the moon's place by an amount visible to the naked eye, is the so-called "annual equation." When the earth is nearer the sun than its mean distance, the sun's disturbing force is, of course, greater than the mean, and the month is *lengthened a little;* during that half of the year, therefore, the moon keeps falling behindhand; and *vice versa* during the half when the sun's distance exceeds the mean. The maximum amount of this inequality is 11′ 9″.00, and its period one anomalistic year.

See Herschel's "Outlines of Astronomy," sections 738 seqq.

There remains one lunar irregularity among the multitude of lesser ones, which is of great interest theoretically, and is still a bone of contention among mathematical astronomers; namely, —

459. *Seventh: The Secular Acceleration of the Moon's Mean Motion.* — It was found by Halley, early in the last century, by a comparison of ancient with modern eclipses, that the month is now

certainly shorter than it was in the days of Ptolemy, and that the shortening has been progressive, apparently going on continuously, —*in sæcula sæculorum*,—whence the name. In 100 years the moon, according to the results of Laplace, gets in advance of its mean place about 10″, *and the advance increases with the square of the time*, so that in a thousand years it would gain nearly 1000″. and in 2000 years 4000″, or more than a degree. The moon at present is supposed to be just about a degree in advance of the position it would have held if it had kept on since the Christian era with precisely the rate of motion it then had. If this acceleration were to continue indefinitely, the ultimate result would be that the moon would fall upon the earth, as the quickened motion corresponds to a shortened distance.

460. It was nearly 100 years after Halley's discovery before Laplace found its explanation in the decreasing eccentricity of the earth's orbit. Under the action of the other planets this orbit is now growing more nearly circular, without, however, changing the length of its major axis. Thus *its area becomes larger*, and the earth's *average* distance from the sun becomes greater (although the *mean* distance, technically so-called, does not change, the "mean distance" being simply half the major axis). As a result of this rounding up of the earth's orbit, the *average disturbing force of the sun is therefore diminished*, and this diminution allows the month to come nearer the length it would have if there were no sun to disturb the motion; that is to say, the month keeps shortening little by little, and it will continue to do so until the eccentricity of the earth's orbit begins to increase again, some 25,000 years hence.

461. But the theoretical amount of this acceleration, about 6″ in a century, does not agree with the value obtained by comparing the most ancient and modern eclipses, which is about 12″; and this value, again, does not agree with the one derived by comparing modern observations of the moon with those made by the Arabians about a thousand years ago, which, according to recent investigations by Professor Newcomb, indicate an acceleration of only about 8″.

So long as the actual acceleration was considered to be 12″, it was generally supposed that the discrepancy between the theoretical and observed result is due to *a retardation of the earth's rotation by the friction of the tides, and a consequent lengthening of the day*. Evidently if the day and the seconds become a little longer, there will be fewer of them in each month or year, and the *apparent* effect of such a change would be to shorten all really constant astronomical periods by one and the same percentage.

As matters stand to-day it is hardly possible to assert with confidence that there is any real discrepancy to be accounted for between the theoretical and observed values, the latter being considerably uncertain. In Newcomb's "Popular Astronomy" (pp. 96–102) there will be found an interesting and trustworthy discussion of the subject.

Questions like this, and those relating to the remaining discrepancies between the lunar tables and the observed places of our satellite, lie on the very frontiers of mathematical astronomy, and can be dealt with only by the ablest and most skilful analysts.

THE TIDES.

462. Just as the disturbing force due to the sun's attraction affects the motions of the moon in her orbit, so the disturbing forces due to the attractions of the moon and sun acting upon the fluids of the earth's surface produce *the tides.* These consist of the regular rise and fall of the water of the ocean usually twice a day, the average interval between the corresponding high waters of successive days being $24^h\ 51^m$, which is precisely the same as the average interval between two successive passages of the moon across the meridian. This coincidence, maintained indefinitely, of itself makes it certain that there must be some causal connection between the moon and the tides.

463. Definitions. — When the water is rising, it is "*flood*" tide; when falling, it is "*ebb.*" It is "*high water*" at the moment when the tide is highest, and "*low water*" when it is lowest. "*Spring tides*" are the highest tides of the month (which occur near the times of new and full moon), while "*neap tides*" are the smallest, which occur when the moon is in quadrature. The relative heights of the spring and neap tides are about as 7 to 4. At the time of spring tides the interval between the corresponding tides of successive days is less than the average, being only about $24^h\ 38^m$, and then the tides are said to "*prime.*" At neap tides the interval is $25^h\ 6^m$, which is greater than the mean, and the tides "*lag.*"

The "establishment" of a port is the mean interval between the time of high water at that port and the next preceding passage of the moon across the meridian. At New York, for instance, this "establishment" is $8^h\ 13^m$, although the actual interval varies about 22 minutes on each side of the mean at different times of the month.

That the moon is largely responsible for the tides is also shown by the fact that the tides, at the time when the moon is in perigee, are nearly twenty per cent higher than those which occur when she is in

apogee. The highest tides of all happen when a *new or full moon occurs at the time the moon is in perigee, especially if this occurs about January 1st, when the earth is nearest to the sun.* Since, as we shall see, the "tide-raising" force varies inversely as the *cube* of the distance, slight variations in the distance of the moon and sun from the earth make much greater variations in the height of the tide —greater nearly in the ratio of 3 to 1.

464. The Tide-Raising Force. — This is the *difference* between the attractions of the sun and moon (mainly the latter) on the main body of the earth, and the attractions of the same bodies on particles at different parts of the earth's surface. The tide-raising force is but a very small part of the whole attraction.

The amount of this disturbing force for a particle at any point on the earth's surface can be found approximately by the same geo-

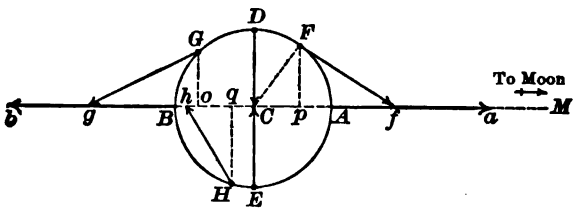

Fig. 151. — The Moon's Tide-Raising Force on the Earth.

metrical construction which was used for the lunar theory (Art. 441). Draw a line from the moon through the centre of the earth. At the points A and B, Fig. 151, where the moon is directly over head or under foot, the tide-raising force is directly opposed to gravity, and equals nearly $\frac{1}{30}$ of the moon's whole attraction, since the line Aa represents the disturbing force on the same scale as the line from A to the moon represents the moon's attraction, and this line, AM, is about sixty times the earth's radius, while Aa is just double it, because Ca has to be taken equal to $3 \times CA$ (Art. 444).

Since the moon's mass is only about $\frac{1}{80}$ of the earth's, and its distance is sixty radii of the earth, this *lifting force* under the moon, *expressed as a fraction of the earth's gravity*, equals

$$\frac{1}{30} \times \frac{1}{80} \times \frac{1}{3600} = \frac{1}{8\,640\,000};$$

i.e., a body weighing *four thousand tons* loses about *one pound* of its weight when the moon is over head or under foot.

At D and E, anywhere on the circle of the earth's surface which is 90° from A and B, the moon's disturbing force *increases* the

weight of a body by just half this amount, the disturbing force being measured by the lines DC and EC. At a point F, situated anywhere on a circle drawn around either A or B with a radius of 54° 44′, the *weight* of a body is neither increased nor decreased, but it is urged towards A or B with a horizontal force expressed by the line Ff, which force is equal to about $\frac{1}{12000000}$ of its weight.

The tidal forces at G and H are expressed by the lines Gg and Hh, each resolvable into vertical and horizontal components.

465. The same result for the lifting-force directly under the moon may be obtained more exactly as follows. The distance from the moon to the centre of the earth is sixty times the earth's radius, and therefore the distance from the moon to the points A and B respectively will be 59 and 61. The moon's attraction at A, C, and B, expressed as fractions of the earth's gravity, will be as follows:—

$$\text{Attraction of moon on particle at } A = g \times \frac{\frac{1}{80}}{59^2} = 0.0000035910 \times g.$$

$$\text{Attraction of moon on particle at } C = g \times \frac{\frac{1}{80}}{60^2} = 0.0000034723 \times g.$$

$$\text{Attraction of moon on particle at } B = g \times \frac{\frac{1}{80}}{61^2} = 0.0000033593 \times g.$$

Hence,
$$A - C = 0.0000001187\, g = \frac{1}{8\,424000}\, g.$$
$$C - B = 0.0000001130\, g = \frac{1}{8\,835000}\, g.$$

This is more correct than the preceding, which is based on an approximation that considers the moon's distance as *very large* compared with the earth's radius, while it is really only sixty times as great, and sixty is hardly a "very large" number in such a case.

Attempts have been made to *observe* directly the variations in the force of gravity produced by the moon's action, but they are too small to be detected with certainty by any experimental method yet contrived. Both Darwin and Zöllner found that other causes which they could not get rid of produced disturbances more than sufficient to mask the whole action of the moon.

466. It is worth while to note in this connection that the maximum lifting-force due to the attraction of a distant body varies inversely as the *cube* of its distance, as is easily shown, thus:—calling D the distance of the disturbing body from the earth's centre, and r the earth's radius, we have

$$\text{Attraction at } A = \frac{M}{(D-r)^2}; \quad \text{attraction at } C = \frac{M}{D^2}.$$

$$\text{Tide-raising force at } A = M\left\{\frac{1}{(D-r)^2} - \frac{1}{D^2}\right\} = M\left\{\frac{2Dr - r^2}{D^2(D^2 - 2Dr + r^2)}\right\}$$
$$= M\left\{\frac{2Dr - r^2}{D^4 - 2D^3r + D^2r^2}\right\} = M\left\{\frac{2r}{D^3}\right\}, \text{ nearly,}$$

when r is a small fraction of D.

467. It is very apt to puzzle the student that the moon's action should be a *lifting* force at B as well as at A (Fig. 151). He is likely to think of the earth as fixed, and the moon also fixed and attracting the water upon the earth, in which case, of course, the moon's attraction, while it would decrease gravity at A, would increase it at B.

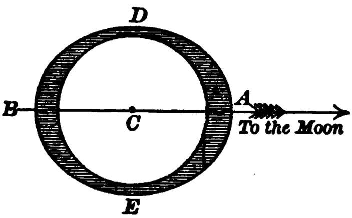

Fig. 152.—The Statical Theory of the Tides.

The two bodies are not fixed, however. Let him think of the three particles at A, C, and B, Fig. 152, as unconnected with each other, and falling freely towards the moon; then it is obvious that they would separate; A would fall faster than C, and C than B. Now imagine them connected by an elastic cord. It is obvious that they will still draw apart until the tension of the cord prevents any further separation. Its tension will then measure the "lifting force" of the moon which tends to draw both the particles A and B away from C.

468. The Sun's Action. — This is precisely like that of the moon, except that the sun's distance, instead of being only sixty times the earth's radius, is nearly 23,500 times that quantity. Since the tide-raising power varies as the *cube* of the distance inversely, while the attracting force varies only with the inverse *square*, it turns out that although the sun's attraction on the earth is nearly 200 times as great as that of the moon, its *tide-raising power is only about two-fifths as much*. When the sun is over head or under foot, his disturbing force diminishes gravity by about $\frac{1}{19\,600000}$.

469. Statical Theory of the Tides. — If the earth were wholly composed of water, and if it kept always the same face towards the moon (as the moon does towards the earth), so that every particle on the earth's surface was always subjected to the same disturbing force from the moon, then, leaving out of account the sun's action, a permanent tide would be raised upon the earth, distorting it into a lemon-shaped form with the point towards the moon. It would be permanently high water at the points A and B (Fig. 152) directly under the moon, and low water all around the earth on the circle 90° from these points, as at D and E. The difference of the level of the water at A and D would in this case be about two feet.

The sun's action would produce a similar tide superposed upon the lunar tide and having about two-fifths of the same elevation. If the two tide summits should coincide, the resulting elevation of the high water would be the sum of the two separate tides. If the sun were 90° from the moon, the waves would be in opposition, and the height of the tide would be decreased, the solar tide partly filling up the depression at the low water due to the moon's action.

Suppose now the earth to be put in rotation. It is easy to see that these tidal waves would *tend* to move over the earth's surface, following the moon and sun at a certain angle dependent on the inertia of the water, and with a westward velocity precisely equal to that of the earth's eastward rotation, — about a thousand miles an hour at the equator. But it is also evident that on account of the varying depth of the ocean, and the irregular form of the shores, the tides could not maintain this motion, and that the actual result must become exceedingly complicated. In fact, the statical theory becomes utterly unsatisfactory in regard to what actually takes place, and it is necessary to depend almost entirely upon the results of observation, using the theory merely as a guide in the discussion of the observations.

Yet while this statical theory of the tides worked out by Newton is certainly inadequate, and in some respects incorrect, it easily furnishes the explanation of some of the most prominent of the peculiarities of the tides.

470. The Priming and Lagging of the Tides. — About the time of new and full moon, as has been stated before (Art. 463), the interval between the corresponding tides of successive days is about thirteen minutes less than the average of $24^h\ 51^m$, while a week later it is about as much longer. The reason is found in the combination of the solar and lunar tides.

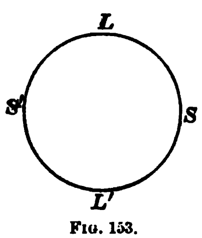

FIG. 153.

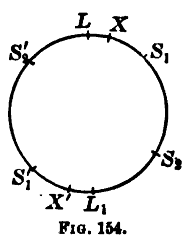

FIG. 154.

Priming and Lagging of the Tide.

On the days of new and full moon the two tides coincide, and the tide wave has its crest directly under the moon, or rather at the normal distance behind the moon which corresponds to the "establishment" of the port of observation.

At quadrature the crest of the solar tide will be just 90° from the crest of the lunar wave, but it will leave the summit of the *combined wave* just where it would be if there were no solar wave at all: evidently there is no possible reason why the smaller wave at S and S' should displace the crest of the wave at L (Fig. 153) towards the right that would not also require its displacement towards the left; it will therefore simply *lower* the wave at L *without displacing it* one way or the other. But when the solar tide wave SS' (Fig. 154) has its crest at S_1 and S_1', 45° from L and L', as it will do about three days after new or full moon, then its combination with the lunar wave will make the crest of the combined wave take position at a point X between the two crests, and about half an hour of time ahead (*west*) of the lunar tide; so that at that time of the month high water will occur about half an hour *earlier* than if there were no solar tide (since the tide waves travel westward). And this half-hour has to be gained by diminishing the interval between the successive tides for the three preceding days. Similar reasoning shows that when the solar tide crest falls at S_2 and S_2', the combined tide wave will be *east* of the lunar wave, and come later into port.

471. Effect of the Moon's Declination and Diurnal Inequality. — In high latitudes on the Pacific Ocean, twice a month, when the moon is farthest north or south of the celestial equator, the two tides of the day are very different in magnitude. When the moon's declination is zero, there is no such difference: nor is there ever any difference at ports which are near the earth's equator.

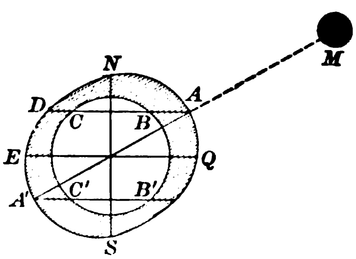

FIG. 155. — The Diurnal Inequality.

Fig. 155 makes it clear why it should be so. When the moon's declination is zero, things are as in Fig. 152 (Art. 469), and the two tides of the same day are sensibly equal at ports in all latitudes. When the moon is at her greatest northern declination, say 28°, the two tide summits will be at A and A' in Fig. 155; the tide which occurs at B when the moon is overhead will be great, while the tide in the corresponding southern latitude at B' will be small. The tides which occur twelve hours later will be small at the northern station, then situated at C, and large at the southern station, then at C'. For a port on the equator at E or Q there will be no such difference. In the Atlantic Ocean the difference is hardly noticeable, because, as we shall see very soon, the tides in that ocean are mainly (not entirely) due to tide waves propagated into it from the Pacific and Indian Oceans around the Cape of Good Hope.

472. The Wave Theory of the Tides. — If the earth were entirely covered with deep water, except a few little islands projecting here and there to serve for observing stations, the tide waves would run

around the globe *regularly*. According to Darwin the tide-crests, if the depth exceeded 14 miles, would keep exactly under the moon (considering only the lunar tide). If the depth were somewhat less, the tide-crests on the equator would follow the moon at an angle of 90°, while those near the poles would maintain their old relation, and at some intermediate latitude there would be a tideless belt of conflicting currents. In the actual ocean, comparatively shallow and of varying depth, the case becomes hopelessly complicated. The continents of North and South America, with the southern antarctic continent, make a barrier almost complete from pole to pole, leaving only a narrow passage at Cape Horn; and the varying depth of the water and the irregular contours of the shores are such that it is quite impossible to determine by theory what the course and character of the tide wave must be. We must depend upon observation; and observations are inadequate, because, with the exception of a few islands, our only possible tide stations are on the shores of continents where local circumstances largely control the phenomena.

473. Free and Forced Oscillations. — If the water in the ocean is suddenly disturbed (as for instance, by an earthquake), and then left to itself, a "free" wave will be formed, which, if the horizontal dimensions of the wave are large as compared with the depth of the water, will travel at a rate depending solely on the depth. The velocity of such a free wave is given by the formula $v = \sqrt{gh}$; that is, it is equal *to the velocity acquired by a body in falling through half the depth of the ocean.*

Thus a depth of	25	feet	gives	a	velocity of	19 +	miles	per	hour.
	100	"	"	"	"	" 39	"	"	"
	10,000	"	"	"	"	" 388	"	"	"
	40,000	"	"	"	"	" 775	"	"	"
	67,200	(12¾ miles)			"	" 1000	"	"	"
	90,000	"	"	"	"	" 1165	"	"	"

Observations upon the waves caused by certain earthquakes in South America and Japan have thus informed us that between the coasts of those countries the Pacific averages between two and one-half and three miles in depth.

474. Now, as the moon in its diurnal motion passes across the American continent each day, and comes over the Pacific Ocean, it starts such a "parent" wave in the Pacific, and the wave once started moves on nearly (but not exactly) like an earthquake wave. Not exactly, because the velocity of the earth's rotation being about

1050 miles an hour at the equator, the moon runs relatively westward faster than the wave can naturally follow, and so for a while slightly accelerates it. A second tidal wave is produced daily twelve hours later when the moon passes *underneath*. The tidal wave is thus, *in its origin, a forced oscillation, while in its subsequent travel it is pretty nearly a free one.*

475. Co-Tidal Lines. — These are lines drawn upon the surface of the ocean connecting those places which have their high water at the same moment of Greenwich time. They mark the crest of the tide wave for each hour of Greenwich time; and if we could draw them with certainty upon the globe, we should have all necessary information as to the motion of the wave. Unfortunately we can obtain no direct knowledge as to the position of these lines in mid-ocean; we only get a few points here and there on the coasts and on islands, so that a great deal necessarily remains conjectural. Fig. 156 is a reduced copy of such a map, borrowed with some modifications from that given in Guyot's "Physical Geography."

476. Course of Travel of the Tidal Wave. — On studying the map we find that the main or "parent" wave starts twice a day in the Pacific, off Callao, on the coast of South America. This is shown on the chart by a sort of oval "eye" in the co-tidal lines, just as a mountain summit is shown on a topographical chart by an "eye" in the contour lines. From this point the wave travels northwest through the deepest water of the Pacific at the rate of about 850 miles per hour, reaching Kamtchatka in about ten hours. To the west and southwest the water is shallower and the travel slower, — only 400 to 600 miles per hour, — so that the wave arrives at New Zealand in about twelve hours. Passing on by Australia, and combining with the small wave which the moon raises directly in the Indian Ocean, the resultant tide crest reaches the Cape of Good Hope in about twenty-nine hours, and enters the Atlantic. Here it combines with the tide wave, twenty-four hours younger, which has "backed" into the Atlantic around Cape Horn, and it is modified also by the *direct tide* produced by the moon's action upon the waters of the Atlantic. The resultant tide crest then travels *northward* through the Atlantic at the rate of nearly 700 miles per hour. It is about forty hours old when it first reaches the coast of the United States in Florida, and our coast is so situated that it arrives at all the principal ports within two or three hours of that time. It is forty-one or forty-two hours old when it arrives at New York and Boston. To reach London it has to travel around the northern end of Scotland and through the North Sea, and is nearly sixty hours old when it arrives at that port and the ports of the German Ocean, — Hamburg, etc.

In the great oceans there are thus three or four tide crests travelling

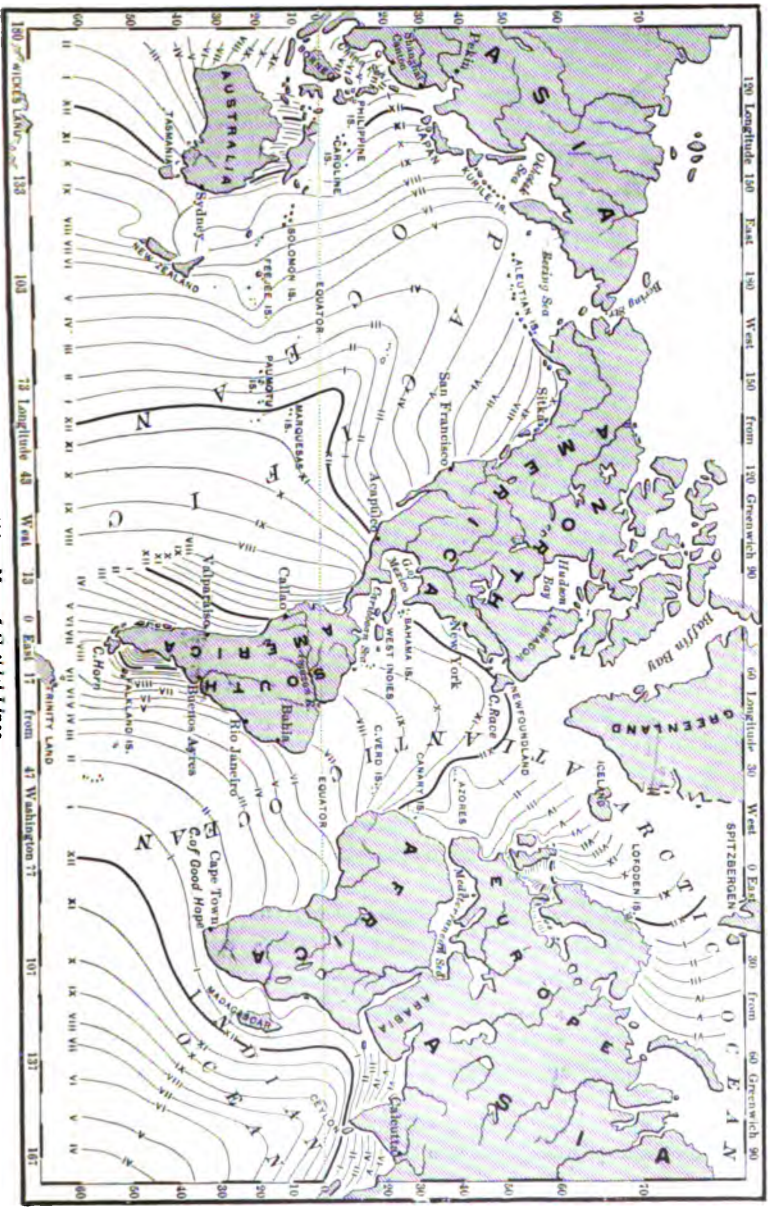

FIG. 156. — Map of Cotidal Lines.

simultaneously, following each other nearly in the same track, but with continual minor changes, owing to the variations in the relative positions of the sun and moon and their changing distances and declinations. If we take into account the tides in rivers and sounds, the number of simultaneous tide crests must be at least six or seven; that is, the high water at the extremity of its travel, up the Amazon River, for instance, must be at least three or four days old, reckoned from its birth in the Pacific.[1]

477. Tides in Rivers. — The tide wave ascends a river at a rate which depends upon the depth of the water, the amount of friction, and the swiftness of the stream. It may, and generally does, ascend until it comes to a *rapid, where the velocity of the water is greater than that of the wave.* In shallow streams, however, it dies out earlier.

Contrary to what is usually supposed, it often ascends to an elevation far above that of the highest crest of the tide wave at the river's mouth. In the La Plata and Amazon it goes up to an elevation at least one hundred feet above the sea-level. The velocity of the tide wave in a river seldom exceeds ten or twenty miles an hour, and is usually less.

478. Height of Tides. — In mid-ocean the difference between high and low water is usually between two and three feet, as observed on isolated deep-water islands in the Pacific; but on the continental shores the height is usually much greater. As soon as the tide wave

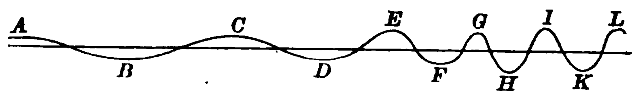

FIG. 157. — Increase in Height of Tide on approaching the Shore.

touches bottom, so to speak, the velocity is diminished and the height of the wave is increased, something as in the annexed figure (Fig. 157). Theoretically the height varies *inversely as the fourth root of the depth.* Thus, where the water is 100 feet deep, the tide wave should be twice as high as at the depth of 1600 feet.

Where the configuration of the shore forces the wave into a corner, it sometimes becomes very high. At the head of the Bay of Fundy, tides of seventy feet are not very uncommon, and an altitude of nearly a hundred feet is said to be occasionally attained.

At Bristol, England, in the mouth of the Severn the tide rises fifty feet, and sometimes ascends the river (as it also does the Seine, in France, and

[1] We are greatly indebted to Loomis's discussion of the subject in his "Elements of Astronomy."

the Amazon) as a *breaking* wave, called the "bore" or "eiger" (French, *mascaret*), with a nearly vertical front five or six feet in height, crested with foam, and very dangerous to small vessels. On the east coast of Ireland, opposite to Bristol, the tide ranges only about two feet.

In mid-ocean the water has no progressive motion, but near the land it has, running in at the flood to fill up the bays and cover the flats, and then running out again at the ebb. The velocity of these tidal *currents* must not be confounded with that of the tide wave itself.

479. Reflection and Interference. — The tide wave when it reaches the shore is not entirely destroyed, especially if the coast is bold and the water deep; but is partly reflected, and the reflected wave goes back into the ocean to meet and modify the new tide wave which is coming in. Of course, in such a case we get "interferences," so that on islands in the Pacific only a few hundred miles apart we find great differences in the heights of the tides. At one place the direct waves and the waves reflected from the shores of Asia and South America may conspire to give a tide of three or four feet, or nearly double its normal value, while at another they nearly destroy each other.

There are places, also, which are reached by tides coming by two different routes. Thus on the east coast of England and Scotland the tide waves come both around the northern end of Scotland and through the Straits of Dover. In some places on this coast we have, therefore, a tide of nearly double height, while at others not very far away there will be hardly any tide at all; and at intermediate points there are sometimes *four* distinct high waters in twenty-four hours. As a consequence of this reflection and interference of the tide waves it follows that if the tide-raising power were suddenly abolished, the tides would not immediately cease, but would continue to run for several days, and perhaps weeks, before they gradually died out.

480. Effect of the Varying Pressure of the Barometer, and of the Wind. — When the barometer at a given port is lower than usual, the level of the water is generally higher than the average, at the rate of about one foot for every inch of the mercury in the barometer; and *vice versa* when it is higher than usual.

When the wind blows into the mouth of a harbor, it drives in the water of the ocean by its surface friction, and may raise the water several feet. In such cases the time of high water, contrary to what might at first be supposed, is *delayed*, sometimes as much as fifteen or twenty minutes.

This result depends upon the fact that the water runs into the harbor for a longer time than it would do if the wind were not blow-

ing. The normal depth of the water on the bar is reached *before* the predicted time, so that at the predicted time the water is deeper than it would be if there were no wind, but the *maximum* depth is not attained until some time later. Of course, the results are the opposite when the wind blows out of the harbor: the time of high water comes earlier, and the depth of water on the bar at the predicted time of high water is less than it otherwise would be.

481. Tides in Lakes and Inland Seas. — These are small and difficult to detect. Theoretically, the range between high and low water in a land-locked sea should bear about the same ratio to the rise and fall of the tide in mid-ocean that the length of the sea does to the diameter of the earth. Variations in the direction of the wind and the barometric pressure cause continual oscillations in the water-level which, even in a quiet lake, are much larger than the true tides; so that it is only by taking a long series of observations, and discussing them with reference to the moon's position in the sky, that it is possible to separate the real tide from the effects of other causes. In Lake Michigan, at Chicago, a tide of about one and three-quarters inches has thus been detected, the "establishment" of the port being about thirty minutes. In Lake Erie, at Buffalo and Toledo, the tide is about three-quarters of an inch. On the coasts of the Mediterranean the tide averages about eighteen inches, attaining a height of three or four feet at the head of some of the bays.

482. The Rigidity of the Earth. — Lord Kelvin has endeavored to make the tides the criterion of the rigidity of the earth's core. Evidently if the solid parts of the earth were fluid, there would be no observable tide anywhere, since the whole surface would rise and fall together. If the earth were semi-solid, so to speak (that is, viscous, and capable of yielding more or less to the forces tending to change its form), the tides would be observable, but to a less degree than if the earth's core were rigid. And with this further peculiarity — since a viscous body requires time to change its form, waves of *short period* would be observable upon the semi-solid earth nearly to their full extent, while those of *long period* would almost entirely disappear, owing to the slow yielding of the earth's crust. Now the actual tide wave, as observed, is really made up of a multitude of component tide waves of different periods, ranging from half a day upwards. According to the "principle of forced vibrations" every regularly recurring periodic change in the forces which act on the surface of the ocean must produce a tide of greater or less magnitude, and of exactly corresponding period.

We have, for instance, the semi-diurnal, solar, and lunar tides; then the two monthly tides due to the change in the moon's distance and declination, and the two annual tides due to the changes of the sun's distance and declination, not to speak of the nineteen-year tide due to the revolution of the moon's nodes.

A thorough analytical discussion of thirty-three years' tidal observations at different parts of the world has been made under the direction of Lord Kelvin by Mr. George Darwin, with the result that not only do the short waves show themselves, but *the waves of long period are found to manifest themselves with almost their full theoretical value.* Lord Kelvin concludes that the earth as a whole "*must be more rigid than steel, but perhaps not quite so rigid as glass.*"[1] This result is at variance with the prevalent belief of geologists that the core of the earth is a molten mass, and has led to much discussion which we cannot deal with here.

483. Effect of the Tides on the Earth's Rotation.—If the tidal motion consisted merely in the upward and downward motion of the particles of the ocean to the extent of two feet or so twice a day, it would involve a very trifling expenditure of energy; and this is the case with the mid-ocean tide. But near the land this almost insensible mere oscillatory motion is transformed into the bodily travelling of immense masses of water, which flow in upon the shallows and then out again to sea with a great amount of fluid friction; and this involves the expenditure of a very considerable amount of energy which is dissipated as heat. From what sources does this energy come? The answer is that it must be derived *mainly from the earth's energy of rotation*, and the necessary effect is to diminish that energy by lessening the speed of the rotation. Compared with the earth's whole stock of rotational energy, however, the loss of it by tidal friction, even in a century, is very small, and the effect on the length of the day is extremely slight.

The reader will recall the remarks upon the subject of the secular acceleration of the moon's mean motion a few pages back (Art. 461).

While it is certain that the tidal friction *tends* to lengthen the day, it does not follow that the day really grows longer. There are counteracting causes:—for example, the earth's radiation of heat into space, and the consequent shrinkage of her volume.

[1] This is substantially confirmed by recent researches of S. S. Hough upon the variation of latitude.

As matters stand we do not know whether, *as a fact*, the day is really longer or shorter than it was a thousand years ago. The change, if any has really occurred, can hardly be as great as $\frac{1}{1000}$ of a second.

484. Effect of the Tide on the Moon's Motion. — Not only does the tide diminish the *earth's* energy of rotation directly by the tidal friction, but, theoretically, it also communicates a minute portion of that energy to the *moon*. It will be seen that a tidal wave, situated as in Fig. 158, would slightly accelerate the moon's motion, the attraction of the moon by the tidal protuberance F being slightly greater than that of the tide wave at F' — a difference tending to increase the moon's velocity and so to increase the major axis of its orbit. The effect is therefore to make the moon *recede* from the earth, and to *lengthen* the month.

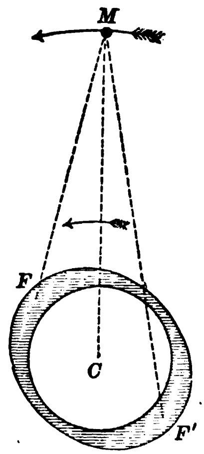

FIG. 158.
Effect of the Tide on the Moon's Motion.

Upon this interaction between the tides and the motions of the earth and moon Professor George Darwin has founded his theory of "*tidal evolution*"; namely, that the satellites of a planet, having separated from it millions of years ago, have been made to recede to their present distances by just such an action. A similar action is invoked by Dr. See to explain the elongated orbits of double stars. (Art. 877.)

An excellent popular statement of the theory will be found in the closing chapter of Ball's "Story of the Heavens," and in his little book entitled "Time and Tide." The original papers of Mr. Darwin in the "Philosophical Transactions" are of course severely mathematical.

CHAPTER XIV.

THE PLANETS: THEIR MOTIONS, APPARENT AND REAL.—THE PTOLEMAIC, TYCHONIC, AND COPERNICAN SYSTEMS. —THE ORBITS AND THEIR ELEMENTS.—PLANETARY PERTURBATIONS.

485. For the most part, the stars keep their relative configurations unchanged, however much they alter their positions in the sky from hour to hour. The "dipper" remains always a "dipper" in whatever part of the heavens it may be. But while this is true of the stars in general, certain of the heavenly bodies, and among them those that are the most conspicuous of all, form an exception. The sun and moon continually change their places, moving eastward among the stars; and certain others, which to the eye appear as very brilliant stars, also move,[1] though not in quite so simple a way.

486. These bodies were named by the Greeks the "*planets*"; that is, "wanderers." The ancient astronomers counted seven of them. They reckoned the sun and moon, and in addition Mercury, Venus, Mars, Jupiter, and Saturn.

Venus and Jupiter are at all times more brilliant than any of the fixed stars. Mars at times, but not usually, is nearly as bright as Jupiter; and Saturn is brighter than all but a very few of the stars. Mercury is also bright, but seldom seen, because always near the sun.

At present the sun and moon are not reckoned as planets; but the roll includes, in addition to the five other bodies known by the ancients, the earth itself, which Copernicus showed should be counted among them, and also two new bodies of great magnitude (though inconspicuous because of their distance) which have been discovered in modern times; then there is in addition a host of so-called "*asteroids*" which circulate in the otherwise vacant space between the planets Mars and Jupiter.

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[1] When we speak of the motion of the planets, the reader will understand that the *diurnal* motion is not taken into account. We speak of their motions *among the stars*.

487. The list of the planets in the order of distance from the sun stands thus at present: Mercury, Venus, the Earth, Mars, Jupiter, Saturn, Uranus, and Neptune; and between Mars and Jupiter, in the place where a planet would naturally be expected to revolve, there are at present known over 460 little planets, which probably represent a single one, somehow "spoiled in the making," so to speak, or burst into fragments.

The planets are all dark bodies, shining only by reflected sunlight, — globes which, like the earth, revolve around the sun in orbits nearly circular, moving all in the same direction, and (with some exceptions among the asteroids) nearly in the common plane of the ecliptic and sun's equator. All of them but the inner two and the asteroids are also attended by "satellites." Of these the earth has one (the moon), Mars two, Jupiter five, Saturn nine,[1] Uranus four, and Neptune one; *i.e.*, so far as at present known; for although it is hardly probable, it is not at all impossible that others may yet be found.

488. Relative Distances of Planets from the Sun: Bode's Law. — There is a curious approximate relation between the distances of the planets from the sun, which makes it easy to remember them. It is usually known as Bode's Law, because Bode first brought it prominently into notice in 1772, though Titius of Wittenberg seems to have discovered and enunciated it some years earlier. The law is this: Write a series of 4's. To the second 4 add 3; to the third add 3 × 2, or 6; to the fourth, 3 × 4, or 12; and so on, doubling the added number each time, as in the accompanying scheme.

4	4	4	4	4	4	4	4	4
	3	6	12	24	48	96	192	384
4	7	10	16	[28]	52	100	196	388
☿	♀	⊕	♂	(1)	♃	♄	♅	♆

The resulting numbers, divided by 10, are pretty nearly the true mean distances of the planets from the sun, in terms of the radius of the earth's orbit. In the case of Neptune, however, the law breaks down utterly, and is not even approximately correct.

For the present, at least, the law is to be regarded as a mere coincidence. Its explanation may perhaps ultimately be found in the process by which the system was developed.

The general expression for the *n*th term of the series is $4 + 3 \times 2^{(n-2)}$; but it does not hold good of the first term, which is simply 4, instead of being $5\frac{1}{2}$, *i.e.*, $(4 + 3 \times 2^{-1})$, as it should be.

[1] See note on page 406.

489. Table of Names, Distances, and Periods.

Name.	Symbol.	Distance.	Bode.	Diff.	Sid. Period.	Syn. Period.
Mercury	☿	0.387	0.4	−0.013	88^d or 3^m	116^d
Venus	♀	0.723	0.7	+0.023	224.7^d or $7\frac{1}{2}^m$	584^d
Earth	⊕	1.000	1.0	0.000	$365\frac{1}{4}^d$ or 1^y	. . .
Mars	♂	1.523	1.6	−0.077	687^d or $1^y\ 10\frac{1}{2}^m$	780^d
Mean Asteroid		2.650	2.8	−0.150	$3^y.1$ to $8^y.0$	various
Jupiter	♃	5.202	5.2	+0.002	$11^y.9$	399^d
Saturn	♄	9.539	10.0	−0.461	$29^y.5$	378^d
Uranus	♅ & ♅	19.183	19.6	−0.417	$84^y.0$	370^d
Neptune	♆	30.054	38.8	−8.746 !	$164^y.8$	$367\frac{1}{2}^d$

The column headed "Bode" gives the distance according to Bode's law; the column headed "Diff.," the difference between the true distance and that given by Bode's law.

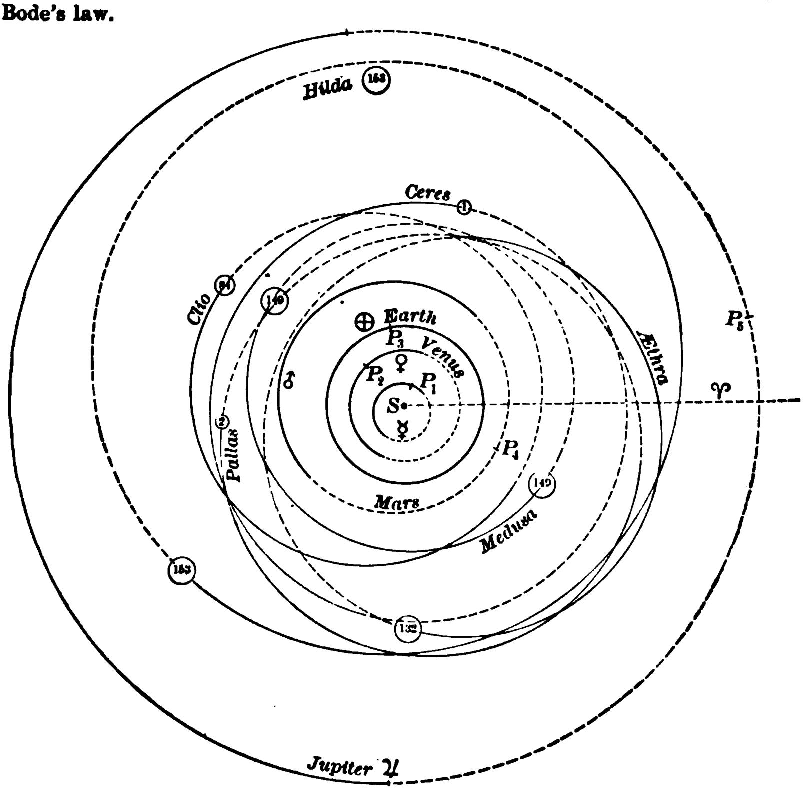

Fig. 159. — Plan of the Orbits of the Planets inside of Saturn.

490. Fig. 159 shows the smaller orbits of the system (including the orbit of Jupiter) drawn to scale, the radius of the earth's orbit being taken as one centimetre. On this scale the diameter of Saturn's orbit would be $19^{cm}.08$, that of Uranus would be $38^{cm}.36$, and that of Neptune, $60^{cm}.11$. The nearest fixed star on the same scale would be about a mile and a quarter away. The dotted half of each orbit is that which lies below, *i.e.*, south of, the plane of the ecliptic. The place of perihelion of each planet's orbit is marked with a *P*. The orbits of five of the asteroids are also given.

491. Periods. — The *sidereal period* of a planet is the time of its revolution around the sun from a star to the same star again, as *seen from the sun.* The *synodic period* is the time between two successive conjunctions of the planet with the sun, as *seen from the earth.* The sidereal and synodic periods are connected by the same relation as the sidereal and synodic months (Art. 232); namely, —

$$\frac{1}{S}=\frac{1}{P}-\frac{1}{E},$$

in which E, P, and S are respectively the periods of the earth and of the planet, and the planet's synodic period, and the numerical difference between $\frac{1}{P}$ and $\frac{1}{E}$ is to be taken without regard to sign.

$\frac{1}{P}$ is the planet's *mean daily motion* expressed as a fraction of the whole circumference, while $\frac{1}{E}$ is the earth's motion; and the equation simply states that the daily *synodic motion* $\left(\frac{1}{S}\right)$, *is the difference of the other two motions.* The two last columns of the table in Article 489 give the approximate periods, both sidereal and synodic, for the different planets.

492. Apparent Motions. — As viewed from a distant point on the line drawn through the sun, perpendicular to the plane of the ecliptic, the planets would be seen to travel in their nearly circular orbits with a regular motion. As seen from the earth the apparent motion is much more complicated, being made up of their real motion around the sun combined with an apparent motion due to the earth's own movement.

493. Law of Relative Motion. — The motion of a body relative to the earth can be very simply stated. *It is always the same as if the body had, combined with its own motion, another motion, identical with that of the earth, but reversed.*

The proof of this is simple. Let E, Fig. 160, be the earth, and P the planet, its direction and distance being given by the line EP. Let E have a motion which will take it to E' in a unit of time, and P a motion which will take it to P' in the same time. Then at the end of a unit of time the distance and direction of P from E will be given by the line $E'P'$. But if we suppose E to remain at rest, and give to P a motion Pe equal to EE' but

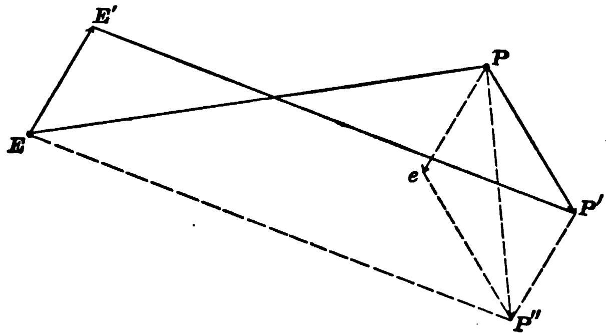

FIG. 160. — The Relative Motions of Two Bodies.

opposite in direction, and combine this motion with PP' by drawing the parallelogram of motions, we shall get P'' for the resulting place of P at the end of the unit of time; and because the line EP'' is parallel and equal to $E'P'$ (as follows from the construction), the point P'', as seen from E, would occupy, in the celestial sphere, precisely the same position as P' seen from E'; since all parallel lines pierce the sphere at one and the same optical point (Art. 7).

If, therefore, the earth moves in a circle, every body really at rest will *appear* to move in a circle of the same size as the earth's orbit, but keeping in such a part of its circle as always to have its motion precisely opposite to the earth's own real motion at the moment. We shall have occasion to use this principle very frequently.

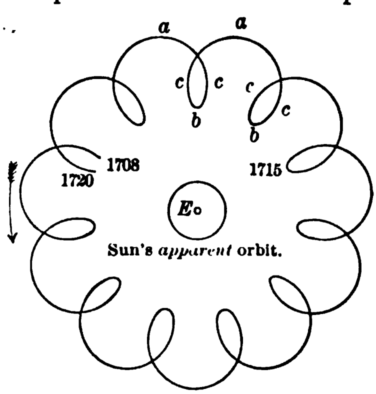

FIG. 161.
Geocentric Motion of Jupiter from 1708 to 1720. (Cassini.)

494. Geocentric Motion of a Planet in Space.—The "*geocentric*" motion of a planet is its motion *relative to the earth regarded as a fixed centre*, and is therefore made up of two motions, — that of a body moving once a year around the circumference of a circle equal

to the earth's orbit, while the centre of this circle itself goes around the sun upon the real orbit of the planet, and with a periodic time equal to that of the planet. In other words, its distance and direction from the earth are always just what they would be if the earth were at rest while the planet itself moved in this complicated manner. As the result of this combination of motions the relative, or "*geocentric*," orbit of a planet is a looped curve, which, if the real orbits were perfectly circular, would be exactly an *epicycloid*. Since, however, the orbits are slightly oval the loops actually vary somewhat in size and distance from each other. Jupiter, for instance, appears to move as in Fig. 161, making eleven loops in each revolution, the smaller circle having a diameter of about one-fifth that of the larger one, upon which its centre moves, since the diameter of Jupiter's orbit is about five times that of the earth's. (For fuller illustration see Appendix, Art. 1009.)

495. Apparent Motions of the Planets upon the Celestial Sphere. Direct and Retrograde Motions and Stationary Points. — As a

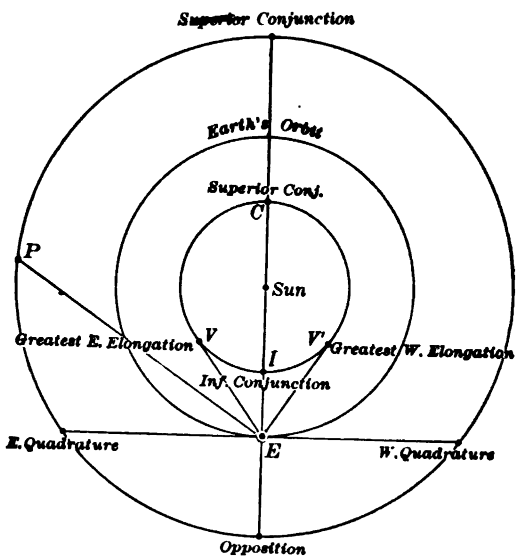

FIG. 162. — Planetary Configurations and Aspects.

consequence of this looped motion we have a peculiar back-and-forth movement of the planets among the stars. Starting from the time when the sun is between us and the planet, — the time of superior

conjunction,[1] as it is called, because the planet is then *above* the sun, *i.e.*, further from the earth (at *a* in its *geocentric* orbit, Fig. 161), — the planet moves eastward among the stars for a certain time, continually increasing its longitude (and also its right ascension) until at last its apparent motion slackens and it becomes *stationary* at the points marked *c, c, c,* in the geocentric orbit, Fig. 161. The elongation of this stationary point from the sun depends upon the size of the planet's orbit compared with that of the earth.

Then it reverses its motion and moves westward, or "*retrogrades*," for a while, *the middle of the arc of retrogression being passed at the time when the earth and planet are in line with the sun, and on the same side of it* at the points marked *b, b,* in the geocentric orbit, — the ends of the "loops" where the distance of the planet from the Earth is a minimum. If the planet is one of the outer ones, it will then be opposite to the sun in the sky like the full moon, and is said to be "*in opposition*." If it is one of the inferior planets (Venus or Mercury), it will then be in "*inferior conjunction*," as it is called, between the earth and sun.

After the planet has completed its arc of retrogression, it again becomes stationary, turns upon its course, and once more advances eastward among the stars, until the synodic period is completed by its re-arrival at superior conjunction.

Both in the number of degrees passed over, and in the time spent in this motion, the eastward or "*direct*" motion always exceeds the retrograde. In the case of the remoter planets the excess is small — from 3° to 10°; in the case of the nearest ones, Mars and Venus, it is many times greater.

As observed with a *sidereal clock*, all the planets come *later* to the meridian each night when moving *direct*, since their right ascension is then increasing; but *vice versa*, of course, when they are *retrograding*.

496. Motions in Latitude. — If their orbits lay precisely in the same plane as that of the earth, the planets would always keep exactly upon the ecliptic. In fact, however, they deviate from that

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[1] We give Fig. 162 to illustrate the meaning of the different terms, *Opposition, Quadrature, Inferior and Superior Conjunction,* and *Greatest Elongation.* *E* is the position of the earth, the inner circle being the orbit of an *inferior* planet, while the outer circle is the orbit of a *superior* planet. In general, the angle *PES* (the angle at the earth between lines drawn from the earth to the planet and to the sun) is the planet's *elongation* at the moment. For a superior planet it can have any value from zero to 180°; for an inferior it has a maximum value that the planet cannot exceed, depending upon the diameter of its orbit.

circle to the extent of 4° or 5°, and Mercury sometimes as much as 8°; so that their paths among the stars form loops and kinks. Fig.

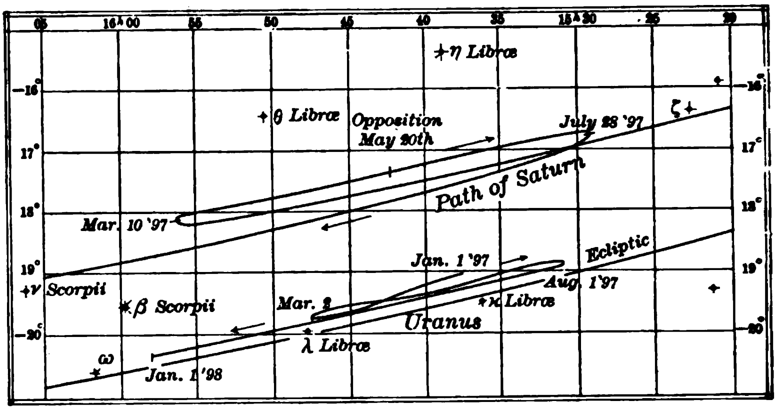

Fig. 163. — Motion of Saturn and Uranus in 1897.

163 shows, as an example, the apparent motions of Saturn and Uranus for 1897. In the case of Mars the loops are usually much more intricate.

497. Motion of the Planets with Respect to the Sun's Place in the Sky. Change of Elongation. — The visibility of a planet depends mainly upon its angular distance, or "*elongation*," from the sun, because when near the sun the planet will be above the horizon only by day, and cannot usually be seen. As regards their motions, considered from this point of view, there is a marked difference between the inferior planets and the superior.

498. Behavior of a Superior Planet. — The superior planets drop always steadily *westward* with respect to the sun's place in the heavens, continually increasing their western elongation, or decreasing their eastern: they therefore *invariably come earlier to the meridian every successive night*, as observed by a time-piece *keeping solar time.*

Beginning at superior conjunction, the planet is then moving eastward among the *stars* with its greatest speed; but even then its eastward motion is not so great as the sun's, and so the planet *relatively* falls westward. After a while it will have fallen behind by 90°, and will then be in western quadrature, and on the meridian at sunrise; at the end of half its synodic period it will have lost 180°, and will be just opposite the sun at sunset, being then at its least possible distance from the earth, and at its greatest brilliance. At this time the difference between the times of its daily culminations is

also the greatest possible, and may be as much (in the case of Mars) as six minutes, by which amount it arrives at the meridian earlier each successive night. After opposition the planet is higher in the sky each night at sunset until it reaches eastern quadrature, when it is 90° east of the sun, and therefore on the meridian at sunset. Thence it drops back, falling more and more slowly westwards towards the sun, until the synodic period is completed by a new conjunction.

499. Motion of an Inferior Planet.—The inferior planets appear, on the other hand, to *vibrate* across the sun, moving out equal distances on each side of it, but making the westward swing much quicker than the eastern.

The reason of this difference is obvious from Fig. 162. Matters take place with respect to the earth, sun, and planet as if the earth were at rest and the planet revolving around the sun once in a *synodic* (not sidereal) period. Now, since the distance between the points of greatest elongation, V and V', is less through inferior conjunction I, than from V' around to V through C, the time ought to be correspondingly shorter, as it is.

500. The Ptolemaic System. — The ancient astronomers, for the most part, never doubted the fixity of the earth, and its position in the centre of the celestial universe, though there are some reasons to think that Pythagoras may have done so. Assuming this and the actual diurnal revolution of the heavens, Ptolemy, who flourished at Alexandria about 140 A.D., worked out the system which bears his name. His Μεγάλη Σύνταξις (or *Almagest* in Arabic) was for fourteen centuries the authoritative "Scripture of Astronomy." He showed that all the apparent motions of the planets could be accounted for by supposing each planet to move around the circumference of a circle called the *"epicycle,"* while the centre of this circle, sometimes called the *"fictitious planet,"* itself moved on the circumference of another and larger circle called the *"deferent."* It was as if the real planet was carried on the end of a crank-arm which turned around the fictitious planet as a centre, in such a way as to point towards or from the earth at times when the planet is in line with the sun.

In the case of the superior planets the revolution in the epicycle was made once a year, so that the "crank-arm" was always parallel to the line joining earth and sun, while the motion around the deferent occupied what we now call the planet's period. Fig. 164 represents the Ptolemaic System, except that no attention is paid to dimensions, the "deferents" being spaced

at equal distances. It will be noticed that the epicycle-radii which carry at their extremities the planets Mars, Jupiter, and Saturn are all always parallel to the line that joins the earth and sun. In the case of Venus and Mercury this was not so. Ptolemy supposed that the deferent circles for these planets lay *between* the earth and the sun, and that the "fictitious planet" in both

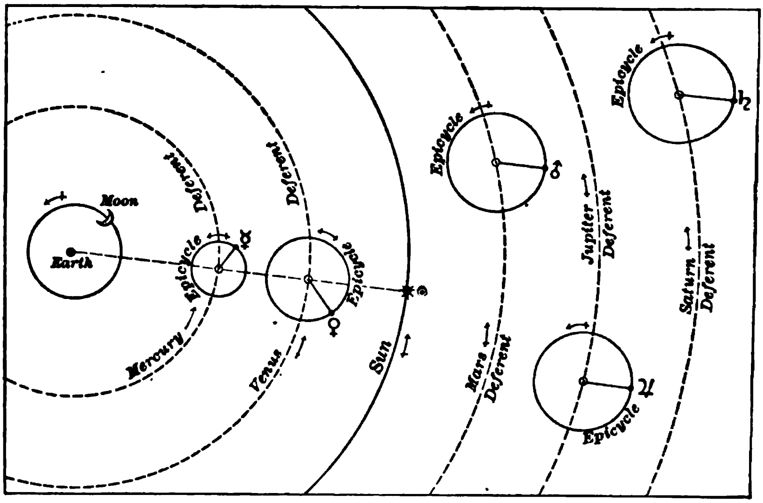

FIG. 164. — The Ptolemaic System.

cases revolved in the *deferent* once a year, always keeping exactly between the earth and the sun: the motion in the *epicycle* in this case was completed in the time of the planet's period, as we now know it. He ought to have seen that, for these two planets, the deferent was really the orbit of the sun itself, as the ancient Egyptians are said to have understood.

501. To account for some of the irregularities of the planets' motions it was necessary to suppose that both the deferent and epicycle, though circular, are *eccentric*, the earth not being exactly in the centre of the deferent, nor the "fictitious planet" in the exact centre of the epicycle. In after times, when the knowledge of the planetary motions had become more accurate, the Arabian astronomers added epicycle upon epicycle until the system became very complicated. King Alphonso of Spain is said to have remarked to the astronomers who presented to him the Alphonsine tables of the planetary motions, which had been computed under his orders, that "if he had been present at the creation he would have given some good advice."

502. Some of the ancient astronomers attempted to account for the planetary and stellar motions in a mechanical way by means of what were called the "*crystalline spheres.*" The planet Jupiter, for instance, was supposed to

be set like a jewel on the surface of a small globe of something like glass, and this itself was set in a hollow made to fit it in the thick shell of a still larger sphere which surrounded the earth. Thus the planets were supported and carried by the motions of these invisible crystalline spheres; but this idea, though prevalent, was by no means universally accepted.

503. Copernican System.—Copernicus (1473–1543) asserted the diurnal rotation of the earth on its axis, and showed that it would fully account for the apparent diurnal revolution of the stars. He also showed that nearly all the known motions of the planets could be accounted for by supposing them to revolve around the sun, with the earth as one of them, in orbits circular, but slightly out of centre. His system, as he left it, was *nearly* that which is accepted to-day, and Fig. 159 may be taken as representing it. He was, however, obliged to retain a few small epicycles to account for certain of the irregularities.

So far, no one dared to doubt the exact circularity of celestial orbits. It was metaphysically improper that *heavenly* bodies should move in any but *perfect* curves, and the circle was regarded as the only perfect one. It was left for Kepler, some sixty-five years later than Copernicus, to show that the planetary orbits are *elliptical*, and to bring the system substantially into the form in which we know it now.

504. Tychonic System. — Tycho Brahe, who came between Copernicus and Kepler, found himself unable to accept the Copernican system for two reasons. One reason was that it was unfavorably regarded by the clergy, and he was a good churchman. The other was the scientific objection that if the earth moved around the sun, the *fixed stars all ought to appear to move in a corresponding manner* (Art. 492), each star describing annually an oval in the heavens of the same apparent dimensions as the earth's orbit itself, seen from the star. Technically speaking, they ought to have an "*annual parallax.*" His instruments were by far the most accurate that had so far been made, and he could detect no such parallax; hence he concluded, not illogically, but incorrectly, that the earth must be at rest. He rejected the Copernican system, placed the earth at the centre of the universe, according to the then received interpretation of Scripture, made the sun revolve around the earth once a year, and then (this was the peculiarity of his system) made all the planets except the earth revolve around the sun.

This theory just as fully accounts for all the motions of the planets as the Copernican, but breaks down absolutely when it encounters the aberration of light, and the annual parallax of the stars, which we can *now* detect with our modern instruments, although Tycho could not with his. The Tychonic system never was generally accepted; the Copernican was very soon firmly established by Kepler and Newton.

505. Elements of a Planet's Orbit. — These are certain numerical quantities which describe the orbit with precision, and furnish the means of finding the planet's place in the orbit at any given time, whether past or future, so far as that place depends upon the attraction of the sun alone. Those usually employed are seven in number, as follows: —

1. The semi-major axis, a.
2. The eccentricity, e.
3. The inclination to the ecliptic, i.
4. The longitude of the ascending node, ☊.
5. The longitude of perihelion, π.
6. The epoch, E.
7. The period P, or daily motion μ.

506. Of these, the first five pertain to the orbit itself, regarded as an ellipse lying in space with one focus at the sun, while two are necessary to determine the planet's place in the orbit.

The *semi-major axis*, a (CA in Fig. 165), defines the *Size* of the

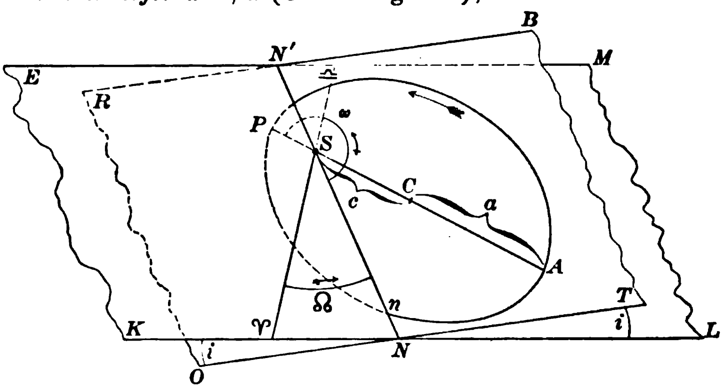

FIG. 165. — The Elements of a Planet's Orbit.

orbit, and may be expressed either in "astronomical units" (the earth's mean distance from the sun is the astronomical unit) or in miles.

The *Eccentricity* defines the orbit's *Form*. It is a mere numerical quantity, being the fraction $\frac{c}{a}$ (usually expressed decimally), obtained by dividing the distance between the sun and the centre of the orbit by the semi-major axis. In some computations it is convenient to use, instead of the decimal fraction itself, the angle ϕ which has e for its sine; *i.e.*, $\phi = \sin^{-1}e$.

The third element, i, is the *Inclination* between the plane of the planet's orbit and that of the earth. In the figure it is the angle KNO, the plane of the ecliptic being lettered $EKLM$.

The fourth element, ☊ (*the Longitude of the ascending node*), defines what has been called the "*aspect*" of the orbit-plane; *i.e.*, the direction in which it faces. The line of nodes is the line NN' in the figure, the intersection of the two planes of the orbit and ecliptic; and the angle ♈SN is the longitude of the ascending node, the line S♈ being the line drawn from the sun to the first of Aries. The planet, moving around its orbit in the plane $ORBT$, and in the direction of the arrow, passes from the lower or southern side of the plane of the ecliptic to the northern at the point n, which, as seen from S, is in the same direction as N.

The fifth, and last, of the elements which belong strictly to the orbit itself is π, the so-called *Longitude of the perihelion*, which defines the *direction* in which the major axis of the ellipse (the line PA) lies on the plane $ORBT$. It is not strictly a *longitude*, but equals the sum of the two angles ☊ and ω; *i.e.*, ♈SN (in the plane of the ecliptic) $+$ NSP (in the plane of the orbit and reckoned in the direction of the planet's motion). NSP exceeds 180° in the figure. It is quite sufficient to give ω alone, and in the case of cometary orbits this is usually done.

507. If we regard the orbit as an oval wire hoop suspended in space, these five elements completely define its position, form, and size. The *plane* of the orbit is fixed by the elements numbered three and four, the *position* of the orbit in this plane by number five, the *form* of it by number two, and finally its *magnitude* by number one.

508. To determine where the planet will be at any subsequent date we need two more elements.

Sixth. *The Periodic Time,* — we must have the *sidereal period, P,* or else *the mean daily motion, μ,* which is simply 360° divided by the number of days in P.

Seventh. And finally; we must have a starting-point, the "*Epoch,*" so-called; *i.e.*, the longitude of the planet as seen from the sun, at some given date, usually Jan. 1st, 1850 or 1900, or else some precise date at which the planet passed the perihelion or node.

509. If it were not for perturbations caused by the mutual interaction between the planets, these elements would never change, and could be used

directly for computing the planet's place at any date in the past or in the future; but, excepting a and P, they do change on account of such interaction and accordingly it is usual to add in tables of the planetary elements columns headed $\Delta\Omega$, $\Delta\pi$, Δi, and Δe, giving the amount by which the quantities Ω, π, i, and e respectively change in a century.

510. If Kepler's harmonic law were strictly true, we should not need both a and P, because we should have

$$(\text{Earth's Period})^2 : P^2 :: 1^3 : a^3, \text{ or } P = a^{\frac{3}{2}},$$

P being expressed in years and a in astronomical units. But since the exact form of the equation is

$$P_1^2(1 + m_1) : P_2^2(1 + m_2) :: a_1^3 : a_2^3 \text{ (Art. 417)},$$

it is necessary to regard P and a as independent, and give both of them.

511. Geocentric Place. — Our *observations* of a planet's place are necessarily "*geocentric*," or earth-centred; they give us, when properly corrected for refraction and parallax, the planet's *right ascension* and *declination* as seen from the centre of the earth, and from them, if desired, the corresponding geocentric *longitude* and *latitude* are easily obtained by the method explained in Article 180.

511*. Interpolation of Observations. — It often happens that we want the place at some moment of time when the planet could not be directly observed, as, for instance, in the day time. If we have a series of observations of the planet made about that time, the place for the exact moment is readily deduced by a process of *interpolation*, and with an accuracy actually exceeding that of any single observation of the series.

Graphically it is done by simply plotting the observations actually made. Suppose, for instance, we want the right ascension of Mars for 8 A.M. on June 3, and have meridian-circle observations made at 10 o'clock P.M. on June 1, at $9^h\ 55^m$ on June 2, at $9^h\ 50^m$ on June 3, and so on. We first lay off the times of observation as abscissas along a horizontal line taken as the time-scale, and then lay off the observed right ascensions as ordinates at points corresponding to the times. Then we draw a smooth curve through the points so determined, and from this curve we can read off directly, the right ascension corresponding to any desired moment. The declination can be treated in the same way. Of course, what can be done graphically can be done still more accurately by computation.

512. Heliocentric Place. — The *heliocentric* place of a planet is the place as seen from the sun; and when we know the longitude of the node of a planet's orbit and its inclination, as well as the planet's dis-

tance from the sun, this heliocentric place can at once be deduced from the geocentric by a trigonometrical calculation. The process is rather tedious, however, and its discussion lies outside the scope of this work.

(The reader is referred to Watson's "Theoretical Astronomy," p. 86. An elementary geometrical treatment of the reduction is also given in Loomis's "Treatise on Astronomy," p. 211.)

513. Determination of the Period of a Planet. — This can be done in two ways:

First. *By observation of its node-passage.* When the planet is passing its node, it is in the plane of the ecliptic, and the earth being also always in that plane, the planet's latitude, *both geocentric and heliocentric*, will be zero, no matter what may be the place of the earth in its orbit. (At any other point of the planet's orbit except the node its apparent latitude would not be thus independent of the earth's place, but would vary according to its distance from the earth.) If, then, we observe the planet at two successive passages of the same node, the interval between the moments when the latitude becomes zero will be the planet's period, — *exactly*, if the node is stationary; very *approximately*, even if the node is not absolutely stationary, as none of the nodes actually are.

There are two difficulties with this method.

(*a*) In the case of Uranus the period is eighty-four years, and in that of Neptune 164 years — too long to wait.

(*b*) Since the orbits all cross the ecliptic at a very small angle, so that the latitude remains near zero for a number of days, it is extremely difficult to determine the precise minute and second when it is exactly zero; and slight errors in the declinations observed will produce great errors in the result.

514. Second. *By the mean synodic period of the planet.* The synodic period is the interval between two successive oppositions or conjunctions of the planet, the opposition being the moment when the planet's longitude differs from that of the sun by 180°.

This angle between the planet and sun cannot well be measured directly, but we can make with the meridian circle a series of observations both of the planet's right ascension and declination for several days before and after the date of opposition, and reduce the observations to latitude and longitude. The sun will be observed, of course, at noon, and the planet near midnight; but from the solar observations we can deduce the longitudes of the sun corresponding to the exact moments when the planet was observed. From these we find the difference of longitude between the planet and the sun at the time of each planetary observation; and finally from these differences

of longitude, we find the precise moment when that difference was exactly 180°, or the moment of opposition. This can be ascertained within a very few seconds of time if the observations are good.

Since the orbits are not strictly circular, the interval between two successive observations will not be the *mean* synodic period, but only an approximation to it; but when we know it *nearly*, we can compare oppositions many years apart, and by dividing the interval by the known number of entire synodic periods (which is easily determined when we know the approximate length of a single period) we get the mean synodic period very closely, — especially if the two oppositions occur at about the same time of the year. Having the synodic period, the true sidereal period at once follows from the equation

$$\frac{1}{P}=\frac{1}{E}-\frac{1}{S}.$$

515. To find the Distance of a Planet in Terms of the Earth's Distance. — When we know the planet's sidereal period, this is easily done by means of two observations of the planet's "*elongation*"

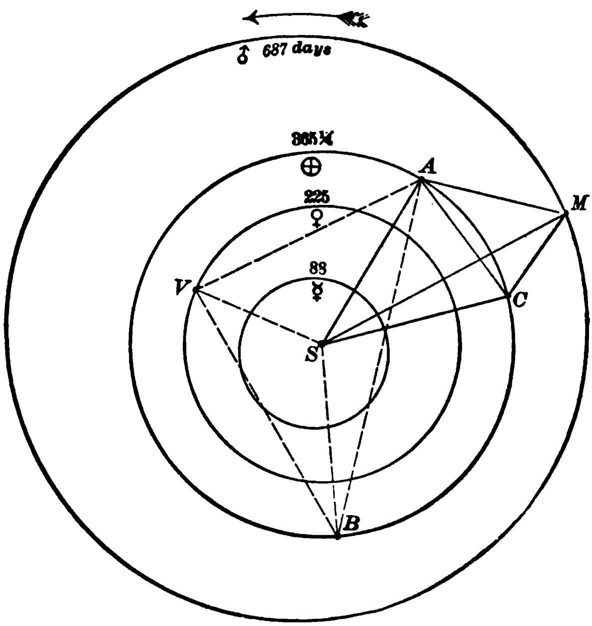

FIG. 166. — Determination of the Distance of a Planet from the Sun.

taken at an interval equal to its periodic time. The "elongation" of a planet is the difference between its longitude and that of the

sun, and a series of meridian-circle observations of sun and planet will furnish these differences of longitude for any selected moment included within the term of observation.

To find the distance of the planet Mars, for instance, we must therefore have two observations separated by an interval of 687 days. Suppose the earth to have been at A (Fig. 166) at the moment of the first observation. Then at the time of the second observation she will be at the point C, the angle ASC being that which the earth will describe in the next $43\frac{1}{2}$ days, which is the difference between two complete years (or $730\frac{1}{2}$ days) and the 687-day interval between the two observations.

The angles SCM and SAM are the "elongations" of the planet from the sun, and are given directly by the observations. The two sides SA and SC are also given, being the earth's distance from the sun at the dates of observation. Hence we can easily solve the quadrilateral, and find the length of SM, as well as the angle ASM.

This angle determines the planet's *heliocentric* longitude at M, since we know the direction of SA, the longitude of the earth at the time of observation.

The student can follow out for himself the process by which, from two elongations of Venus, SAV and SBV, observed at an interval of 225 days, the distance of Venus from the sun (or SV) can be obtained.

516. In order that this method may apply with strict accuracy it is necessary that at the moment of observation M should be in the same plane as A, S, and C; that is, at the node. If it is not so, the process will give us, not the true distance of the planet itself from the sun, but that of the "projection" of this distance on the plane of the ecliptic; *i.e.*, the distance from the sun to the point m (Fig. 167), where the perpendicular from the planet would strike that plane. But when we have determined Am and the angle mAM, the planet's geocentric latitude, we easily compute Mm; and from Sm and Mm we get the true distance SM and the heliocentric latitude of the planet MSm.

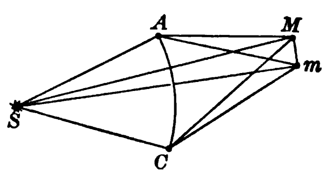

FIG. 167.

517. From a series of pairs of observations distributed around the planet's orbit it would evidently be possible to work out the orbit completely. It was in this way that Kepler showed that the orbits of the planets are ellipses, and deduced their distances from

the sun; and his third, or harmonic law, was then discovered simply by making a comparison between the distances thus found and the corresponding periods.

518. Mean Distance of an Inferior Planet by Means of Observations of its Greatest Elongation. — By observing from the earth the greatest elongation SEV (Fig. 168) of one of the inferior planets, its distance from the sun can very easily be deduced if we regard the orbit as a circle; for the triangle SVE will be right-angled at V, and $SV = SE \times \sin SEV$.

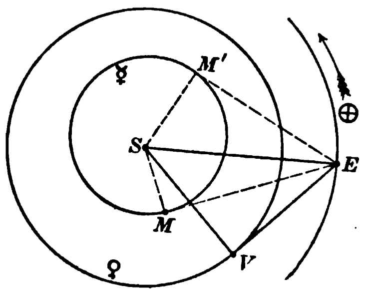

Fig. 168.

Distance of an Inferior Planet determined by Observations of its Greatest Elongation.

In the case of Venus the orbit is so nearly circular that the method answers very well, the greatest elongation never differing much from 47°. Mercury's orbit is so eccentric that the distance thus obtained from a single elongation might be very wide of the true mean distance. Since the greatest elongation, SEM, varies all the way from 18° to 28°, it would be necessary to observe a great many elongations, and take the average result.

519. Deduction of the Orbit of a Planet from Three Observations. — When one has command of a great number of observations of a planet running back many years, and can select such as are convenient for his purpose, as Kepler could from Tycho's records, it is comparatively easy to find the elements of a planet's orbit; but when a new planet is discovered, the case is different. The problem first arose practically in 1801, when Ceres, the first of the asteroids, was discovered by Piazzi in Sicily, observed for a few weeks and then lost in the sun's rays at conjunction, before other astronomers could be notified of the discovery, in those days of slow communication, made slower and more uncertain by war.

Gauss, then a young man at Göttingen, attacked the problem, and invented the method which, with slight modifications, is now universally used in such cases.

We do not propose to enter into details, but simply say that *three absolutely accurate observations of a planet's right ascension and declination are ordinarily sufficient to determine its orbit.* Three observations, made only as accurately as is now possible, with intervals of two or

three weeks between them, will give a very good *approximation* to the orbit; and it can then be corrected by further observations.

520. "Since there are five independent variables in the general equation (in space) of a conic having a given focus — the sun — it is necessary to have five conditions in order to determine them. Three are given by the observations themselves; viz., the *directions* of the body as seen from the earth at three given instants; a fourth is supplied by the 'law of equal areas,' since the sectors included between the three radii vectores must be proportional to the elapsed times; finally, the fifth is imposed by the requirement that the changes in the speed of the body must correspond to the variations in the length of the radius vector, in accordance with the known intensity of the sun's attraction."

(The student is referred to Gauss's "Theoria Motus," or to Watson's "Theoretical Astronomy," or to Oppolzer's great work on "The Determination of Orbits," for the full development of the subject.)

521. Planetary Perturbations. — The attraction of the planets for each other disturbs their otherwise elliptical motion around the sun. As in the case of the lunar theory the disturbing forces are, however, always relatively small, but not for the same reason. The sun's disturbing force is small because its *distance from the moon is nearly four hundred times that of the earth.* In the planetary theory the disturbing bodies are often nearer to the disturbed than is the sun itself, as, for instance, in the disturbance of Saturn by Jupiter at certain points of their orbits; but the *mass of the disturbing body in no case is as great as $\frac{1}{1000}$ part of the sun's mass*, and for this reason the disturbing force arising from planetary attraction is never more than a small fraction of the sun's attraction.

The greatest disturbing force which occurs in the planetary system (except in the case of some of the asteroids) is that of Jupiter on Saturn at the time when the planets are nearest: it then amounts to $\frac{1}{128}$ of the sun's attraction. When these two planets are most remote from each other, it amounts to $\frac{1}{357}$. There is no other case where the disturbing force is as much as $\frac{1}{1000}$ of the sun's attraction (again excepting the asteroids disturbed by Jupiter).

522. In any special case the disturbing force can be worked out on precisely the same principles that lie at the foundation of the diagram by which the sun's disturbing force upon the moon was found (Art. 441, Fig. 147); but the resulting diagram will look very different, because the disturbing body is relatively very near the disturbed orbit.

The planetary perturbations which result from the "integration" of the effects of the disturbing forces, *i.e.*, from their continual action through long intervals of time, divide themselves into two great classes, — the *Periodic* and the *Secular*.

523. Periodic Perturbations. — These are such as depend on the positions of the planets in their orbits, and usually run through their course in a few revolutions of the planets concerned. For the most part they are very small. Those of Mercury never amount to more than 15″, as seen from the sun. Those of Venus may reach about 30″, those of the earth about 1′, and those of Mars about 2′. The mutual disturbances between Jupiter and Saturn are much larger, amounting respectively to 28′ and 48′; while those of Uranus are again small, never exceeding 3′, and those of Neptune are not more than half as great as that. In the case of the asteroids, which are powerfully disturbed by Jupiter, the periodical perturbations are enormous, sometimes as much as 5° or 6°.

524. Long Inequalities. — The periodic inequalities of the planets are so small, because, as a rule, there is a nearly complete compensation effected at every few revolutions, so that the accelerations balance the retardations. The line of conjunction falls at random in different parts of the orbits, and when this is the case, no considerable displacement of either planet can take place. But when the periodic times of two planets are *nearly commensurable*, their line of conjunction will fall very near the same place in the two orbits for a considerable number of years, and the small unbalanced disturbance left over at each conjunction will then accumulate in the same direction for a long time. Thus, five revolutions of Jupiter roughly equal two of Saturn; and still more nearly, seventy-seven of Jupiter equal thirty-one of Saturn, in a period of 913 years. From this comes the so-called "*long inequality*" of Jupiter and Saturn, amounting to 28′ in the place of Jupiter and 48′ in that of Saturn, and requiring more than 900 years to complete its cycle. Between Uranus and Neptune there is a large inequality with a period of over 4000 years.

In the case of the earth and Venus there is a similar "long inequality" with a period of 235 years, amounting, however, to less than 3″ in the positions of either of the planets.

525. Secular Inequalities. — These are inequalities which depend not on the position of the planets in their orbits, but *on the relative position of the orbits* themselves, with reference to each other, — the way, for instance, in which the *lines of nodes* and *apsides* of two neighboring orbits lie with reference to each other. Since the planetary orbits change their positions very slowly, these perturbations,

although in the strict sense of the word periodic also, are very slow and majestic in their march, and the periods involved are such as stagger the imagination. They are reckoned in myriads and hundreds of thousands of years. From year to year they are insignificant, but with the lapse of time become important.

526. Secular Constancy of the Periods and Mean Distances. — It is a remarkable fact, demonstrated by Lagrange and La Place about 100 years ago, that *the mean distances and periods are entirely free from all such secular disturbance.* They are subject to slight *periodic* inequalities having periods of a few years, or even a few hundred years: but in the *long run* the two elements never change. They suffer no perturbations which depend on the position of the orbits themselves, but only such as depend on the positions of the planets in their orbits.

527. Revolution of the Nodes and Apsides. — The *nodes and perihelia*, on the other hand, move on continuously. The lines of apsides of all the planets (Venus alone excepted) *advance*, and the nodes of all without exception (except possibly some of the asteroids), *regress* on the ecliptic.

The quickest moving line of apsides — that of Saturn's orbit — completes its revolution in 67,000 years, while that of Neptune requires 540,000. The swiftest line of nodes is that of Uranus, which completes its circuit in less than 37,000 years, while the slowest — that of Mercury — requires 166,000 years.

528. The Inclinations of the Orbits. — These are all slowly changing — some increasing, and others decreasing; but as La Place and Leverrier have shown, all the changes are confined within narrow limits for all the larger planets: they oscillate (though not in regular periods), but the oscillations are never extensive. ✓

It is not certain that this is so with the asteroids, some of which have inclinations to the ecliptic of 25° and 30°: it is possible that some of *these* inclinations may change by a very considerable amount.

529. The Eccentricities. — These also are slowly changing in the same way as the inclinations, some increasing and some decreasing; and their changes also are closely restricted. The periods of the alternate increase and decrease are always many thousand years in length but, as in the case of the eccentricities, they are very irregular: there is no *isochronous*, pendulum-like swing such as many have imagined.

The asteroids are again to be excepted; the eccentricities of their orbits may change considerably.

530. Stability of the Planetary System. — About the end of the eighteenth century La Place and Lagrange succeeded in proving that *the mutual attraction* of the planets could never destroy the system, nor even change the elements of the orbit of any one of the larger planets to an extent which would greatly alter its physical condition.

The nodes and apsides revolve continuously, it is true, but that change is of no importance. The distances from the sun and the periods do not change at all in the long run; while the inclinations and eccentricities, as has just been said, confine their variations within narrow limits.

531. The "Invariable Plane" of the Solar System. — There is no reason, except the fact that *we* live on the earth, for taking the plane of the *earth's* orbit (the plane of the ecliptic) as the fundamental plane of the solar system. There is, however, in the system an "*invariable plane,*" the position of which remains forever unchanged by any mutual action among the planets, as was discovered by La Place in 1784. This plane is defined by the following conditions, — *that if from all the planets perpendiculars be drawn to it* (*i.e.*, to speak technically, if the planets be "projected" upon it), *and then if we multiply each planet's mass by the area which the planet's projected radius vector describes upon this plane in a unit of time, the sum of these products will be a maximum.* The ecliptic is inclined about 2° to this invariable plane, and has its ascending node nearly in longitude 286°.

532. La Place's Equations for the Inclinations and Eccentricities. — La Place demonstrated the two following equations, viz.:

$$(1)\ \Sigma\,(m\sqrt{a} \times e^2) = C. \qquad (2)\ \Sigma\,(m\sqrt{a} \times \tan^2 i) = C'.$$

Equation (1) may be thus translated: *Multiply the mass of each planet by the square root of the semi-major axis of its orbit, and by the square of its eccentricity; add these products for all the planets, and the sum will be a constant quantity C, which is very small.* It follows that no eccentricity can become very large, since e^2 *in the equation is essentially positive:* there can therefore be *no counterbalancing of positive and negative eccentricities;* and if the eccentricity of one planet increases, that of some other planet or planets must correspondingly decrease.

The second equation is the same, merely substituting $\tan^2 i$ for e^2, i being *the inclination of the planet's orbit to the invariable plane.*

The constant in this case also is small, though of course not the same as in the preceding equation.

533. Work of Poincaré. — This has recently given a new aspect to the question of the stability of the system. Poincaré has shown that the assumptions as to the convergence of the series used in previous calculations are unwarranted, and that therefore the conclusions reached are unsound. It is no longer absolutely certain that gravitational perturbations may not ultimately prove destructive in some remote future. There are also other conceivable destructive forces, — the action of a resisting medium, for instance, or the entrance into the system of great bodies from outer space.

Exercises on Chapter XIV.

1. What is the mean daily gain of the earth on Mars as seen from the sun, *i.e.*, the synodic motion of Mars, assuming their sidereal periods as 365.25 days for the earth, and 687 days for Mars. = 27'.7

2. Find the synodic period of Venus, her sidereal period being 225 days. (See Art. 490.) = 586

3. Given the synodic period of a planet as three years, what is its sidereal period?

Ans. { Three-quarters of a year, or
One and a half years.

4. Given a synodic period of four years, find the sidereal period.

5. What would be the sidereal period of a planet which had its synodic period equal to the sidereal? *Ans.* Two years.

6. Within what limits of distance from the sun must lie all planets having synodic periods longer than two years? (Apply Kepler's third law after finding the sidereal periods that would give a synodic period of two years.)

Ans. { 0.763 Astron. units, or 70 895000 miles, and
1.588 " " " 147 500000 miles.

7. A brilliant starlike object was seen about 7 P.M. on April 1 exactly at the east point of the horizon. Could it have been a ~~real star or one of~~ the planets? If not, why not?

8. Mercury was at inferior conjunction on Feb. 8, 1896, at 1 P.M. On May 6, at fifteen minutes after noon (exactly one sidereal period later), its elongation from the sun was observed to be 18° 50′ East. Find the distance of the planet from the sun at that time in Astron. units, the earth's orbit being regarded as circular. (See Art. 515.)

The fact that the first observation was made at conjunction greatly simplifies the calculation.

Ans. { Distance from the sun = 0.335 Astron. units.
The planet was near perihelion.

CHAPTER XV.

THE PLANETS: METHODS OF FINDING THEIR DIAMETERS, MASSES, ETC. — THE "TERRESTRIAL PLANETS" AND ASTEROIDS. — INTRA-MERCURIAL PLANETS AND THE ZODIACAL LIGHT.

IN discussing the individual peculiarities of the planets, we have to consider a multitude of different data; for instance, their *diameters*, their *masses*, and *densities*, their *axial rotation*, their *surface-markings*, their *reflecting power* or "*albedo*," and their *satellite systems*.

534. Diameter. — The apparent diameter of a planet is ascertained by measurement with some kind of micrometer (Art. 73). For this purpose the "double-image" micrometer has an advantage over the wire micrometer because of the effect of irradiation, and by the fact that in measuring, the observer's attention is concentrated upon a single point instead of being directed to two.

When we bring two wires to touch the two limbs of the planet, Fig. 169, *a*, the bright image of the planet is always measured too large, because every bright object appears somewhat extended by its physiological action upon the retina of the eye. This is known as *irradiation* — well exemplified at the time of new moon, when the bright crescent appears to be much larger than the "old moon" faintly visible by earth-shine. With small instruments this error is often considerable, but it may be reduced to some extent by using a sufficiently bright illumination of the field of view.

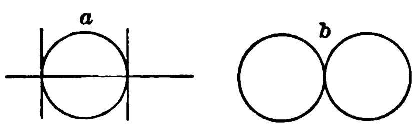

FIG. 169.
Micrometer Measures of a Planet's Diameter.

With the double-image micrometer, the observer in measuring has to bring in contact two discs of equal brightness, as in Fig. 169, *b*; and in this case the irradiation almost vanishes at the point of contact.

The diameter thus measured is, of course, only the *apparent* diameter, to be expressed in seconds of arc, and varies with every change of distance. To get the real diameter in linear units, we have

$$\text{Real diameter} = \frac{\Delta \times D''}{206265},$$

in which Δ is the distance of the planet from the earth, and D'' the diameter in seconds of arc. If Δ is given only in astronomical units, the diameter comes out, of course, in terms of that unit. To get the diameter in miles, we must multiply by the value of this unit in miles; that is, by the sun's distance from the earth.

535. Extent of Surface and Volume. — Having the diameter, the *surface*, of course, is proportional to its *square*, and is equal to the earth's surface multiplied by $\left(\frac{s}{\rho}\right)^2$, in which s is the semi-diameter of the planet and ρ that of the earth.

The *volume* equals $\left(\frac{s}{\rho}\right)^3$ in terms of the earth's volume. (The student must be on his guard against confounding the *volume* or *bulk* of a planet with its *mass*.)

The nearer the planet, other things being equal, the more accurately the above data can be determined. The error of $0''.1$ in measuring the apparent diameter of Venus, when nearest, counts for less than thirteen miles in the real diameter of the planet; while in Neptune's case it would correspond to more than 1300 miles. The student must not be surprised, therefore, at finding considerable discrepancies in the data given for the remoter planets by different authorities.

536. Mass of a Planet which has a Satellite. — In this case its mass is easily and accurately found by observing the period and distance of the satellite. We have the fundamental equation

$$(M+m)=\left(\frac{4\pi^2}{G}\right)\left(\frac{r^3}{t^2}\right),$$

in which M is the mass of the planet, m that of its satellite, r the radius of the orbit of the satellite, and t its period.

The formula is derived as follows: From the law of gravitation the accelerating force which acts on the satellite is given by the equation

$$f = G\frac{M+m}{r^2}$$

(Art. 417), in which M is the mass of the planet and m that of the satellite. From the law of circular motion (Art. 411, Eq. b) we have

$$f = 4\pi^2\left(\frac{r}{t^2}\right);$$

whence (equating the two values of f) we have

$$\frac{M+m}{r^2} = \left(\frac{4\pi^2}{G}\right)\left(\frac{r}{t^2}\right);$$

and finally

$$(M+m) = \left(\frac{4\pi^2}{G}\right)\left(\frac{r^3}{t^2}\right).$$

This demonstration is strictly good only for circular orbits; but the equation is equally true, and can be proved, for elliptical orbits, if for r we put a, the semi-major axis of the satellite's orbit.

For many purposes a proportion is more convenient than this equation, since the equation requires that M, m, r, and t be expressed in properly chosen units in order that it may be numerically true. Converting the equation into a proportion, we have

$$(M+m) : (M_1+m_1) = \frac{r^3}{t^2} : \frac{r_1^3}{t_1^2};$$

or, in words, *the united mass of a body and its satellite is to the united mass of a second body and its satellite as the cube of the distance of the first satellite divided by the square of its period is to the cube of the distance of the second satellite divided by the square of its period.* This enables us at once to compare the masses of any two bodies which have attendants revolving around them.

The mass of the moon is so considerable as compared with that of the earth (about $\frac{1}{80}$) that it will not do to neglect it; but in all other cases the satellite is less than $\frac{1}{1000}$ of the mass of its primary, and need not be taken into account.

537. Examples. — (1) Required the mass of the *sun* compared with that of the *earth*. The proportion is

$$(S + \text{earth}) : (E + \text{moon}) = \frac{(93000000)^3}{(365\frac{1}{4})^2} : \frac{(239000)^3}{(27.4)^2}.$$

The quantities in the last term of the proportion are of course the distance and period of *the moon*; and it is to be remembered that for the period of the moon we must use, not the *actual* sidereal period, but the period *as it would be if the moon's motion were undisturbed*, — a period about an hour shorter.

(2) Compare the mass of the earth with that of Jupiter, whose remotest satellite has a period of 16¾ days, and a distance of 1,160,000 miles. We have

$$(E + \text{moon}) : (J + \text{satellite}) = \frac{(239000)^3}{(27.4)^2} : \frac{1167000^3}{16.7^2},$$

which gives the mass of Jupiter about 316 times as great as that of the earth and moon together, or 318 times the mass of the earth alone.

538. It is customary to express the mass of a planet as a certain fraction of the sun's mass, and the proportion is simply

$$\text{Sun} : \text{Planet} = \frac{R^3}{T^2} : \frac{r^3}{t^2},$$

whence $$\text{Planet's mass} = \text{Sun's mass} \times \left(\frac{r}{R}\right)^3 \times \left(\frac{T}{t}\right)^2,$$

where T and R are the *planet's* period and distance from the sun. Since R and r can both be determined in astronomical units without any necessity for knowing the length of that unit in miles, the *masses of the planets in terms of the sun's mass* are independent of any knowledge of the solar parallax. But to compare them with the *earth*, we must know this parallax, since the moon's distance from the earth, which enters into the equations, is found by observation in *miles* or in *radii of the earth*, and not in astronomical units.

In order to make use of the satellites for this purpose we must determine by micrometrical observations their distances from the planets and their periods.

539. Mass of a Planet which has no Satellite. — When a planet has not a satellite, the determination of its mass is a very difficult and troublesome problem, and can be solved only by finding some perturbation produced by the planet, and then ascertaining, by a sort of "trial and error" method, the mass which would produce that perturbation. Venus disturbs the earth and Mercury, and from these perturbations her mass is ascertained. Mercury disturbs Venus, and also one or two comets which come near him, and in this way we get a rather rough determination of his mass.

540. Density. — The density of a body as compared with the earth is determined simply by dividing its mass by its volume; *i.e.*,

$$\text{Density} = \frac{m}{\left(\frac{s}{\rho}\right)^3}.$$

For example, Jupiter's *diameter* is about eleven times that of the earth (*i.e.* $\left(\frac{s}{\rho}\right) = 11$), so that his *volume* is 11^3, or 1331 times the earth's. His *mass*, derived from satellite observations, is about 316 times the earth's. The

density, therefore, equals $\frac{316}{1331}$, or about 0.24, of the earth's density, or about $1\frac{1}{3}$ times that of water, the earth's density being 5.58 (Art. 171).

541. The Surface Gravity. — The force of gravity on a planet's surface as compared with that on the surface of the earth is important in giving us an idea of its physical condition. If r is the radius of the planet in terms of the earth's radius, then

$$\text{Surface gravity, or } \gamma, = \frac{m}{\left(\frac{s}{\rho}\right)^2}, = \frac{m}{\left(\frac{s}{\rho}\right)^3} \times \left(\frac{s}{\rho}\right),$$

i.e., it equals the planet's *density*, multiplied by its *diameter* expressed in terms of the earth's diameter.

$$\text{For Jupiter, therefore, } \gamma = \frac{316}{11^2} = 11 \times \text{density} = 11 \times 0.24 = 2.64 \text{ nearly.}$$

That is, a body at Jupiter would weigh 2.6 times as much as at the earth's surface.

542. The Planet's Oblateness. — The "oblateness" or "polar-compression" is the *difference* between the equatorial and polar diameters divided by the equatorial diameter. It is, of course, determined, when it is possible to determine it at all, simply by micrometric measurements of the difference between the greatest and least diameters. The quantity is always very small and the observations delicate.

543. The Time of Rotation, when it can be determined, is found by observing the passage of some spot visible in the telescope across the central line of the planet's disc. In reducing the observations they must be corrected for changes in the planet's direction from the earth, and also for variations of distance which affect the time in which light reaches us. In some cases the rotation period has been determined by observation of regular changes in the planet's brightness.

544. The Inclination of the Axis is deduced from the same observations which are used in obtaining the rotation-period. It is necessary to determine with the micrometer the paths described by different spots as they move across the planet's disc. It is possible to ascertain it with accuracy for only a very few of the planets: Mars, Jupiter, and Saturn are the only ones that furnish the needed data.

545. The Surface Peculiarities and Topography of the surface are studied by the telescope. The observer makes drawings of any

markings which he may see, and by their comparison is at last able to discriminate between what is temporary and what is permanent on the planet.[1] Mars alone, thus far, permits us to make a map of its surface.

546. Spectroscopic Peculiarities and Albedo. — The characteristics of the planet's atmosphere can be to some extent studied by means of the spectroscope, which in some few cases shows the presence of water-vapor and other absorbing media, by dark bands in the planet's spectrum. The "*albedo*," or reflecting power of a planet's surface is determined by photometric observations, comparing it with a real or artificial star, or with some other planet.

547. The Satellite System of a Planet. — The principal data to be ascertained are the *distances* and *periods* of the satellites, and the observations are made by measuring the *apparent* distances and directions of the satellites from the centre of the planet with the wire micrometer (Art. 73). Observations made at the times when the satellite is near its elongation are especially valuable in determining the distance.

If the planet and earth were at rest, the satellite's path would appear to be an ellipse, unaltered in dimensions during the whole series of observations; but since the earth and planet are both moving, it becomes a complicated problem to determine the satellite's true orbit from the *ensemble* of observations.

548. With the exception of the moon and the outer satellite of Saturn, all the satellites of the planetary system move almost exactly in the plane of the equator of the primary; and all but the moon and the seventh satellite of Saturn (Hyperion) move in orbits almost circular. La Place and Tisserand have shown that the equatorial protuberance of a planet compels any satellite which is not very remote from its primary to move nearly in the equatorial plane, but the almost perfect circularity of the orbits is not yet explained. When there are a number of satellites in a system, interesting problems arise in connection with their mutual disturbances; and in a few cases it becomes possible to determine a satellite's mass as compared with that of its primary. In several instances satellites show peculiar variations in their brightness, which are supposed to indicate that they make an axial rotation in the time of one revolution around the primary, in the same way as our moon does.

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[1] Photography is beginning to be applied, and with some success, at the Lick Observatory, in the case of Jupiter.

549. Humboldt's Classification of the Planets. — Humboldt has divided the planets into two groups: the *terrestrial* planets, so-called, and the *major* planets. The terrestrial planets are Mercury, Venus, the earth, and Mars. They are bodies of the same order of magnitude, ranging from 3000 to 8000 miles in diameter, not very different in density (the earth being the largest and probably the densest of them), and are probably roughly alike in physical constitution, and covered with water and air. But we hasten to say that the differences in the amount of heat and light which they receive from the sun, and in the force of gravity upon their surfaces, and probably in the density of their atmospheres, are such as to bar any positive conclusions as to their being the abode of life resembling the forms of life with which we are acquainted on the earth.

550. The four major planets, Jupiter, Saturn, Uranus, and Neptune, are much larger bodies (ranging in diameter between 30,000 and 90,000 miles), are much less dense, and so far as we can make

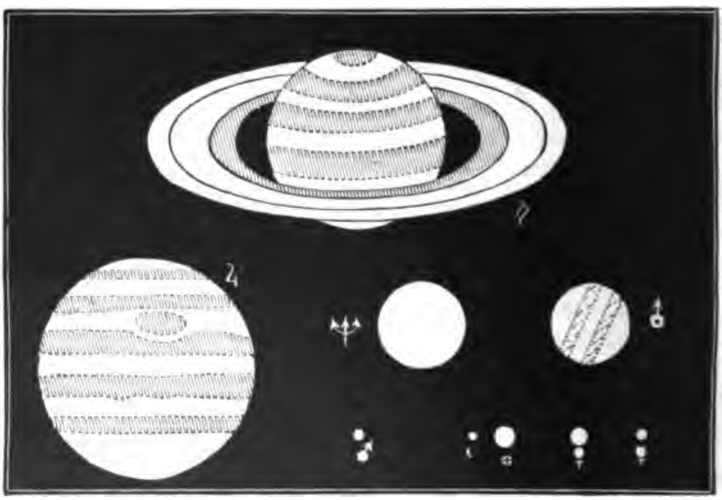

FIG. 170.—Relative Sizes of the Planets.

out, present to us only a surface of cloud, and may not have anything solid about them. There are some reasons for suspecting that they are at a high temperature; in fact, that Jupiter is a sort of *semi-sun;* but this is by no means yet certain.

As for the multitudinous asteroids, the probability is that they represent a single planet of the terrestrial group which, as has been intimated, failed for some reason in its evolution, or else has

been broken to pieces. All of them united would not make a planet one-fourth the mass of the earth.

Fig. 170 shows the relative sizes of the different planets.

In what follows, all the numerical data, so far as they depend on the solar parallax, are determined on the assumption that that parallax is 8″.80, and that the sun's mean distance is 92,897000 miles.

MERCURY.

551. There is no record of the discovery of the planet. It has been known from remote antiquity; and we have recorded *observations* running back to B.C. 264.

For a time the ancient astronomers seem to have failed to recognize it as the same body on the eastern and western sides of the sun, so that the Greeks had for a time two names for it, — Apollo when it was morning star, and Mercury when it was evening star. According to Arago, the Egyptians called it Set and Horus, and the Hindoos also gave it two names.

It is so near the sun that it is comparatively seldom seen with the naked eye; but when near its greatest elongation it is easily enough visible as a brilliant star of the first magnitude low down in the twilight, perhaps not quite so bright as Sirius, but certainly brighter than Arcturus. It is usually visible for about a fortnight at each elongation, and is best seen in the evening at such eastern elongations as occur in March and April. In Northern Europe it is much more difficult to observe than in lower latitudes, and Copernicus is said never to have seen it. Tycho, however, obtained a considerable number of observations. For the most part, of course, observations upon it are made in the daytime.

552. It is exceptional in the solar system in a great variety of ways. It is the *nearest* planet to the sun, *receives the most light and heat*, is *the swiftest* in its movement, and (excepting some of the asteroids) *has the most eccentric orbit*, with the *greatest inclination* to the ecliptic. It is also the *smallest* in diameter and has the least mass, asteroids again excepted.

553. Distance, Light, and Heat. — Its *mean distance* from the sun is 36,000000 miles, but the eccentricity of its orbit is so great (0.205), that the sun is seven and one-half millions of miles out of its centre, and the actual distance of the planet from the sun ranges all the way from 28,500000 to 43,500000, while its velocity in its orbit

varies from thirty-five miles a second at perihelion to only twenty-three at aphelion. On the average it receives 6.7 times as much light and heat as the earth; but the heat received at perihelion is to that at aphelion in the ratio of 9 to 4. For this reason there must be two seasons in its year due to the changing distance, even if the equator of the planet is parallel to the plane of its orbit, which would preclude seasons like our own. If the planet's equator is inclined at an angle like the earth's, then the seasons must be very complicated.

554. Period. — The *sidereal period* is very nearly 88 days, and the *synodic period*, or the time from conjunction to conjunction again, is about 116 days. The greatest elongation ranges from 18° to 28°, and occurs about twenty-two days before and after the inferior conjunction, or about thirty-six days before and after the superior conjunction. The planet's arc of retrogression is about 12° (considerably variable), and the stationary point is very near the greatest elongation.

555. Inclination. — The inclination of the orbit to the ecliptic is about 7°, but the greatest geocentric latitude (that is, the planet's greatest distance from the ecliptic as seen from the earth) is never quite so great.

556. Diameter, Surface, and Volume. — *The apparent diameter* ranges from 5″ to about 13″, according to its distance from us; the least distance from the earth being about 57,000000 miles (93 − 36), while the greatest is about 129,000000 (93 + 36). The *real diameter* is very near 3000 miles, not differing from that more than fifty miles either way. It is not easy to measure, and the "probable error" is perhaps rather larger than would have been expected. With this diameter, its *surface* is $\frac{1}{7}$ of the earth's, and its *volume* $\frac{1}{18.5}$.

557. Mass, Density, and Surface Gravity.—Its mass is very difficult to determine, since it has no satellite, and the values obtained by La Place, Encke, Leverrier, and others, range all the way from $\frac{1}{5}$ of the earth's mass to $\frac{1}{30}$. The planet is so small and so near the sun that its effect in disturbing the other planets is very slight, and the "probable error" of the mass determined from these perturbations is correspondingly large.

In his recent work upon the "Fundamental Elements of Astronomy," Newcomb settles upon a value of $\frac{1}{8944000}$ of the sun's mass, or $\frac{1}{21}$ of the earth's. Harkness gets $\frac{1}{25}$ of the earth's. Assuming Newcomb's value, the density of Mercury comes out about seven-eighths that of the earth; and its surface gravity a little less than *one-third.* If we take Harkness' figures the density is only 0.72, and its superficial gravity, 0.27. But none of the results thus far obtained are to be regarded as more than rough approximations to the truth. The data are not sufficient to furnish accurate determinations.

558. Its *Albedo,* or reflecting power, as determined by Zöllner is very low — only 0.13, somewhat inferior to that of the moon.

In 1878 Mr. Nasmyth observed the planet in the same field of view with Venus; and although Mercury was then not much more than half as far from the sun as Venus, and therefore four times as brightly illuminated, it appeared to be less luminous in the telescope. "Venus was like silver, Mercury like zinc or lead."

In the proportion of light given out at its different phases, it behaves like the moon, flashing out strongly near the full, as if it had a surface of the same rough structure as that of our satellite. Like the moon and Mars also, but in contrast with Jupiter, the illuminated edge of its disc is always brighter than the centre.

559. Telescopic Appearance and Phases. — Seen by the telescope, the planet looks like a little moon, showing phases precisely similar to those of our satellite. At inferior conjunction the dark side is towards us; at superior conjunction the illuminated surface. At

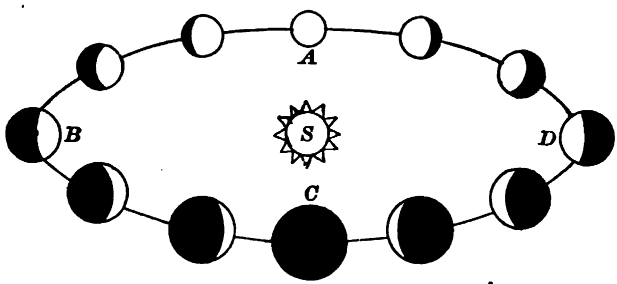

FIG. 171. — Phases of Mercury and Venus.

the greatest elongation it appears like a half-moon. Between superior conjunction and greatest elongation it is gibbous, while between inferior conjunctions and the elongations it shows the crescent phase. Fig. 171 illustrates the phases of both Mercury and Venus.

As a rule Mercury is so near the sun that it can be observed only in the daytime, but with proper precautions in screening the object glass of the telescope from direct sunlight, the observation is not very difficult. The surface presents very little of interest, there being no well-defined markings, though there are sometimes indistinct shadings which perhaps indicate permanent geographical features. It has been attempted to deduce from these the length of the planet's day, and many years ago Schroeter, a German astronomer, a contemporary of the elder Herschel, obtained $24^h\ 05^m$ as a result, which until recently remained practically uncontradicted, though also unconfirmed. In 1889, however, Schiaparelli, the Italian astronomer, announced that he had ascertained that the markings do not move sensibly upon the planet's disc, in the course of several hours even, and therefore, that the time of rotation must be much longer than a day, and he finds as a result the remarkable fact that *the planet rotates on its axis only once during its orbital period of 88 days; and keeps the same face always turned towards the sun*, behaving in this respect, just as the moon does towards the earth. Owing to the eccentricity of the orbit, however, the planet has a large 'libration' (Art. 250), amounting to nearly $23\frac{1}{2}°$ on each side of the mean position; *i.e.*, seen from a favorable position on the planet's surface, the sun instead of rising and setting daily, as with us, would oscillate about 47° back and forth in the sky, every 88 days. This asserted discovery is extremely important, and has excited great interest. There is little doubt that it is correct; but the necessary observations are very difficult, and the only direct confirmation thus far is from the Flagstaff observations of Lowell in 1896, who gets a result perfectly agreeing with that of Schiaparelli.

560. Atmosphere. — The evidence upon this subject is not conclusive. Its atmosphere, if it has one, must, however, be much less dense than that of Venus. No ring of light is seen surrounding the disc of the planet when it enters the limb of the sun at the time of a transit, while in the case of Venus such a ring, due to the atmospheric refraction, is very conspicuous. On the other hand, Huggins and Vogel, who have examined the spectrum of the planet, report that certain lines in the spectrum, due to the presence of water-vapor, were decidedly stronger than in the spectrum of the air (illuminated by sunshine), which formed the background for the planet, making it probable that it has an atmosphere containing water-vapor like the atmosphere of the earth, but probably less extensive and dense.

561. Transits. — Usually at the time of inferior conjunction the planet passes north or south of the sun, the inclination of its orbit being 7°; but if the conjunction occurs when the planet is very near its node, it will cross the disc of the sun and be visible upon it as a small black spot — not, however, large enough to be seen without a telescope, as Venus can under similar circumstances.

At this time we have the best opportunity for measuring the diameter of the planet; but unless special precautions are taken, the measured diameter under these circumstances is likely to be *too small*, on account of the irradiation of the surrounding background, which encroaches upon the planet's disc.

Since the planet's nodes are in longitudes 227° and 47°, and are passed by the earth on May 7 and November 9, the transits can occur only near those days. If the orbit of the planet were strictly circular, the "transit limit" (corresponding to an ecliptic limit) would be 2° 10′; but at the May transits the planet is near its aphelion and much nearer the earth than ordinarily, so that the limit is diminished, while the November limit is correspondingly increased. The May transits are in fact less than half as numerous as the November transits.

562. Interval between Transits. — Twenty-two synodic periods of Mercury are pretty nearly equal to 7 years; 41 still more nearly equal 13 years; and 145 almost exactly equal 46 years. Hence, after a November transit, a second one is possible in 7 years, probable in 13 years, and practically certain in 46. For the May transits the repetition after 7 years is not possible, and it often fails in 13 years.

The first transit of Mercury ever observed was by Gassendi, Nov. 7, 1631.

The last transit (visible in the U. S.) occurred on Nov. 7, 1894.

The following list gives the transits of the coming century. An asterisk denotes that the whole transit will be visible in the U. S.; a dagger, that a part of it can be seen.

†1907, Nov. 12;	†1914, Nov. 6;	*1924, May 7;
1927, Nov. 8;	1937, May 10;	1940, Nov. 12;
*1953, Nov. 13;	*1960, Nov. 6;	†1970, May 9;
†1973, Nov. 9;	1986, Nov. 12;	1999, Nov. 14.

The transits of Mercury are of no particular astronomical importance, except as giving accurate determinations of the planet's place, by means of which its orbit can be determined. Newcomb has also recently made an investigation of all the recorded transits, for the purpose of testing the uniformity of the earth's rotation. They seem to indicate certain small irregularities in that motion, but hardly make the fact certain.

VENUS.

563. The next planet in order from the sun is Venus, the brightest and most conspicuous of all; the earth's twin sister in magnitude, density, and general constitution, if not also in age, as to which we have no knowledge. Like Mercury, it had two names among the Greeks, — *Phosphorus* as morning star, and *Hesperus* as evening star. It is so brilliant that it is easily seen by the naked eye in the day-time for several weeks when near its greatest elongation; sometimes it is bright enough to catch the eye at once, but usually it is seen by daylight only when one knows precisely where to look for it.

(There is no good reason to suppose that it is the "*Star of Bethlehem*," though some have imagined this to be the case.)

564. Distance, Period, and Inclination of Orbit. — Its mean distance from the sun is 67 200000 miles. The eccentricity of the orbit is the smallest in the planetary system (only 0.007), so that the greatest and least distances of the planet from the sun differ from the mean only 470000 miles each way. Its orbital velocity is twenty-two miles per second.

Its *sidereal period* is 225 days, or seven and one-half months, and its *synodic period* 584 days — a year and seven months. From superior conjunction to elongation on either side is 220 days, while from inferior conjunction to elongation is only 71 or 72 days. The arc of retrogression is 16°.

The inclination of its orbit is only $3\frac{1}{2}°$.

565. Diameter, Surface, and Volume. — The apparent diameter ranges from 67″ at the time of inferior conjunction to only 11″ at the superior. This great difference depends, of course, upon the enormous change in the distance of the planet from the earth. At inferior conjunction the planet is only 26 000000 miles from us (93 — 67). No other body ever comes so near the earth except the moon, and occasionally a comet. Its greatest distance at superior conjunction is 160 000000 miles (93 + 67), so that the ratio between the greatest distance and the least is more than 6 to 1.

The *real diameter* of the planet is 7700 (± 30) miles. Its *surface*, as compared with that of the earth, is ninety-five per cent; its *volume* ninety-two per cent.

566. Mass, Density, and Gravity. — By means of the perturbations she produces, the *mass* of Venus is found, according to Newcomb, to be about eighty-two per cent of the earth's; hence her *density* is eighty-eight per cent, and her *superficial gravity* eighty-five per cent of the earth's.

567. Phases. — The telescopic appearance of the planet is striking on account of her great brilliance. When about midway between greatest elongation and inferior conjunction she has an apparent diameter of 40″, so that, with a magnifying power of only forty-five, she looks exactly like the moon four days old, and of precisely the same apparent size.

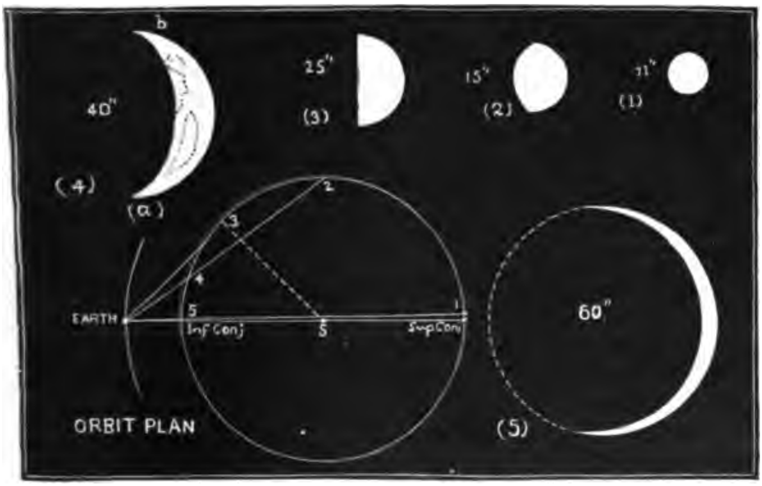

FIG. 172. — Telescopic Appearances of Venus.

Very few persons, however, would think so on their first view through the telescope, for a novice always underrates the apparent size of a telescopic object: he instinctively adjusts his focus as if looking at a picture only a few inches away, instead of projecting the object visually into the sky.

According to the theory of Ptolemy, Venus could never show us more than half her illuminated surface, since according to his hypothesis she was *always between us and the supposed orbit of the sun* (Art. 500). Accordingly, when in 1610 Galileo discovered that she exhibited the gibbous phase as well as the crescent, it was a strong argument for Copernicus. Galileo announced his discovery in a curious way, by publishing the anagram, —

"Haec immatura a me jam frustra leguntur; o. y."

Some months later he furnished the translation,—

"Cynthiæ figuras æmulatur Mater Amorum,"

which is formed by merely transposing the letters of the anagram. His object was to prevent any one from claiming to have anticipated him in this discovery, as had been done with respect to his discovery of the sun spots.

Fig. 172 represents the disc of the planet as seen at four points in its orbit. 1, 3, and 5 are taken at superior conjunction, greatest elongation, and near inferior conjunction respectively, while 2 and 4 are at intermediate points.

568. Maximum Brightness. — The planet attains its maximum brilliance thirty-six days before and after inferior conjunction, at a distance of about 38° or 39° from the sun, when its phase is like that of the moon about five days old. It then casts a strong shadow, and, as has already been said, is easily visible by day with the naked eye.

569. Surface Markings. — These are not at all conspicuous. Near the limb of the planet, which is always much brighter than the central parts (as is also the case with Mercury and Mars), they can never be well seen, although sometimes when Venus was in the crescent phase, intensely bright spots have been reported near the cusps, as at *a* and *b* in No. 4, Fig. 172. These may perhaps be *ice-caps* like those which are seen on Mars. Near the "terminator," which is less brilliant and less sharply defined than the limb, irregular darkish shadings are sometimes seen, such as are indicated by the dotted lines in the figures, but without any distinct outline. They may be continents and oceans dimly visible, or they may be mere atmospheric objects; observations do not yet decide.

Mr. Lowell, however, alone among observers thus far, describes a very different aspect according to his Flagstaff studies in 1896. He makes out an obvious and permanent system of markings, consisting of rather narrow dark streaks, nearly straight, radiating in a spoke-like manner from a sort of "hub" near the centre of the planet's disc. They seemed to him to be quite sharp in outline, but *dim*, as if seen through a luminous atmosphere of considerable depth. He even goes so far as to offer a map of the planet, with names appended to some of the principal features. He attributes his success, not so much to the power of his twenty-four inch telescope (one of the last and most perfect of Clark's productions), as to the excellence of the atmospheric conditions at Flagstaff. He maintains, very reasonably it seems, that in the observation of objects like the finer markings on the discs of Mercury, Venus, and Mars, *steadiness of the image* is even more important than telescopic power or acuteness of vision. At the same time the fact that the markings on Venus are seen only with low magnifying power is

suspicious, and it remains to be seen whether his results will be confirmed and accepted.

570. Rotation of the Planet. — The rotation-period of the planet is still a subject of dispute. Schroeter, from his observations of shadings noted upon its surface, deduced a "day" of $23^h\ 21^m$, and some more recent observers support his conclusion. On the other hand Schiaparelli, while he does not profess to have yet determined the period with precision, considers that his observations disprove Schroeter's result, and show that the rotation-period must be long, and probably 225 days, identical with the planet's orbital period, as in the case of Mercury. Mr. Lowell's observations of 1896 confirm this conclusion, and are indeed decisive if they are accepted as correct.

It is not unlikely that the spectroscope may contribute to the final settlement of the question: if the rotation is rapid the dark lines in the spectrum must be displaced at the edges of the planet's disc (Art. 321, note) by an amount that can be measured.[1]

De Vico, fifty years ago, concluded that the planet's equator makes an angle of 54° with the plane of its orbit, and the statement is still found in many text-books, though it is probably incorrect. If the bright spots referred to in Art. 569 are really "polar caps" the inclination must be small.

No sensible difference has been ascertained between the different diameters of the planet, a fact which favors Schiaparelli's rotation-period. If it were really as much flattened at the poles as the earth is, there should be a measurable difference of 0″.2 between the polar and equatorial diameters.

571. Mountains. — From certain irregularities occasionally observed upon the terminator, and especially from the peculiar blunted form of one of the cusps of the crescent, various observers have concluded that there are numerous high mountains upon the surface of the planet. Schroeter assigned to some of those near the southern pole the extravagant altitude of twenty-five or thirty miles, but the evidence is entirely insufficient to warrant any confidence in the conclusion.

572. Albedo. — According to Zöllner the *Albedo* of the planet is 0.50, which is about three times that of the moon, and almost four times that of Mercury. It is, however, exceeded by the reflecting power of the surfaces of Jupiter and Uranus, while that of Saturn

[1] Up to 1904 the results on the whole rather favor the long period, but are hardly conclusive.

appears to be about the same. This high reflecting power probably indicates that the surface is mostly covered with cloud, as few rocks or soils could match it in brightness. Lowell, however, denies the existence of anything like a nearly continuous cloud veil such as has been generally supposed.

Contribution from the Mt Wilson Observatory, No. 249.

573. Evidences of Atmosphere. — When the planet is near the sun, the horns of the crescent extend notably beyond the diameter, and when very near the sun, a thin line of light has been seen by several observers, especially Professor Lyman of New Haven, to complete the whole circumference. This is due to refraction of sunlight by the planet's atmosphere, a phenomenon still better seen as the planet is entering upon the sun's disc at a transit, when the black disc is surrounded by a beautiful ring of light. From the observations of the transit of 1874, Watson concluded that the planet's atmosphere must have a depth of about fifty-five miles, that of the earth being usually reckoned at forty miles. Later observations,[1] however, indicate an atmosphere less extensive than our own, and that the luminous twilight ring is due rather to *diffuse reflection* than to refraction. The planet's spectrum usually shows the lines of water vapor, but they may be telluric (Art. 314).

Lights on Dark Portion. — Many observers have also reported faint lights as visible at times on the dark portion of the planet's disc. These cannot be accounted for by reflection, but must originate on the planet's surface; they recall the Aurora Borealis and other electrical manifestations on the earth.

574. Satellites. — No satellite is known, although in the last century a number of observers at various times thought they had found one.

In most cases they observed small stars near the planet, which we can now identify by computing the place occupied by the planet at the date of observation. It is not, however, *impossible* that the planet may have some very minute and near attendants like those of Mars, which may yet be brought to light by means of the great telescopes of the future, or by photography. Of course the extreme brilliance of the planet, and the fact that the necessary observations can be made only in strong twilight, render the discovery of such objects, if they exist, very difficult.

575. Transits. — Occasionally Venus passes between the earth and the sun at inferior conjunction, giving us a so-called "*transit.*"

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[1] See note on page 377.

She is then visible (even to the naked eye) as a black spot on the disc, crossing it from east to west.

As the inclination of the planet's orbit is nearly $3\frac{1}{2}°$, the "*transit limit*" is small (about 4°), and the transits are therefore very rare phenomena. The sun passes the nodes of the orbit on June 5 and December 7, so that all transits must occur on or near those dates. When Venus crosses the sun's disc *centrally*, the duration of the transit is about *eight hours*. Taking the mean diameter of the sun as 32′, or $\frac{1}{675}$ of a circumference, and the planet's synodic period as 584 days, the geocentric duration of a central transit should be $\frac{277}{723}$[1] $\times \frac{1}{675} \times 584^d$, which equals 0.332 days, or $7^h\ 58^m$.

576. Recurrence of Transits. — Five synodic, or thirteen sidereal, revolutions of Venus are very nearly equal to eight years, the difference being only a little more than one day; and still more nearly, in fact almost exactly, 243 years are equal to 152 synodic, or 395 sidereal, revolutions. If, then, we have a transit at any time, we *may have* another at the same node *eight* years earlier or later. Sixteen years before or after it would be impossible, and no other transit can occur *at the same node* until after the lapse of *two hundred and thirty-five* or *two hundred and forty-three* years.

If the planet crosses the sun nearly centrally, the transit will not be accompanied by another at an eight-year interval, but the planet will pass either north or south of the sun's disc, at the conjunctions next preceding and following. If, however, as is now the case, the transit path is near the northern or southern edge of the sun, then there will be a companion transit across the opposite edge of the disc eight years before or after. Thus, if we have a pair of *June* transits, separated by an eight-year interval, it will be followed by another pair at the same node in 243 years; and a pair of *December* transits will come in about halfway between the two pairs of June transits. After a thousand years or so from the present time the transits will cease to come in pairs, as they have been doing for 2000 years.

577. Transits of Venus have occurred or will occur on the following dates: —

Dec. 7, 1631	Dec. 9, 1874	June 5, 1761	June 8, 2004
Dec. 4, 1639	Dec. 6, 1882	June 3, 1769	June 6, 2012

[1] If Venus (with her actual rate of motion) were at the same distance from the earth as from the sun, the duration of a central transit would be $\frac{584^d}{675}$; but at conjunction she is nearer in the ratio of 277 to 723, and the duration is correspondingly shortened.

The special interest in these transits consists in the use that has been made of them for the purpose of finding the sun's parallax, a subject which will be discussed later on (Chap. XVI.).

The first observed transit, in 1639, was seen by only two persons, — Horrox and Crabtree, in England. The four which have occurred since then have been extensively observed in all parts of the world where they were visible, by scientific expeditions sent out for the purpose by different nations. The transits of 1769 and 1882 were visible in the United States. Fig. 173 shows the track of Venus across the sun's disc at the two transits of 1874 and 1882

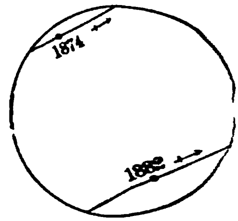

FIG. 173.
Transit of Venus Tracks.

MARS.

This planet is also prehistoric as to its discovery. It is so conspicuous in color and brightness, and in the extent and apparent capriciousness of its movement among the stars, that it could *not* have escaped the notice of the very earliest observers.

578. Orbit. — Its *mean distance* from the sun is 141,500000 miles, but the *eccentricity* of the orbit is so considerable (0.093) that the distance varies about 13,000000 miles. The *light and heat* which it receives from the sun is somewhat less than half of that received by the earth. The *inclination* of its orbit is small, 1° 51'. The planet's *sidereal* period is 687 days, or $1^y\ 10\frac{1}{2}^{mo}$, which gives it an average orbital velocity of fifteen miles per second. Its *synodic* period is 780 days, or $2^y\ 1\frac{2}{3}^{mo}$. It is the longest in the solar system, that of Venus (584 days) coming next. Of the 780 days, it moves eastward during 710, and retrogrades during 70, through an arc of 18°.

579. At opposition its *average* distance from the earth is 48,600000 miles (141,500000 miles minus 92,900000 miles). When the opposition occurs near the planet's perihelion, this distance is reduced to 35,050000 miles; if near aphelion, it is increased to over 61,000000. At conjunction the average distance from the earth is 234,400000 miles (141,500000 plus 92,900000).

The apparent diameter and brilliancy of the planet, of course, vary enormously with these great changes of distance.

If we put R for the planet's distance from the sun, and Δ for its distance from the earth, its brightness, neglecting the correction for *phase*, should

equal $\frac{1}{R^2\Delta^2}$. We find from this that, taking the brightness at conjunction as unity (at which time the planet is about as bright as the pole-star), it is more than twenty-three times brighter at the *average* opposition, and fifty-three times brighter if the opposition occurs at the planet's perihelion. At an unfavorable opposition Mars, as has been said, may be 61,000000 miles distant, and its brightness then is only about twelve times as great as at conjunction, — the difference between favorable and unfavorable oppositions being more than *four to one*.

These favorable oppositions occur always in the latter part of August (at which date the earth passes the line of apsides of the planet), and at intervals of fifteen or seventeen years. The last was in 1892, and the next will be in 1907. A reference to Fig. 159 will show how great is the difference between the planet's opposition distance from the earth under varying circumstances.

580. Diameter, Surface, and Volume. — The apparent diameter of the planet ranges from 3″.6 at conjunction, to 25″.0 at a favorable opposition. Its *real* diameter is very closely 4200 miles, — the error may be twenty miles one way or the other. This makes its surface 0.28, and its volume 0.147 (equal to $\frac{1}{7}$) of the earth's.

581. Mass, Density, and Gravity. — Observations upon its satellites give its mass as $\frac{1}{9.4}$ compared with that of the earth. This makes its *density* 0.73 and *superficial gravity* 0.38; that is, a body which weighs 100 pounds on the earth would have a weight of 38 pounds on the surface of Mars.

582. Phases. — Since the orbit of the planet is outside that of the earth, it never comes between us and the sun, *and can never show the crescent phase;* but at quadrature enough of the unilluminated portion is turned towards the earth to make the disc clearly *gibbous* like the moon three or four days from full. Fig. 174 shows its maximum phase accurately drawn to scale.

FIG. 174.

Greatest Phase of Mars.

583. The "Albedo" of the Planet. — According to Zöllner's observations this is 0.26, which is considerably higher than that of the moon ($\frac{1}{6}$), and just double that of Mercury.

584. Rotation. — The planet's time of rotation is $24^h\ 37^m\ 22^s.67$. This very exact determination has been made by Kaiser and Bak-

huyzen, by comparing drawings of the planet which were made more than 200 years ago by Huyghens with others made recently.

It is obvious that observations made a few days or weeks apart will give the time or rotation with only approximate accuracy. Knowing it thus approximately, we can then determine, without fear of error, the *whole number of rotations* between two observations separated by a much longer interval of time. This will give a second and closer approximation to the true period; and with this we can carry our reckoning over centuries, and thus finally determine the period within a very minute fraction of a second. The number given is not uncertain by more than $\frac{1}{50}$ of a second, if so much.

585. The Inclination of the Planet's Equator to the Plane of its Orbit. — This is very nearly 24° according to Lowell, not very different from the inclination of the earth's equator; so far, therefore, as depends upon that circumstance, its seasons should be substantially the same as our own.

586. Polar Compression. — There is a slight but sensible flattening of the planet at the poles. The earlier observers found for the polar compression values as large as $\frac{1}{40}$, and even $\frac{1}{10}$. These large values, however, are inconsistent with the existence of any extensive surface of liquid upon the planet, and more recent observations of the writer show the polar compression to be about $\frac{1}{220}$. This result is substantially confirmed by the still later measures of Lowell (who gets $\frac{1}{190}$), and by the computations of H. Struve based on the perturbations of its nearer satellite. It is, moreover, almost exactly what would be expected from a planet constituted as we suppose Mars to be.

587. Telescopic Appearance and Surface-Markings. — The fact that we are able to determine the time of rotation so accurately of course implies the existence of identifiable markings upon the surface. Viewed through a powerful telescope, the planet's disc, as a whole, is ruddy, or orange-colored, and is specially bright around the limb, but not at the "terminator," if there is any considerable

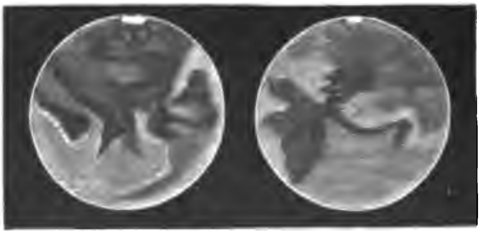

FIG. 175. — Telescopic Views of Mars.

phase. The central portions of the disc present greenish and purplish patches of shade, for the most part not sharply defined, though some of the markings have outlines reasonably distinct. On watching the planet for only a few hours even, the markings pass on across the disc, and are replaced by others. Some of them are permanent, and recur at regular intervals with the same form and appearance, while others appear to be only clouds which for a time veil the surface below, and then clear away. But these are extremely rare as compared with clouds upon the earth.

The most noticeable features may be divided broadly into three classes, —

First, *white patches,* two of which, near the planet's poles, are usually conspicuous, and are generally supposed to be sheets of *snow* and *ice,* since they behave just as would be expected if such were the case. During the planet's northern summer the northern cap dwindles away, while the southern one rapidly increases, and *vice versa* during the southern summer. At times the southern cap is more than 1800 miles across, and four or five months later it sometimes entirely disappears.

Second, *patches of bluish gray or greenish shade,* covering usually about three-eighths of the planet's surface. They lie for the most part in the southern hemisphere and mainly in the equatorial region, forming, in a small telescope, a sort of darkish belt around the planet. These until very lately have been almost universally admitted to be sheets of water, and have received the names of "seas," "gulfs," etc. But recent observations make this doubtful, and suggest that they are more probably regions covered with vegetation, and that no great bodies of water exist on Mars.

Third, *extensive regions of various shades of orange,* covering more than half the surface, especially in the northern hemisphere; it is generally agreed that these are land, probably deserts of sand and rock.

Fig. 175 gives an idea of the planet's general telescopic appearance, though with no attempt at minute accuracy. It fails also in not showing how all the markings fade out at some distance from the brilliant edge of the disc, an effect doubtless due to the planet's atmosphere.

588. Recent Discoveries. The Canals and their "Gemination." Oases. Seasonal Changes. — *a.* Besides the conspicuous markings already mentioned, there are others, difficult to observe, but of great interest and significance. In 1877 and 1879 Schiaparelli announced

the discovery of a great number of fine, dark straight lines, or "canals," as he called them, crossing the ruddy portions of the disc in all directions; and in 1881 he announced further that many of these *became double* at times, like the two parallel tracks of a railway. His observations have since been confirmed and added to by various eminent astronomers in Europe and America, especially by Perrotin at Nice and Lowell in Arizona. But others, equally eminent and apparently under equally favorable conditions, fail to see the reported features. At present, from numerous experiments recently made upon maps and models viewed at different distances, it seems probable that illusions enter into the case to some extent,—illusions resulting from the unconscious misinterpretation of actual markings imperfectly seen. The so-called "canals" doubtless exist, though probably not exactly as represented upon the map. There are, however, special reasons to suspect the reality of their "gemination." And yet it is by no means impossible that time may bring a complete confirmation of Schiaparelli's results.

b. According to several observers, especially the Flagstaff astronomers, the canals are not limited to the ruddy portions of the surface, but in some cases *extend across the dusky regions also.* This observation, if correct, is of great importance in showing that the so-called "seas" cannot be bodies of water. But it is difficult, and needs to be more fully confirmed before final acceptance.

c. At the intersection of the canals (of which over 180 are now catalogued and located on Mr. Lowell's map of the planet) small, round dark spots are observed at certain times. These, at first called "lakes," are now regarded by him as "*oases.*"

d. It is found also that the extent and darkness of the so-called "seas" varies greatly with the Martial seasons. Speaking generally, the "seas" of each hemisphere are darker and larger at the time when their respective polar caps begin to shrink, and they become smaller, and more definite in outline when the caps are on the point of vanishing.

e. Mr. Lowell's observations appear to show that there are few if any *high mountain* peaks or ranges, though there are indications of a few elevations perhaps two or three thousand feet in height. The planet's surface seems to be remarkably level as compared with the earth's.

589. Atmosphere and Temperature. — At one time it was supposed that Mars possessed a very dense atmosphere which gave it its ruddy color; but considerations based on the planet's low surface gravity (not quite 38 per cent of the earth's) as well as the

direct evidence of observation, show that this cannot be the case. There is no doubt, however, that it has *some* atmosphere, since the occasional presence of thin veils of cloud, as well as the deposition and dissipation of the polar caps, can be explained only by its presence; but its height and density must be much less than that of our own. Some of the earlier spectroscopic observers considered that they had found certain evidence of the presence of water-vapor, but later observations, especially those of Campbell at the Lick Observatory, render the case doubtful, to say the least.

As to the *temperature* of Mars, we have no certain knowledge. On the one hand we know that on account of the planet's distance from the sun the intensity of solar radiation upon its surface must be less than here in the ratio of 1 to $(1.524)^2$, *i.e.*, only about 43 per cent as great as with us: its "solar constant" must be less than 13 calories against our 30. Then, too, the low density of its atmosphere, probably less at the planet's surface than on the tops of our highest mountains, would naturally assist to keep down the temperature to a point far below the freezing-point of water. But on the other hand things certainly *look* as if the polar caps were really masses of *snow* and *ice* deposited from vapor in the planet's atmosphere, and as if these actually melted during the Martian summer, sending floods of water through the channels provided for them, and causing the growth of vegetation along their banks. We are driven, therefore, to suppose either that the planet has sources of heat, internal or external, which are not yet explained; or else, as long ago suggested, that the polar "snow" may possibly be composed of something else than frozen *water*. The problem is a perplexing one, and it is earnestly to be hoped that before very long we may come into possession of some heat measurer sufficiently delicate to give us direct evidence as to the warmth or coldness of the planet's surface. (See also Art. 915.)

589*. Speculations of Flammarion and Lowell.—These astronomers, and many others, practically ignore the temperature difficulty, and unhesitatingly assume that the polar caps are composed of snow and ice, that they melt in spring and summer, and that the water thus liberated makes its way towards the equator over the planet's mountainless plains, partially obscuring for several weeks the well-known features which at other times are conspicuous. According to Mr. Lowell the dark regions formerly supposed to be seas are regions, possibly marshy, more or less covered with *vegetation*, while the ruddy portions are Saharan deserts intersected with water-channels, which he regards as *artificial* (in part, at least) and arranged for purposes of irrigation. When the water reaches these, verdure springs up along their

course, and these streaks of vegetation are what we recognize as the "canals," the water-channels themselves being far too narrow to be visible in our telescopes.[1] When they cross each other "oases" are formed. As to the "gemination" of the canals, no clear explanation appears as yet, though suggestions have been offered that it may be due to some mode of growth, or some treatment of the crops produced by the irrigation.

589. Habitability of Mars.** — It may be said with some confidence that on Mars the conditions, different as they must be from our own, are still more nearly earthlike than on any other of the heavenly bodies *which we can see* with our present telescopes.

And yet, as already pointed out, unless the planet has unknown sources of heat, the temperature must be too low to permit anything like terrestrial life. Nor, with one possible exception, is there the slightest evidence of the existence of intelligent beings upon it. Mr. Lowell argues from the straightness of the canals (some of them over 2000 miles in length), and from the accuracy with which several of them converge at certain oases, that they

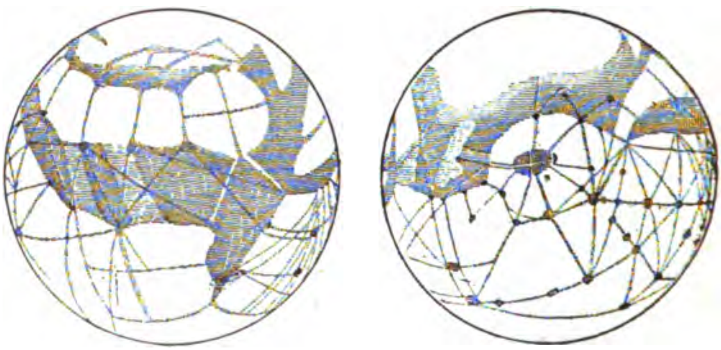

FIG. 176. — Mars, Lowell, 1895.

must have been intelligently engineered. (See Fig. 176.) If his observations on this point are not illusions based on misconception his conclusion is perhaps not unnatural though by no means necessary; but where seeing is difficult at best, it is easy for one to imagine that he sees what he thinks he ought to see.

589*. Maps of Mars.** — Numerous maps of the planet have been constructed by various astronomers since the first was drawn by Maedler in 1830. Fig. 176* is from one published by Schiaparelli in 1888, and shows most of his canals, and the gemination of such of them as exhibit that phenomenon. There can be no doubt as to

[1] When Mars is nearest us, at a distance of 35 500000 miles, we view it, even with a magnifying power of 1000, only as we see the moon with a field-glass magnifying between six and seven times; of course at a distance (optical) of 35500 miles no really minute details can possibly be distinguished.

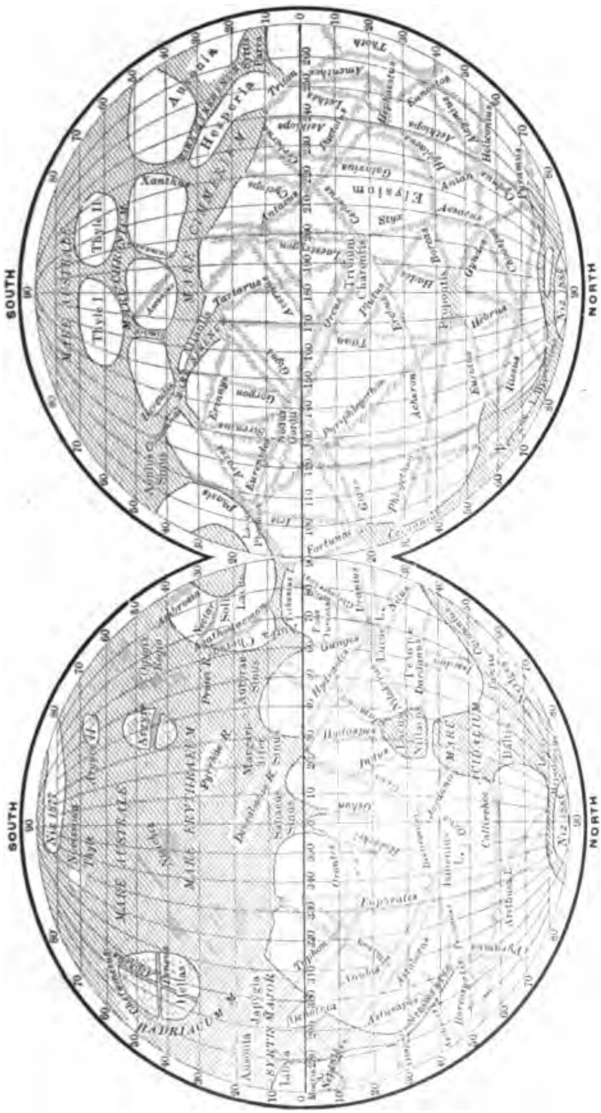

FIG. 176*. — Chart of Mars as observed from 1877 to 1888. (Schiaparelli.)

the substantial correctness of the principal features, but as to minor details there may be considerable uncertainty; nor must it be forgotten that certain important features notably change their size and appearance with the progress of the planet's seasons. In 1904–5 Lampland at Professor Lowell's Flagstaff Observatory obtained photographs of the planet, which, for the first time, show nearly all the features visible in ordinary telescopes. The nomenclature of Areography is still in an unsettled condition, but the system of Schiaparelli, based on ancient geography, is now more generally accepted than any other, and will probably prevail.

590. Satellites. — There are two satellites which were discovered in August, 1877, by Professor Hall at Washington, with the *then* new 26-inch telescope. They are exceedingly minute, and can be seen only with the most powerful instruments. The outer *one*, Deimos, is at a distance of 14600 miles from the centre of the planet, and has a period of $30^h\ 18^m$, while the inner one, Phobos, is at a distance of only 5800 miles, and its month is but $7^h\ 39^m$ long, not one-third of the *day* of Mars. Owing to this fact it rises in *the west* every night for the "*Marticoli*" (if there are any people there) and sets in the *east*, after about $5\frac{1}{2}^h$.

Deimos does not do this; it rises in the east like other stars, but its orbital eastward motion among the stars is so nearly equal to *its* diurnal motion westward, that it is nearly 132 hours between two successive risings. This is more than four of its months, so that *it* undergoes all its changes of phase four times in the interval.

Of course, both the satellites are frequently eclipsed. Their orbits appear to be exactly circular, and they move exactly in the plane of the planet's equator; and they *keep* so, maintained in their relation to the equator by the action of the "equatorial bulge" upon the planet.

591. As givers of moonlight they do not amount to much. Their diameters are too small to be measured with any micrometer; but from their apparent "*magnitude*" (*i.e.*, brightness), as seen from the earth, and assuming that their surfaces have the same reflective power as that of the planet, Professor Pickering has estimated the diameter of Phobos, which is the larger one, as about seven miles. and that of Deimos, as five or six. The light given by Phobos to the inhabitants of Mars would be about $\frac{1}{60}$ of our moonlight; that of Deimos about $\frac{1}{1200}$. Mr. Lowell's estimates of their diameters are considerably greater, — 10 miles for Deimos, and 25 for Phobos.

The period of Phobos is by far the shortest period in the solar system. Its rapidity of revolution raises important questions as to the theory of the development of the solar system, and requires modification of the views which had been held up to the time of their discovery. If the nebular hypothesis is true, a shortening of

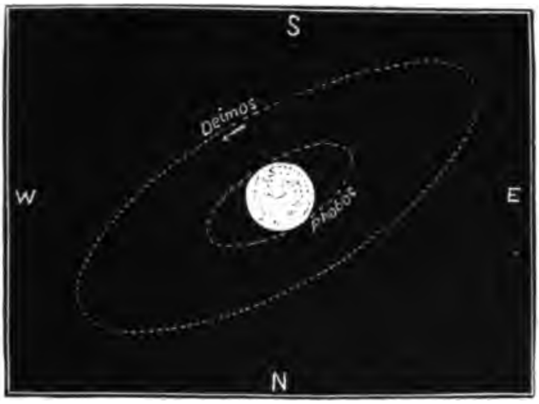

FIG. 177. — Orbits of the Satellites of Mars.

the satellite's period, or a lengthening of the planet's day, must have occurred since the satellite came into being, since that hypothesis will not account for the existence of a satellite having a period shorter than the diurnal rotation of its primary. Fig. 177 is a diagram of the satellite orbits as they appeared from the earth in 1888. It is reduced from the American Nautical Almanac for that year.

THE ASTEROIDS, OR MINOR PLANETS.

592. These are a group of small planets circulating in the space between Mars and Jupiter. The name "*asteroid*" was suggested by Sir William Herschel early in the century, when the first ones were discovered. The later term *planetoid* is preferred by some.

It was very early noticed that there is a break in the series of the distances of the planets from the sun. Kepler, indeed, at one time thought he had discovered the true law[1] and the real reason why

[1] His supposed law was as follows: Imagine the sun surrounded by a hollow spherical shell, on which lies the orbit of the earth. Inside of this shell inscribe a regular *icosahedron* (the twenty-sided regular solid), and within that inscribe a second sphere. This sphere will carry upon it the orbit of Venus. Inside of the sphere of Venus inscribe an *octahedron* (the eight-sided solid), and the sphere which fits within it will carry Mercury's orbit. Next, working outwards from the

the planets' distances are what they are. This theory of his was broached twenty-two years, however, before he discovered the harmonic law, and he probably abandoned it when he discovered the elliptical form of the planets' orbits. At any rate, in later life he suggested that it was likely that there was a planet between Mars and Jupiter too small to be seen.

The impression that such a planet existed gained ground when Bode, in 1772, published the law which bears his name, and it was still further deepened when, nine years later, in 1781, Uranus was discovered, and the distance of the new planet was found to conform to Bode's law. An association of twenty-four astronomers, mainly German, was immediately formed to look for the missing planet, who divided the zodiac between them and began the work. Singularly enough, however, the first discovery was made, not by a member of this association, but by Piazzi, the Sicilian astronomer of Palermo, who was then engaged upon an extensive star catalogue. On January 1, 1801, he observed a seventh-magnitude star which by the next evening had unquestionably moved, and kept on moving. He observed it carefully for some six weeks, when he was taken ill; before he recovered, it had passed on towards superior conjunction, and was lost in the rays of the sun. He named it *Ceres*, after the tutelary goddess of Sicily.

When at length the news reached Germany in the latter part of March it created a great excitement, and the problem now was to rediscover the lost planet. The association of planet-hunters began the search in September, as soon as its elongation from the sun was great enough to give any prospect of success. During the summer Gauss devised his new method of computing a planetary orbit, and computed the ephemeris of its path. Very soon after receiving his results, Baron Von Zach rediscovered Ceres on December 31, and Dr. Olbers on the next day, just one year after it was first found by Piazzi.

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earth's orbit, circumscribe around the earth's sphere a *dodecahedron*, circumscribing around it another sphere, and this will carry upon it the orbit of Mars. Around the sphere of Mars circumscribe the *tetrahedron*, or the regular pyramid. The corners of this solid project very far, so that the sphere circumscribed around the tetrahedron will be at a very great distance from the sphere of Mars. It carries the orbit of Jupiter. Finally, the cube, or *hexahedron*, circumscribed around the orbit of Jupiter, gives us in the same way the orbit of Saturn. We thus obtain a series of distances not enormously incorrect (though by no means agreeing with fact even as closely as does Bode's law); and, moreover, the theory had the great advantage to Kepler's mind of accounting for the fact that there are (so far as was then known) but *seven* planets, there being possible but *five* regular solids.

593. In March, 1802, Dr. Olbers, who in looking for Ceres had carefully examined the small stars in the constellation of Virgo, on going over the ground again, found a second planet, which he named Pallas, a body of about the same brightness as Ceres. *Two* having now been found, and Pallas having a very eccentric and much inclined orbit, he conceived the idea that they were fragments of a broken planet, and that other planets of the same group could probably be found by searching near the intersection of their two orbits. Juno, the third, was discovered by Harding at Lilienthal (Schröter's observatory) in 1804; and Vesta, the largest and brightest of the whole group (sometimes visible to the naked eye), was found by Olbers himself in 1807. The search was kept up for several years after this, but no more planets were found because they did not look after small enough stars.

The fifth, Astræa, was discovered in 1845 by Hencke, an amateur astronomer who for fifteen years had been engaged in studying the smaller stars in hopes of just the reward he captured. In 1846 no asteroid was found (the discovery of Neptune was glory enough for that year), but in 1847 three more were brought to light; and since then not a year has passed without adding from one to thirty to the number. At the beginning of 1906 the list given in the Paris "Annuaire" counted up 570, duly identified and "numbered," with about thirty more not then sufficiently observed to warrant complete recognition. Fresh discoveries are made continually, though the new asteroids are mostly very small, — stars fainter than the twelfth magnitude, which require a large telescope to make them even visible. All the brighter ones have evidently been already picked up.

594. Method of Search. — Formerly the search for these objects was conducted by making special star charts of certain regions near the ecliptic selected by the "asteroid-hunter," and afterwards comparing the chart with the heavens, when interlopers would at once be detected, and their nature determined by their motion. The operation was very laborious.

During 1891 a new method of search was introduced. By photographing a portion of the heavens with a camera of wide field, mounted equatorially and moved by clock-work, pictures are obtained in which any planets present can be easily distinguished by their motion during the two or three hours during which the exposure of the plate is continued; while the images of *stars* are round, if the clock-work runs correctly, the planets are apparently elongated into *streaks.* Wolf of Heidelberg and Charlois of Nice have been specially successful in this method of discovery.

Of course, great care must be taken to be sure that the object discovered is a *new* planet, and not one of the multitude already known. Generally it is possible to decide very quickly which of the known planets will be in the neighborhood, and a rough computation will commonly decide at once whether the planet is new. Not always, however, and mistakes in this regard are not very unusual.

The known asteroids have been discovered by comparatively a few observers. Four persons, working by the old method, have discovered more than twenty each. Palisa of Vienna has discovered 83; the late Dr. Peters of Clinton, N.Y., 52; Luther of Dusseldorf, 24; and the late Professor Watson of Ann Arbor, 22. More recently Charlois of Nice has discovered over 100, nearly all by the photographic method; and Max Wolf of Heidelberg, and his assistants a still larger number, all by photography.

These minor planets are mostly *named*, the names being derived from mythology and legend. They are also designated by numbers, and the symbol for each planet is the number written in a circle. Thus, for Ceres the symbol is (1); for Hilda, (153); and so on.

A full list of them, with the elements of their orbits, is published biennially in the "Annuaire du Bureau des Longitudes," Paris.

595. Their Orbits. — The mean distance of the different asteroids from the sun varies greatly and the periods are correspondingly different. Adalberta, (330), has the smallest mean distance[1] (of the planets enumerated in Jan. 1898), *viz.*, 2.09, or 194 270000 miles, with a period of $3^y\ 3^d.1$. Thule, (279), is the remotest, with a mean distance of 4.30 or 400 000000 miles, and a period of $8^y\ 313^d$. According to Svedstrup, the mean distance of the "mean asteroid" is 2.65 (246 000000 miles), and its period about $4\frac{1}{3}$ years. Its distance from the earth at time of opposition would be, of course, 1.65, or 153 000000 miles.

The inclinations of their orbits *average* about 8°; but Pallas, (2), has an inclination of 35°, and Euphrosyne, (31), of $26\frac{1}{2}$°.

Several of the orbits are extremely eccentric. Æthra, (132), has an almost cometary eccentricity of 0.38, and over a dozen others have eccentricities exceeding 0.30. They are distributed quite unequally in the range of distance, there being, as Kirkwood has pointed out, very few at such distances that their periods would be exactly commensurable with that of Jupiter.

596. Diameter and Surface. — These bodies are so small that micrometrical measurements upon them are extremely difficult, and until very recently our estimates of their probable size have been based merely upon their brightness. Pickering, by photometric

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[1] See page 377 for note on Eros.

methods, and *assuming* an "albedo" the same as that of Mars, found for Vesta (the only one ever visible to the naked eye) a diameter of 319 miles. In 1894–95, however, Mr. Barnard with the great Lick telescope succeeded in making micrometric measures of the discs of the four brightest with the following rather surprising results: — Diameter of Ceres, 485 miles; of Pallas, 304; of Juno, 118; and of Vesta, 243. But the percentage of probable error must be pretty large.

The surprise lies, of course, in the great contrast of albedo between Vesta and the other three. As to the rest of the family, it is hardly possible that any one of them can be as much as 100 miles in diameter, and the smallest are probably less than ten miles through, — nothing more than "mountains broke loose."

597. Mass, Density, etc. — As to the individual masses and densities we have no certain knowledge. It is probable that the density does not differ much from the density of the crust of the earth, or the mean density of Mars. If this is so, the mass of Ceres might possibly be as great as $\frac{1}{6000}$ of the earth. On such a planet the force of superficial gravity would be about $\frac{1}{23}$d of gravity on the earth, and a body projected from the surface with a velocity of about 2500 feet a second — that of an ordinary rifle-ball — would fly off into space and never return to the planet, but would circulate around the sun as a planet on its own account. On the smallest asteroids, with a diameter of about ten miles, it would be quite possible to throw a stone from the hand with velocity enough to send it off into space.

598. Aggregate Mass. — Although we can only estimate very roughly the masses of the individual members of the flock, it is possible to get some more certain knowledge of their *aggregate* mass. Leverrier from the motion of the line of apsides of the orbit of Mars demonstrated that the whole amount of matter thus distributed in the space between Mars and Jupiter cannot exceed about *one-fourth* of the mass of the earth. A still later computation by Ravené in 1896, indicates a total mass only about $\frac{1}{15}$ as great as the earth's.

The united masses of those which are already known would make only a very small fraction of such a body. Up to August, 1880, the united bulk of the asteroids then discovered was estimated at $\frac{1}{4000}$ part of the earth's bulk, with a mass probably about $\frac{1}{6000}$ of the earth's.[1] Presumably, therefore, the number of these bodies remaining undiscovered is exceedingly great —

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[1] Barnard's measures (Art. 596) would increase this estimate of bulk and mass, but would not seriously affect the general conclusion.

to be counted by thousands, if not by millions. Most of them, of course, must be much smaller than those which are already known.

599. Forms, Variations of Brightness, and Atmosphere. — We have as yet only scanty knowledge on this point, but Dr. Olbers observed in Vesta certain fluctuations in her brightness which seemed to him to indicate that she is not a globe, but an angular mass, — a splinter of rock. This, however, is not confirmed by the more recent photometric observations of Müller or Pickering.

Since 1900, however, it has been discovered by photometric methods that some of these bodies show regular variations of brightness recurring at intervals of from 5 to 8 hours, due probably to axial rotation. Iris, (7); Sirona, (116); Hertha, (135); Tercidina, (345); Eros, (433); several others are already known to behave in this way, and yet others are suspected.

600. Origin. — With respect to this we can only speculate. Two views have been held, as has been already intimated. One is, that the material, which according to the nebular hypothesis ought to have been concentrated to form a single planet of the class to which the earth belongs, has failed to be so collected, and has formed a flock of small separate masses. It is now very generally believed that the matter which at present forms the planets was once distributed in *rings*, like the rings of Saturn. If so, this ring, or meteoric swarm, would necessarily suffer violent perturbations from the nearness of the enormous planet Jupiter, and so would be under very different conditions from any of the other rings. This, as Peirce has shown, might account for its breaking up into many fragments.

The other view is that a planet about the size of Mars has broken to pieces. It is true, as has been often urged, that this theory in its original form, as presented by Olbers, cannot be correct. No *single* explosion of a planet could give rise to the present assemblage of orbits, nor is it possible that even the perturbations of Jupiter could have converted a set of orbits originally all crossing at one point (the point of explosion) into the present tangle. The smaller orbits are so small that however turned about they lie wholly inside the larger, and cannot be made to intersect them. If, however, we admit a *series* of explosions, this difficulty is removed; and if we grant an explosion at all, there seems to be nothing improbable in the hypothesis that the fragments formed by the bursting of the parent mass would carry away within themselves the same forces and reactions which caused the original bursting; so that they themselves would be likely enough to explode at some time in their later history.

At present opinion is divided between these two theories.

601. The number of these bodies already known is so great, and the prospect for the future is so indefinite, that astronomers are at their wits' end how to take care of this numerous family. To compute the orbit and ephemeris of one of these little rocks is more laborious (on account of the great perturbations produced by Jupiter) than to do the same for one of the major planets; and to keep track of such a minute body by observation is far more difficult. Until recently, the German Jahrbuch has been publishing the ephemerides of such as came within the range of observation each year; but this cannot be kept up much longer, and the probability is that hereafter only the larger ones, or those which present some remarkable peculiarity in their orbits, will be followed up. One little family of them, however, is "endowed." Professor Watson, at his death, left a fund to the American National Academy of Sciences to bear the expense of taking care of the twenty-two which he discovered.

INTRA-MERCURIAL PLANETS AND THE ZODIACAL LIGHT.

It is not at all improbable that there are masses of matter revolving around the sun within the orbit of Mercury.

602. Motion of the Perihelion of Mercury's Orbit. — Leverrier, in 1859, from a discussion of all the observed transits of Mercury, found that the perihelion of its orbit has a movement of nearly 38″ a century over and above what can be accounted for by the action of the known planets, and he calculated that it could be explained by the attraction of a planet, or ring of small planets, revolving inside this orbit nearly in its plane, with a mass about half as great as that of Mercury itself.

It could also be explained on the hypothesis that the force of gravitation instead of varying strictly as $\frac{1}{D^2}$ varies as $\frac{1}{D^{(2+n)}}$, where n is an extremely small quantity; also on the hypothesis that the law of attraction is not exactly the same for bodies in motion as at rest, being slightly *less* in the former case, — the so-called *electro-dynamic* theory of gravitation. But Newcomb finds that while the motions of Mercury's *perihelion* may be explained in these various ways, the motion of his *node* (and that of Venus also) appears to be inconsistent with the existence of such a planetary ring, and the subject is by no means cleared up as yet.

603. Dr. Lescarbault's Observation: Vulcan. — A certain country physician, living some eighty miles from Paris, Dr. Lescarbault, on the publication of Leverrier's result, announced that he had actually seen this planet crossing the sun nine months before, on the 26th of March of that year, 1859. He was visited by Leverrier, who became satisfied of the genuineness

of his observations, and the doctor was duly congratulated and honored as the discoverer of "Vulcan," which name was assigned to the supposed new planet. An interesting account of the matter may be found in Chambers' "Descriptive Astronomy"; and in many of the works published from twenty to twenty-five years ago, as well as in some more recent ones, "Vulcan" is assigned a place in the solar system, with a distance of about 13 000000 miles and a period of 19 + days. Lescarbault described it as having an apparent diameter of about 7″, which would make it over 2500 miles in diameter.

604. Nevertheless, it is nearly certain that Vulcan does not exist. There are various opinions which we need not here discuss as to the explanation of this pseudo-discovery. But the planet, if real, ought since 1859 to have been visible on the sun's face at certain definite times which Leverrier calculated and published; and it has never been seen, though very carefully looked for. Small, round, dark objects have from time to time been indeed reported on the sun's disc, which in the opinion of the observers at the time were not sun spots; but most of these observations were made by amateurs with comparatively little experience, with small telescopes, and with no measuring apparatus by which they could certainly determine whether or not the spot seen moved like a planet. In most of these cases photographs or simultaneous observations made elsewhere by astronomers of established reputation, and having adequate apparatus, have proved that the problematical "dots" were really nothing but ordinary small sun spots, and the probability is that the same explanation applies to the rest.

605. Eclipse Observations. — A planet large enough to be seen distinctly on the sun by a 2½-inch telescope, such as Lescarbault used, would be a conspicuous object at the time of a solar eclipse, and most careful search has been made for the planet on such occasions; but so far, although *stars* of the third and fourth magnitudes, and even of the fifth, have been clearly seen by the observers within a few degrees of the eclipsed sun, no planet has been found.

One apparent exception occurred in 1878. During the eclipse of that year, Professor Watson observed two starlike objects (of the fourth magnitude), which he thought at the time could not be identified with any known stars consistently with his observations. Mr. Swift, also, at the same eclipse, reported the observations of two bright points very near the sun; but these from his statement could not (both) have been identical with Watson's stars. Later investigations of Dr. Peters have shown that the assumption of a very small and very likely error in Professor Watson's circle-readings (which were got in a very ingenious, but rather rough way, without the use of graduations) would enable his stars to be identified with θ and ζ Cancri, and it is almost certain that these were the stars he saw. Mr.

Swift's observations remain unexplained. With this exception, the eclipse observations all give negative results, and astronomers generally are now disposed to consider the "Vulcan question" as settled definitely and adversely.

606. At the same time it is extremely probable that there are a number, and perhaps a very great number, of *intra-Mercurial asteroids.* A body two hundred miles in diameter near the sun would have an angular diameter of only about $\frac{1}{2}''$, as seen from the earth, and would not be easily visible on the sun's disc, except with very large telescopes. It would not be at all likely to be picked up accidentally. Objects with a diameter of not more than forty or fifty miles would be almost sure to escape observation, either at a transit or during a solar eclipse unless, possibly, by photography.

607. Zodiacal Light. — This is a faint, pyramidal haze of light that extends from the sun along the ecliptic. In the evening it is best seen in February, March, and April, because the portion of the ecliptic which lies east of the sun's place is then most nearly perpendicular to the western horizon. During the autumnal months the zodiacal light is best seen in the morning sky for a similar reason. In our latitudes it can seldom be traced more than 90° or 100° from the sun; but at high elevations within the tropics it is said to extend entirely across the sky, forming a complete ring, and there is said to be in it at the point exactly opposite to the sun a patch a few degrees in diameter of slightly brighter luminosity, called the "Gegenschein" or "counter-glow."

The portions of this object near the sun are reasonably bright, and even conspicuous at the proper seasons of the year; but the more distant portions in the neighborhood of the "counter-glow" are so extremely faint that it is only possible to observe them at a distance from cities and large towns, in places where the air is free from smoke, and where the darkness of the sky is not affected by the general illumination due to gas and electric lights.

Its spectrum has been observed by Wright of New Haven and others, and appears to be continuous, showing no *bright* lines — it is too faint to show the *dark* lines of the solar spectrum if they are really present, as is very probable, since the light appears to be partially *polarized* as if *reflected* from minute particles.

It has often been stated that the spectrum of the zodiacal light shows the same bright line which characterizes that of the Aurora Borealis: this is a mistake.

608. The cause of the phenomenon is not certainly known, but at present the theory most generally accepted attributes it to *sunlight reflected by myriads of small meteoric bodies* which are revolving around the sun nearly in the plane of the ecliptic, forming a thin, flat sheet like one of Saturn's rings, and extending far beyond the orbit of the earth. It may be that the denser

portion of this meteoric ring within the orbit of Mercury is the cause of the motion of the perihelion of that planet which Leverrier detected; it is for this reason that we deal with the subject here rather than in connection with meteors. While this theory, however, is at present more generally accepted than any other, it cannot be said to be established. Some are disposed to consider the zodiacal light as a mere extension of the sun's corona, whatever that may be.

Exercises on Chapter XV.

1. On May 2, 1896, the apparent semi-diameter of Jupiter was 17″.75, its distance from the earth being at that time 5.431 Astron. units. Required the planet's diameter compared with that of the earth.

Remember that the solar parallax, 8″.80, is the same as the earth's semi-diameter seen from distance unity.

Ans. $\frac{17.75}{8.80} \times 5.421$, or 10.95 times diameter of the earth.

2. Assuming the preceding measure as exact, what ought to be the apparent semi-diameter of the planet when at a distance of 4.25?

Ans. 22″.63.

3. What must be the mass of the earth to make the moon revolve around it with the same period as now, but at twice its present distance? (See Art. 537.)

Make E the present mass of the earth, and E' the required mass, and apply the equation given in Art. 537.

Ans. $E' = 8E$.

4. How much must the mass of the earth be increased to make the moon, at its present distance, revolve in two days?

Ans. $E' = E \times \left(\frac{27.32}{2}\right)^2, = 186.6\ E.$

5. What reduction of the earth's mass, suddenly produced, would *release* the moon, *i.e.*, transform her orbit into a parabola or hyperbola?

Ans. Any reduction exceeding 50 per cent.

6. At what rate does the elongation of Venus from the sun change at or near the time of superior conjunction? (See Art. 564.)

Ans. $\begin{cases} 38''.81 \text{ hourly, or} \\ 15' 31''.4 \text{ daily.} \end{cases}$

7. At what rate does Venus appear to cross the sun's disc during a transit? (See Art. 575).

Ans. 241″ per hour.

8. At what rates does Mars advance when at conjunction, and retrograde at opposition?

Ans. { At conjunction, 42′.43 daily advance. At opposition, 21′.43 daily regression.

NOTE TO ART. 573.

The observation of Professor Lyman was repeated by Russell at Princeton in December, 1898.

His investigations indicate that *refraction* plays a comparatively small part in the formation of the luminous ring, which appears to be mainly due to *diffusion* of light, like that which produces our own twilight. If due to refraction, the ring should be widest and brightest on the side of the planet farthest from the sun, while the reverse is the case.

Nor do they furnish any evidence that the planet's atmosphere is more dense or extensive than the earth's, but rather the contrary; as might be expected considering her smaller mass, and probably higher temperature.

NOTE TO ART. 595.

EROS. The planet Eros, (433), is, in many ways, the most interesting of the Asteroid group thus far known. It was discovered, photographically, by Witt of Berlin, in August, 1898, and at once attracted attention by its rapid motion. Its mean distance from the sun is not quite 135,480000 miles, — less than that of Mars, and its period is 643 days. The eccentricity of its orbit is 0.223, so that its aphelion distance is 165,670000 miles, and its perihelion distance, 105,290000, exceeding the mean distance of the earth by only about 12 million miles. The orbits, however, are so situated that it cannot come nearer than 13½ million. Still this is only a little more than half the least distance of Venus, and observations of Eros, made at such a time of closest approach, will furnish by far the most precise means known for determining the solar parallax. Unfortunately these close oppositions are rare: one occurred in 1894 (before the planet was discovered), and ~~it will be thirty years before such another opportunity occurs;~~ though in 1901 the conditions were far better than usual, and better than they will be again until 1931 and 1938.

The inclination of the planet's orbit is nearly 11°, and when nearest the earth, as in 1894, it goes into circumpolar declinations, and at the time of opposition it moves almost directly south, its motion in right-ascension being very slight.

The planet is extremely small, probably not exceeding 20 miles in diameter, and seldom visible except in the largest telescopes, though when nearest it is just possible that it may reach the naked-eye limit. As already stated it exhibits at times regular variations of brightness apparently indicating a rotation-period of $5^h\ 16^m$, though a different explanation is possible.

CHAPTER XVI.

THE PLANETS CONTINUED. — THE MAJOR PLANETS: JUPITER, SATURN, URANUS, AND NEPTUNE.

JUPITER.

609. While this planet is not so brilliant as Venus at her best, it stands next to her in this respect, being on the average about five times brighter than Sirius, the brightest of the fixed stars. Jupiter, moreover, being a "superior" planet, is not confined, like Venus, to the neighborhood of the sun, but at the time of opposition is the chief ornament of the midnight sky.

610. Orbit. — The orbit presents no marked peculiarities. The *mean distance* of the planet from the sun is 483,000000 miles. The *eccentricity* of the orbit being nearly $\frac{1}{20}$ (0.04825); the greatest and least distances vary by about 21,000000 miles each way, making the planet's greatest and least distances from the sun 504,000000 and 462,000000 miles respectively. The average distance of the planet from the earth at opposition is 390,000000, while at conjunction it is 576,000000 miles. The minimum opposition distance is only 369,000000, which is obtained when the opposition occurs about October 6, Jupiter being in perihelion when its heliocentric longitude is about 12°. At an aphelion opposition (in April) the distance is 42,000000 miles greater; that is, 411,000000.

The relative brightness of Jupiter at an average conjunction and at the nearest and most remote oppositions is respectively as the numbers 10, 27, and 18. The average brightness at opposition is, therefore, more than double that at conjunction; and at an October opposition the planet is fifty per cent brighter than at an April one. The differences are considerable, but far less important than in the case of Mars, Venus, and Mercury.

The *inclination* of the orbit to the ecliptic is small, — only 1° 19′.

611. Period. — The *sidereal period* is 11.86 years, and the *synodic* is 399 days (a number easily remembered), a little more than a year and a month. The planet's orbital velocity is about eight miles a second.

612. Dimensions. — *The planet's apparent diameter* varies from 50″ at an October opposition (or $45\frac{1}{2}''$ at an April one) to 32″ at conjunction. The form, however, of the planet's disc is not truly circular, the polar diameter being about $\frac{1}{17}$ part less than the equatorial, so that the eye notices the oval form at once. The equatorial diameter in *miles* is 88,200, the polar being 83,000. Its mean[1] diameter, therefore, is 86,500, — almost eleven times that of the earth.

This makes its surface 119 times, and its volume 1300 times, that of the earth. It is by far the largest of the planets in the system; in fact, whether we regard its bulk or its mass, larger than all the rest put together.

613. Mass, Density, etc. — Its *mass* is very accurately known, both by the motions of its satellites, and the perturbations of the asteroids. It is $\frac{1}{1047.4}$ of the sun's mass, or very nearly 318 times that of the earth. Comparing this with its volume, we find its *density* 0.24, less than $\frac{1}{4}$ the density of the earth, and almost precisely the same as that of the sun. Its mean *superficial gravity* comes out 2.64 times that of the earth; that is, a body on Jupiter would weigh $2\frac{5}{8}$ times as much as upon the surface of the earth; but on account of the rapid rotation of the planet and its ellipticity there is a very considerable difference between the force of gravity at the equator and at the pole, amounting to $\frac{1}{5}$ of the equatorial gravity. (On the earth the difference is only $\frac{1}{190}$.)

614. Phases and Albedo. — Its orbit is so much larger than that of the earth that the planet shows no sensible phases, even at quadrature, though at that time the edge farthest from the sun shows a slight darkening.

The reflecting power, or *Albedo*, of the planet's surface is very high, — 0.62 according to Zöllner, that of white paper being only 0.78. The centre of the disc of this planet (and the same is also true of Saturn) is considerably brighter than the limb — just the reverse, as will be remembered, from the condition of things upon the moon, and upon Mars, Venus, and Mercury. This peculiarity of a darkened limb, in which Jupiter resembles the sun, has sug-

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[1] The mean diameter of an oblate spheroid is $\frac{2a+b}{3}$, not $\frac{a+b}{2}$. Of the three axes of symmetry which cross at right angles at the planet's centre, *one* is the axis of rotation, and both the others are equatorial.

gested the idea that it is to some extent *self-luminous.* This, however, is not a necessary consequence, as a nearly transparent atmosphere overlying a uniformly reflecting surface would produce the same effect.

The light which the planet emits, if it emits any, must be very feeble as compared with sunlight, since the satellites, when they are eclipsed by entering the shadow, become totally invisible.

615. Axial Rotation. — The planet rotates on its axis in *about* $9^h\ 55^m$. The time can be given only approximately, not because it is difficult to find and observe distinct markings on the planet's disc, but simply because different results are obtained from different spots, according to their nature and their distance from the planet's equator. Speaking generally, spots near the equator indicate a shorter day than those in higher latitudes, and certain small, sharply defined, bright, white spots, such as are often seen, give a quicker rotation than the dark markings in the same latitude.

According to Williams there are at least nine "belts" of atmospheric current on Jupiter, clearly distinct from each other; the swiftest, at the equator, has a rotation-period of only $9^h\ 50^m\ 20^s$, while that of the slowest is $9^h\ 56^m$. The great red spot has given values ranging from $9^h\ 55^m\ 34^s.9$ (in 1879) to $9^h\ 55^m\ 40^s.7$ (in 1886), and $9^h\ 55^m\ 41^s.4$ (in 1896). The increase has been unmistakable, and is not due to any uncertainty in the observations.

616. The Axis of Rotation and the Seasons. — The plane of the equator is inclined only 3° to that of the orbit, so that as far as the sun is concerned there can be no seasons. The heat and light received from the sun by Jupiter are, however, only about $\frac{1}{27}$ as intense as the solar radiation at the earth, its distance being 5.2 times as great.

617. Telescopic Appearance. — Even in a small telescope the planet is a beautiful object. When near opposition a magnifying power of only 40 makes its apparent size equal to that of the full moon (though, as remarked in connection with Venus, no novice would receive that impression), and with a telescope of 8 or 10 inches aperture, and with a magnifying power of 300 or 400, the disc is covered with an infinite variety of beautiful and interesting details which rapidly shift under the observer's eye in consequence of the planet's swift rotation. The picture is rich in color, also, browns and reds predominating, in contrast with olive-greens and occasional

purples; but to bring out the colors well and clearly requires large instruments. For the most part the markings are arranged in streaks more or less parallel to the planet's equator, as shown by Fig. 178. With a small telescope the markings usually reduce to two dark and comparatively well-defined belts, one on each side of the equator, occupying about the same regions of latitude that the trade-wind zones do upon the earth; and very likely in Jupiter's case similar aerial currents have something to do with the appearance, though upon Jupiter, as has been already said, the solar heat is a comparatively unimportant factor. The markings upon the planet are almost, if not entirely, *atmospheric*, as is proved by the manner in which they change their shapes and relative positions. They are *cloud forms*. It is hardly probable that we ever see anything upon the solid surface of the planet underneath, nor is it even certain that the planet has anything solid about it. In Fig. 178, the upper left-hand figure is from a drawing by Trouvelot made in February, 1872; the second is by Vogel in 1880. The small one below represents the planet as seen in a small telescope.

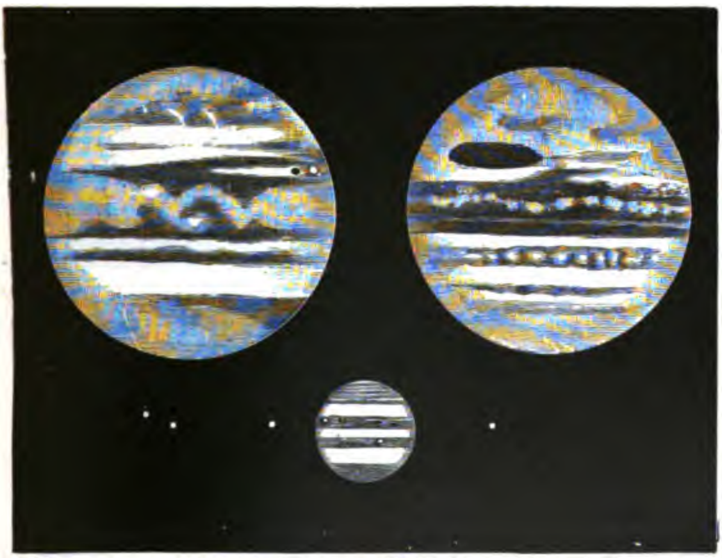

FIG. 178. — Telescopic Views of Jupiter.

618. The Great Red Spot. — While most of the markings on the planet are evanescent, it is not so with all. There are some which

are at least "sub-permanent," and continue for years, not without change indeed, but with only slight changes. The "great red spot" is the most remarkable instance so far. It seems to have been first observed by Prof. C. W. Pritchett of Glasgow, Missouri, in July, 1878, as a pale, pinkish, oval spot some 13″ in length by 3″ in width (30,000 miles by 7000). Within a few months it had been noticed by a considerable number of other observers, though at first it did not attract any special attention, since no one thought of it as likely to be permanent. The next year, however, it was by far the most conspicuous object on the planet. It was of a clear, strong brick-red color, with a length fully one-third the diameter of the planet and a width about one fourth of its length.

For two or three years it remained without much change: in 1882–83 it gradually faded out: in 1885 it had become a pinkish oval *ring*, the central part being apparently occupied with a white cloud. In 1886 it was again a little stronger in color, and the same in 1887, — an object not diffi-

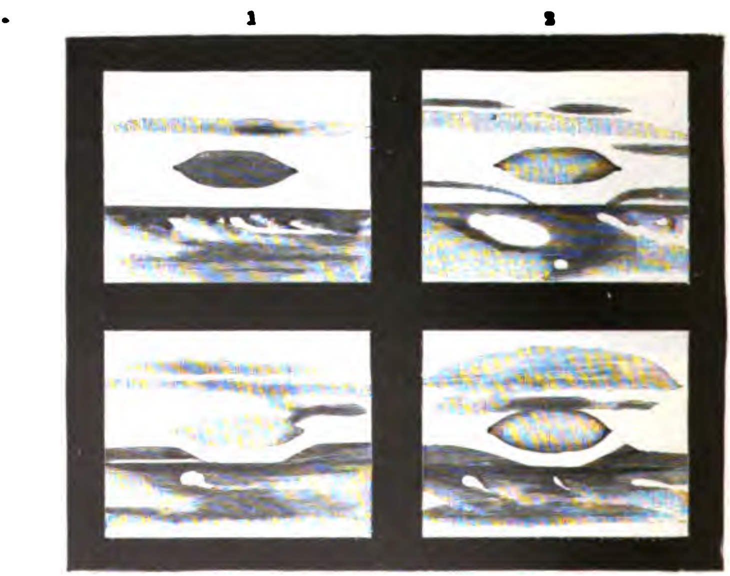

FIG. 179. — Jupiter's "Red Spot." From Drawings by Mr. Denning. 1880–85.

cult to see with a large telescope, but the merest ghost of what it was in 1880. It still persists (1897), though extremely faint, having shortened up a little and lost its pointed ends. It lies at the southern edge of the southern equatorial belt, in latitude about 30°, and for some reason the belt

seems to be "notched out" for it. Even when the spot was palest its *place* was always evident at once from the indentation in the outline of the belt.

It lies in an atmospheric belt which has a rotation-period 22 seconds shorter than its own, so that, to quote the expression of Williams, it "emerges like an island in a river," the current drifting past it at a rate of 12 or 15 miles an hour.

Such phenomena suggest abundant matter for speculation. It must suffice to say that no satisfactory explanation of the phenomena has yet been presented. The unquestionable fact before mentioned (Art. 615), that the time of rotation of the spot has changed by more than 6^s, greatly complicates the subject. Fig. 179, from the drawings of Mr. Denning, represents the appearance of the spot at four different dates; *viz.*, 1, 1880, Nov. 19; 2, 1882, Oct. 30; 3, 1884, Feb. 6; 4, 1885, Feb. 25.

619. Temperature and Physical Constitution. — The rapidity of the changes upon the visible surface implies the expenditure of a considerable amount of heat, and since the heat received from the sun is too small to account for the phenomena which we see, Zöllner, thirty years ago, following the suggestions of Buffon and Kant, practically demonstrated that it must come from within the planet, and that in all probability Jupiter is at a temperature not much short of incandescence, — hardly yet solidified to any considerable extent. Most astronomers suppose the visible features on the planet's surface to be purely atmospheric, but Hough considers that we see the pasty, semi-liquid surface of the globe itself.

620. Atmosphere. — As to the composition of the planet's atmosphere, the spectroscope gives us rather surprisingly little information. We get from the planet a good solar spectrum with the solar lines well marked, but there are no well-defined absorption bands due to the action of the planet's atmosphere. There are, however, some *shadings* in the lower red portion of the spectrum that are probably thus caused. The light, for the most part, seems to come from the upper surface of the planet's envelope of clouds without having penetrated to any depth.

Spectroscopic observations upon the relative shift of the dark lines in the spectrum at the eastern and western limbs, give a very fair determination of its rotation-period (by Doppler's principle).

621. Satellite System. — Jupiter has seven[1] satellites, — four of them the first heavenly bodies ever *discovered* — the first revelation of Galileo's telescope. His earliest observation of them was on Jan. 7, 1610, and in a very few weeks he had ascertained their true character, and determined their periods with an accuracy which is

[1] For the sixth and seventh satellites, see note on page 406.

surprising. The number of the heavenly bodies was now no longer *seven*, and the discovery excited among churchmen and schoolmen a great deal of angry incredulity and vituperation. Galileo called them "the Medicean stars."

These four are usually known as the first, second, etc., in the order of distance from the primary, but they also have names which are sometimes used; *viz.*, Io, Europa, Ganymede, and Callisto. Their relative distances range between 262000 and 1 169000 miles, being very approximately 6, 9, 15, and 26 radii of the planet. Their sidereal periods range between $1^d\ 18\frac{1}{2}^h$ and $16^d\ 16\frac{1}{2}^h$.

The fifth satellite was discovered by Mr. Barnard at the Lick Observatory in September, 1892. It is extremely small, and so near the planet that it is exceedingly difficult to see, and quite out of reach of any telescopes less than 18 or 20 inches in aperture. Its distance from the planet's centre is about 112500 miles, and its period $11^h\ 57.4^m$.

The orbits of all five of these satellites are almost circular, and lie very nearly in the plane of Jupiter's equator.

The satellites slightly disturb each other's motions, and from these disturbances their masses can be ascertained in terms of the planet's mass. The third, which is much the largest, has a mass of about $\frac{1}{11000}$ of the planet's, a little more than double the mass of our own moon. The mass of the first satellite appears to be a little less than $\frac{1}{3}$ as much. The second is somewhat larger than the first, and the fourth is about half as large as the third; *i.e.*, it has about the mass of our own moon. The densities of the first and fourth appear to be not very different from that of the planet itself, while the densities of the second and third are considerably greater.

622. Relation between Mean Motions and Longitudes of the Satellites. — In consequence of their mutual interaction a curious relation (discovered by La Place) exists between the mean motions of the first three satellites. The mean motion is of course 360° divided by T (T being the satellite's period). It appears that the mean motion of the first plus twice the mean motion of the third equals three times that of the second, or

$$\frac{1}{T_1} + \frac{2}{T_3} = \frac{3}{T_2}.$$

A similar relation holds for their longitudes:

$$L_1 + 2L_3 = 3L_2 + 180°;$$

so that they cannot all three come into opposition or conjunction with the sun at once. These relations are permanently maintained by their mutual attractions: *exactly* in the long run, though there are slight perturbations produced by the fourth satellite which disturb the arrangement slightly for short periods. The fourth and fifth satellites do not come into the arrangement.

623. Diameters, etc. — The diameter of the first satellite is a little more than 2400 miles; the second is almost exactly the size of our own moon, *i.e.*, between 2100 and 2200 miles; and the third and fourth have diameters, respectively, of 3600 and 3000 miles, the third, Ganymede, being much larger than either of his sisters. When Jupiter is in opposition, the fourth satellite is sometimes nearly 10½′ away from the planet, or ⅓ of the moon's diameter; and in very clear air can be seen by a sharp eye without telescopic aid. The third, though much larger, never goes more than 6′ from the planet, and it is perhaps doubtful whether it is ever seen with the naked eye, unless when the fourth happens to be close beside it. A good opera-glass will easily show them all as minute points of light. The fifth (new) satellite can hardly exceed 100 miles in diameter.

624. Brightness. — Since the sunlight of Jupiter is only $\frac{1}{27}$ as intense as ours, the moonlight made by the satellites is decidedly inferior to our own, although their reflective power appears to be higher than that of the lunar surface. They differ among themselves considerably in this respect. The fourth satellite is of an especially dark complexion. The others, under similar circumstances, show light or dark according as they have a dark or light portion of the planet for a background. Even the fourth, when crossing the disc, is always seen bright while very near the planet's limb.

625. Markings upon the Satellites. — The satellites show sensible discs when viewed with a large telescope, and all of them but the second sometimes show dark markings upon the surface. These markings, however, are only visible under the most favorable circumstances, and it has not been possible to determine whether they are atmospheric or really geographical, nor to deduce from them with certainty the satellites' periods of rotation.[1] W. Pickering has also reported certain periodical *changes of form* in the first and second satellites, as if they were whirling clouds or meteoric swarms, and not solid bodies. But his observations require confirmation.

626. Variability. — Galileo noticed variations in the brightness of the satellites at different times, and subsequent observers have confirmed his result. In the case of the fourth satellite there seems to be a regular variation depending upon the place of the satellite in its orbit, and suggesting that in its axial rotation it behaves like our own moon, keeping always the same side next its primary. In addition it shows other *irregular* changes in its luminosity: so also do the other satellites according to nearly all authorities, though it is singular that one or two of the best observers do not find any such irregularity indicated by their instrumental photometric observations.

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[1] Mr. Douglas of the Flagstaff Observatory reports in 1897 observations of the markings showing that the third and fourth satellites rotate like the moon, in periods sensibly identical with their orbital revolutions, confirming the earlier conclusion referred to in Article 626.

627. Eclipses and Transits. — The satellites' orbits are so nearly in the plane of the planet's orbit that, excepting the fourth, they all pass through the shadow of the planet, and suffer eclipse at every revolution. At conjunction, also, they cast their shadows upon the planet, and these shadows can easily be seen in the telescope as black dots on the planet's disc, the satellites themselves, which cross the disc about the same time, being much more difficult to observe. The fourth satellite escapes eclipse when Jupiter is far from the node of its orbit. Thus, during 1894 and the first three months of 1895, there were no eclipses of Callisto at all.

Exactly at opposition or conjunction the planet's shadow lies straight behind it out of our sight, so that we cannot at that time

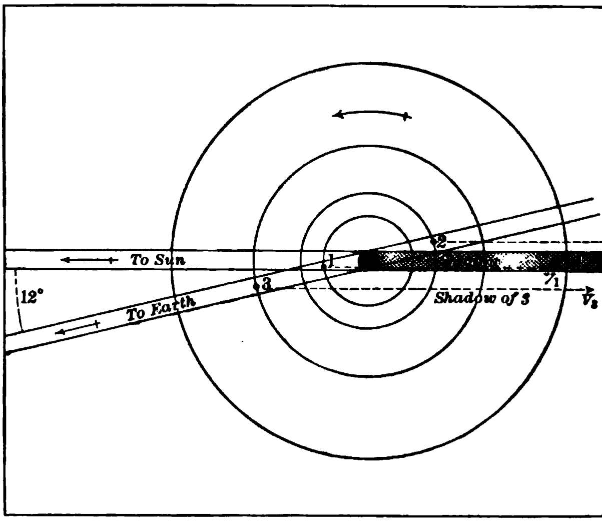

FIG. 180. — Eclipses of Jupiter's Satellites, at Western Elongation.

observe the eclipses, but only their transits across the disc. Before and after these times the shadow lies one side of the planet.

When the planet is at quadrature and the condition of things is as represented in Fig. 180 (which is drawn to scale), the shadow projects so far to one side of the planet that the whole eclipse of all the satellites, except the first, takes place clear of the planet's disc, — both the disappearance and reappearance of the satellite being visible.

628. "Equation of Light." — The most important use that has been made of these eclipses has been to ascertain the time required by light in traversing the distance between us and the sun, the so-

called "*equation of light.*" It was in 1675 that Roemer, the Danish astronomer (the inventor of the transit instrument, meridian circle, and prime vertical instrument,—a man nearly a century in advance of his day), found that the eclipses of the satellites showed a peculiar variation in their times of occurrence, which he explained as due to the time taken by light to pass through space. His bold and original suggestion was rejected by most astronomers for more than fifty years,—until long after his death,—when Bradley's discovery of aberration (Art. 225) proved the correctness of his views.

629. If the planet and earth remained at an invariable distance the eclipses of the satellites would recur with unvarying regularity (their disturbances being very slight), and the mean interval could be determined, and the times tabulated. But if we thus predict the times of eclipses for a synodic period of the planet, then, beginning at the time of opposition, it will be found that as the planet recedes from the earth, the eclipses fall constantly more and more behindhand, and by precisely the same amount for all four of the satellites. The difference between the tabulated and observed time continues to increase until the planet is near conjunction, when the eclipses are more than sixteen minutes late.

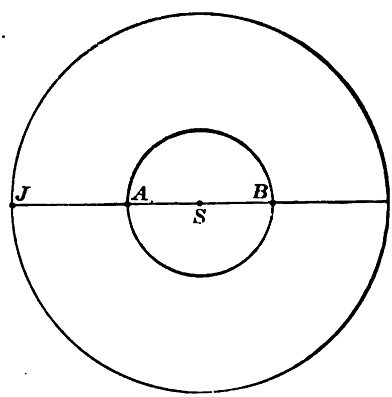

FIG. 181.
Determination of the Equation of Light.

From the insufficient observations at his command, Roemer made the difference twenty-two minutes.

After the conjunction, the eclipses quicken their pace and exactly make up all the loss; so that when opposition is reached once more, they are again on time.

It is easy to see from Fig. 181 that at opposition the planet is nearer the earth than at conjunction by just twice the radius of the earth's orbit; *i.e.*, $JB - JA = 2\,SA$. The whole apparent retardation of the eclipses between opposition and conjunction, should therefore be exactly twice the time required for light to come from the sun to the earth. This time is very nearly 500 seconds, or $8^m\ 20^s$.

Early in the century Delambre, from all the satellite eclipses of which he could then secure observations, found it to be 493ˢ. A few years ago a redetermination by Glasenapp of Pulkowa made it 501ˢ, from fifteen years' observation of the eclipses of the first satellite. The value at present accepted is 499ˢ, and can hardly be erroneous by more than 1ˢ.

630. Photometric Observations of the Eclipses. — The eclipses are *gradual* phenomena, the obscuration of the satellite proceeding continuously from the time it first strikes the shadow of the planet until it entirely vanishes. The moment at which the satellite seems to disappear depends, therefore, on the state of the air and of the observer's eye, and upon the power of his telescope. The same is true of the reappearance; so that the observations are doubtful to the extent of from half a minute for the first satellite (which moves quickly), to a full minute for the fourth. Professor Pickering has proposed to substitute for this comparatively indefinite moment of disappearance or reappearance, *the instant when the satellite has lost or regained just half its normal light*, and he determines this instant by a series of photometric comparisons with one of the neighboring uneclipsed satellites, or with the planet itself.

These comparisons are made with a special photometer devised for the purpose, and planned with reference to rapid reading: by merely turning a small button, the observer is immediately able to make the image of the uneclipsed satellite appear to be of the same brightness as the satellite which is disappearing, and the observations can be repeated very rapidly with the help of special contrivances for recording the times and readings. It is found that this instant of "half-brightness" can be deduced from the set of photometric readings with an error not much exceeding a second or two. Observations of this kind have now been going at Cambridge (U. S.)[1] for several years. A similar plan has also been devised by Cornu, and is being carried out at the Paris Observatory under his direction.

A series of such observations covering the planet's whole period of twelve years, ought to give us a much more accurate determination of the light-equation than we now have.

631. Until 1849 our only knowledge of the velocity of light was obtained by observations of Jupiter's satellites. By assuming as

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[1] Professor Pickering has more recently applied a photographic process to these observations with most gratifying success. A series of pictures is taken, each with an exposure of 10ˢ, the time being recorded on a chronograph, and they determine with great precision the moment when the satellite's brightness had any special value, say fifty per cent of its maximum.

known the earth's distance from the sun, the velocity of light follows when we know the time occupied by light in coming from the sun. At present, however, the case is reversed: we can determine the velocity of light by two independent experimental methods, and with a surprising degree of accuracy; and then, knowing the velocity and the light-equation, we can deduce the distance of the sun.

SATURN.

632. The Orbit and Period. — Saturn is the remotest of the ancient planets, its *mean distance* from the sun being 9.54 astronomical units, or 886000000 miles. The actual distance varies, however, by nearly 50000000 miles on account of the *eccentricity* of its orbit (0.056), which is a little greater than that of Jupiter.

Its nearest approach to the earth at a December opposition (the longitude of its perihelion being 90° 4') is 744 millions of miles, and its greatest distance at a May conjunction is 1028 millions. It is so far from the sun that these changes of distance do not so greatly affect its apparent brightness, as in the case of the nearer planets, the whole range of variation from this cause being less than two to one; that is, at the nearest of all oppositions, the planet is not twice as bright as the remotest of all conjunctions. The changing phases of the rings make quite as great a difference as the variations of distance.

The *orbit is inclined* to the ecliptic about 2½°.

The *sidereal period* of the planet is *twenty-nine and one-half years*, the *synodic period* being 378 days.

The planet itself is unique among the heavenly bodies. The great belted globe carries with it a retinue of eight satellites, and is surrounded by a system of rings unlike anything else in the universe so far as known, the whole constituting the most beautiful and most interesting of all telescopic objects.

633. Diameter, Volume, and Surface.—The apparent mean diameter of the planet varies from 20″ to 14″ according to the distance. We say *mean* diameter because this planet is more flattened at the pole than any other, its ellipticity being nearly ten per cent, though different observers vary somewhat in their results. The equatorial diameter of the planet is about 75000 miles, and its polar about 68000, the mean being very nearly 73000, or a little more than nine times that of the earth. Its *surface* is therefore about eighty-two times, and its *volume* 760 times that of the earth.

634. Mass, Density, and Gravity. — Its *mass* is only ninety-five times the earth's mass, from which follows the remarkable fact that the *density* of Saturn is only *one-eighth that of the earth*, or only *about five-sevenths that of water*. It is by far the least dense of all the planets. The *superficial gravity* is 1.2.

635. Axial Rotation. — It revolves upon its axis in about $10^h 14^m$ according to a determination of Professor Hall, made in 1876 by means of a white spot which suddenly appeared upon its surface, and continued visible for some weeks. His result is substantially confirmed by the observations of Stanley Williams in 1893, which, however, appear to indicate that spots in different latitudes give rotation-periods which differ slightly, but systematically.

The *inclination of the axis to the planet's orbit* is about 27°.

636. Surface, Albedo, and Spectrum. — As in the case of Jupiter, the edges of the disc are not quite so brilliant as the central portions, so that the belts appear to fade out near the limb. These belts are less distinct and less variable than those of Jupiter; and are arranged as shown in Fig. 182, with a very brilliant zone at the equator, though the engraving much exaggerates the contrast. The planet's pole is sometimes marked by a darkish cap of greenish hue.

According to Zöllner, the *Albedo*, or reflecting power of the surface, is 0.52, almost precisely the same as that of Venus, but a little inferior to that of Jupiter. The *spectrum* of the planet is the solar spectrum without any evidence of the presence of water-vapor, so far as can be made out, but with certain unexplained dark bands in the red and orange similar to those observed in the spectrum of Jupiter. The darkest of these bands, however, are not seen in the spectrum of the ring; this might have been expected, since the ring probably has but little atmosphere.

637. The Rings. — The most remarkable peculiarity of Saturn is his *ring-system*. The planet is surrounded by three, thin, flat, concentric rings like circular discs of paper pierced through the centre. Two of them are bright, while the third, the one nearest to the planet, is dusky and comparatively difficult to see. They are generally referred to by Struve's notation as *A*, *B*, and *C*, *A* being the exterior one.

For nearly fifty years this appendage of Saturn was a complete enigma to astronomers. Galileo, in 1610, saw with his little tele-

scope that the planet appeared to have something attached to it on each side, and he announced the discovery that "the outermost planet is triple," — "ultimam planetam tergeminam observavi."

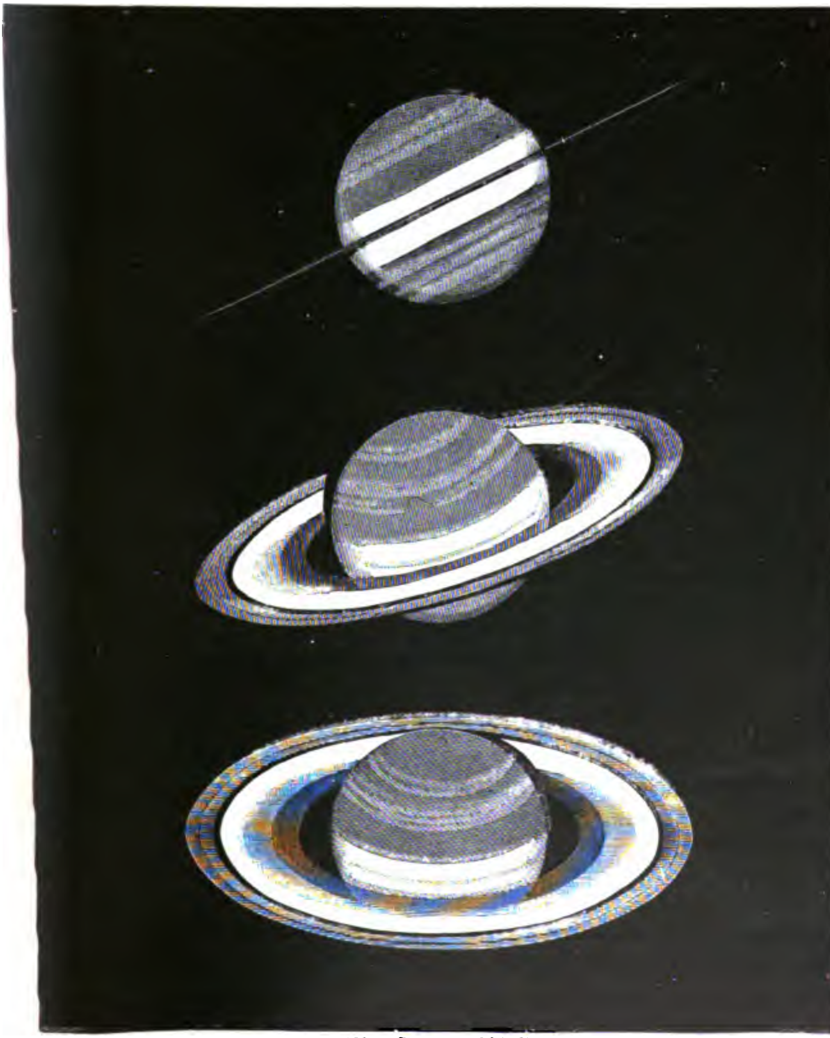

FIG. 182. — Saturn and his Rings.

Not long afterwards the rings were edgewise to the earth so that they became invisible to him; and in his perplexity he inquired "whether Saturn had devoured his children, according to the legend." Huy-

ghens, in 1655, was the first to solve the problem and explain the true structure of the rings. Cassini, twenty years later, discovered that the ring was double,—composed of two concentric portions with a narrow black rift of division between them.

The third, or dusky ring, *C*, is an American discovery, and was first brought to light by W. C. Bond at Cambridge, U. S., in November, 1850. About two weeks later, but before the news had been published in England, it was also discovered independently by Dawes.

For a while there was some question whether it was not really a new formation; but an examination of old drawings shows that Herschel and several other astronomers had previously seen it where it crosses the planet, although without recognizing its character.

638. Dimensions of the Rings.—The outer ring, *A*, has an exterior diameter of 168,000 miles, and is a little more than 10,000 miles wide. The division between it and ring, *B*, is about 1600 miles in width, and apparently perfectly uniform all around. Ring *B* is about 16,500 miles wide, and is much brighter than *A*, especially at its outer edge. At the inner edge it becomes less brilliant, and is joined without any sharp line of demarcation by ring *C*, which is sometimes known as the "*gauze*" or "*crape*" ring, because it is only feebly luminous and is semi-transparent, allowing the edge of the planet to be seen through it. The innermost ring is nearly, perhaps not quite, as wide as the outer one, *A*. There is thus left a clear space of from 9000 to 10,000 miles in width between the planet's equator and the inner edge of the gauze ring, the whole ring system having an external diameter of 168,000 miles, and a width of between 36,000 and 37,000.

The *thickness* of the rings is very small indeed, probably not exceeding 100 miles. If we were to construct a model of them on the scale of 10,000 miles to the inch, so that the outer one would be nearly seventeen inches in diameter, the thickness of an ordinary sheet of writing paper would be about in due proportion. This extreme thinness is proved by the appearances presented when the plane of the ring is directed towards the earth, as it is once in every fifteen years. At that time the ring becomes invisible for several days even to the most powerful telescopes.

639. Phases of the Rings.—The rings are parallel to the equator of the planet, which is inclined about 27° to its orbit, and about 28° to the plane of the ecliptic, the two nodes of the ring being in longitude 168° and 348°, in the constellations of Aquarius and Leo. Now

in the planet's revolution around the sun, the plane of the planet's equator and of the rings always keeps parallel to itself (as shown in Fig. 183), just as does the plane of the earth's equator. Twice, therefore, in the planet's revolution, when the plane of the ring

FIG. 183. — The Phases of Saturn's Rings.

passes through the earth, we see it edgewise;[1] and twice at its maximum width, when it is at the points half-way between the nodes. The angle of inclination being 28°, the apparent width of the ring at the maximum is just about half its length. The last disappearance of the rings was in October, 1891; the next will be in the summer of 1907. Near the time of disappearance the ring appears simply as a thin needle of light projecting on each side of the planet to a distance nearly equal to its diameter. Upon this the satellites are threaded like beads when they pass behind or in front of it.

640. Irregularities of Surface and Structure. — When the rings are edgewise we find that there are notable irregularities upon them. They are not truly plane, nor quite of even thickness throughout.

The same thing is indicated by certain peculiarities sometimes reported in the form of the shadow cast by the planet on the rings; but caution must be used in accepting and interpreting such observations, because illusions

[1] In traversing the earth's orbit the plane of the ring occupies about 359.6 days, during which the earth crosses it either *once* or *three times*, according to circumstances. The ring then becomes absolutely invisible to all existing telescopes for several days; nor can it be seen by any but very powerful instruments during the time while the plane lies between the earth and sun, often for several weeks. There are usually two such "periods of disappearance" during the critical year.

are very apt to occur from the least indistinctness of vision or feebleness of light. The writer has usually found that the better the seeing, the fewer abnormal appearances were noted, and the experience of the Washington observers is the same.

It can hardly be doubted that the details of the rings are continually changing to some extent. Thus the outer ring, *A*, is occasionally divided into two by a very narrow black line known as "Encke's division," although more usually there is merely a darkish streak upon it, not amounting to a real "crack" in the surface.

641. Structure of the Rings. — It is now universally admitted that the rings are not continuous sheets of either solid or liquid matter, but are composed of a swarm of separate particles, each a little independent moon pursuing its own path around the planet. The idea was suggested long ago, by J. Cassini in 1715, and by Wright in 1750, but was lost sight of until Bond revived it in connection with his discovery of the dusky ring. Professor Benjamin Peirce soon afterwards demonstrated that the rings could not be continuous solids; and Clerk Maxwell finally showed that they can be neither solid nor liquid sheets, but that all the known conditions would be answered by supposing them to consist of a flock of separate and independent bodies, moving in orbits nearly circular and in one plane, — in fact, a swarm of meteors.

641*. This "Meteoric Theory" has recently (in 1895) been beautifully confirmed by the spectroscopic observations of Keeler, illustrated in Fig. 183*. Photographs were made of the spectrum of the planet and its rings with the slit of the spectroscope crossing the planet's image, as shown in the figure. At the western limb of the planet and extremity of the ring the motion of rotation was carrying the particles from us, and the displacement of the spectrum lines should be towards the red, according to Doppler's principle (we note also in passing that, since the particles shine by *reflected* sunlight, the displacement is practically *doubled*, being twice as great as if they were self-luminous). On the eastern side the shift is towards the violet. Now, on looking at the diagram of the spectrum, given below the planet, we see that, while at *C* the line in the spectrum is displaced redwards, as it ought to be, *the displacement at the outer edge of the ring is less than at the inner;* and correspondingly at *A*. This shows that, as theory requires, the *outer edge revolves more slowly than the inner.* The fact is made conspicuous by its effect upon the *inclination* of the lines: while in the spectrum of the ball the lines slope upwards towards the right, in the ring-spectrum they slope the other way. The observation is very delicate, as the whole width of the spectrum

was not quite a millimetre (the figure being magnified nearly fifty times); but Keeler's results have since been fully confirmed by Deslandres, Belopolsky, and Campbell.

An independent photometric confirmation has been derived by Seeliger from the way in which the apparent brightness of the rings varies with their phases; and another from the behavior of Iapetus (the outer satellite), as observed by Barnard in 1892 while undergoing eclipse. The satellite vanished completely in passing through the shadow of the ball and bright rings, but reappeared when immersed in that of the semi-transparent dusky ring.

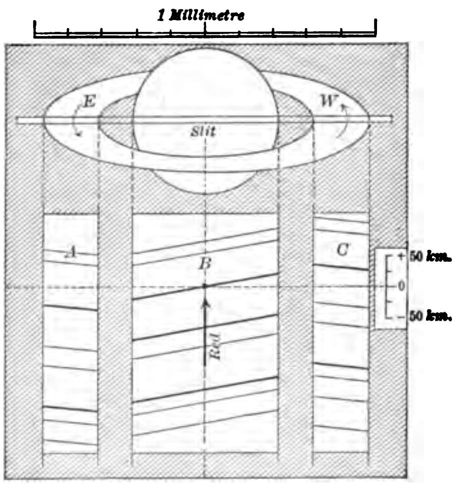

FIG. 183*. — Spectroscopic Observation of Saturn's Ring (Keeler).

The investigations of Hermann Struve show that the *mass of the rings is inappreciable:* they produce no observable effect upon the motion of the satellites. To use his graphic expression, "they seem to be composed of immaterial light," — mere dust-films or wreaths of fog.

642. Stability of the Ring. — If the ring were solid it would certainly not be stable, and the least disturbance would bring it down upon the planet; nor is it certain that even the swarm-like structure makes it forever secure. It is impossible to say positively that the rings may not after a time be broken up. A few years ago there was much interest in a speculation which Struve published in 1851. All the measures which he could obtain up to that date appeared to show that a change was actually in progress, and that the inner edge of the ring was extending itself towards the planet. His latest series of measurements (in 1885) does not, however, confirm this theory. They show no considerable change since 1850, and the measurements of other observers agree with his in this respect.

The researches of Professor Kirkwood of Indiana make it probable that the divisions in the ring are due to the perturbations produced by the satellites. They occur at distances from the planet where the period of a small body would be precisely commensurable with the periods of a number of the satellites. It will be remembered that similar gaps are found in the distribution of the asteroids, at points where the period of an asteroid would be commensurable with that of Jupiter.

643. Satellites. — Saturn has ten[1] of these attendants. The largest of them was discovered by Huyghens in 1655. It appears as a star of the ninth magnitude, and is easily observable with a three-inch telescope. Four others were discovered by Cassini before 1700, two by Sir William Herschel near the end of the last century, and one, Hyperion, by W. C. Bond of Cambridge, in September, 1848, and independently by Lassell at Liverpool two days later. (For Phœbe and Themis, the two newest, see note on page 406.)

The range of the system is enormous. Iapetus has a distance of 2,225000 miles, with a period of 79 days, nearly as long as that of Mercury. There is a remarkable variation in the brightness of this satellite. On the western side of the planet it is fully twice as bright as upon the eastern, which practically demonstrates that, like our own moon, it keeps the same face towards the planet at all times, one-half of its surface being much more brilliant than the other.

Mimas, the nearest and smallest of the satellites, coasts around the edge of the ring at a distance from it of only 34,000 miles, or 118,000 from the planet's centre, having a period of only $22\frac{1}{2}$ hours. This satellite is so small and so near the planet that it can be seen only by very large telescopes and under favorable conditions.

Titan, as its name suggests, is by far the largest of the family. Its distance is about 770,000 miles, and its period a little less than 16 days. It is probably 3000 or 4000 miles in diameter, and according to Stone, its mass is $\frac{1}{4600}$ of Saturn's.

644. Peculiar Behavior of Hyperion. — Hyperion has a distance of 934,000 miles, and a period of $21\frac{1}{4}$ days. Under the action of Titan its orbit is rendered considerably eccentric, and *its line of apsides always keeps itself in the line of conjunction with Titan,* retrograding in a way which at first seemed to defy theoretical explanation, but turns out to be only a "new case in celestial mechanics," and a necessary result of the disturbance by Titan. Mimas also undergoes a very considerable disturbance, which alternately accelerates and retards it to the extent of nearly 60° of its orbit.

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[1] Until Herschel's time it was customary to distinguish the satellites as first, second, etc., in order of distance from the planet; but as Herschel's new satellites were within the orbits of those which were known before, their discovery confused matters, and the confusion became worse confounded when the eighth appeared. They are now usually designated by names assigned by Sir John Herschel as follows, beginning with the most remote, namely: Iapĕtus (Hyperion), Titan; Rhea, Dione, Tethys; Enceladus, Mimas. It will be noticed that these names, leaving out Hyperion, which was undiscovered when they were assigned, form a line and a half of a regular Latin pentameter.

The orbit of Iapetus is inclined about 10° to the plane of the rings, but all the other satellites move exactly in their plane, and all the five inner ones move in orbits sensibly circular. The orbits of Iapetus, Hyperion, and Titan have a slight eccentricity.

URANUS.

645. As the satellites of Jupiter were the first heavenly bodies to be "discovered," so Uranus was the first "discovered" planet, all the other planets then known having been known from prehistoric antiquity. On March 13, 1781, the elder Herschel, in sweeping over the heavens systematically with a seven-inch reflector made by himself, came upon an object which, by its disc, he saw at once was not an ordinary star. In a day or two he had ascertained that it moved, and announced the discovery as that of a *comet*. After a short time, however, it became obvious from the computations of Lexell, that its orbit was nearly circular, that its distance was enormous, and that its path did not at all resemble that ordinarily taken by a comet; and within a year its planetary character was recognized and it was formally admitted as a new member of the solar system. The name of *Uranus*, suggested by Bode, finally prevailed over other appellations (Herschel himself called it the Georgium Sidus, in honor of the king), with the symbol ♅ or ⛢. The former is still generally used by English astronomers.

The discovery of a new planet, a thing then utterly unprecedented, caused great excitement. The king knighted Herschel, gave him a pension, and furnished him with the funds for constructing his great forty-foot reflector of four feet aperture, with which he afterwards discovered the two inner satellites of Saturn. It was found on reckoning back from the date of Herschel's discovery that the planet had been several times before observed as a star by astronomers who narrowly missed the honor which fell to the more fortunate and diligent Herschel. Twelve such observations had been made by Lemonnier alone.

646. Orbit.—The *mean-distance* of Uranus from the sun is very nearly 1800 millions of miles, and the *eccentricity* a trifle less than that of Jupiter's orbit, amounting to about 83,000000. The *inclination* of the orbit to the plane of the ecliptic is very slight, only 46′. The planet's *periodic time* is 84 years, and the *synodic period* (from opposition to opposition) $369^d\,16^h$. The *orbital-velocity* is $4\frac{1}{8}$ miles per second.

647. Appearance and Magnitude. — Uranus is distinctly visible to the naked eye on a dark night as a small star of the so-called sixth magnitude. It is so remote, its orbit having a diameter more than 19 times that of the earth's, that there is very little change in its appearance, and it makes no practical difference whether it is at opposition or quadrature.

In the telescope it shows a sea-green disc of about 4″ in apparent diameter, corresponding to a *real diameter* of 32,000[1] miles. Its *surface* is about 16 times, and its *volume* about 66 times greater than that of the earth, so that the earth compares in size with Uranus about as the moon does with the earth. The *mass* of Uranus is 14.6 times that of the earth, and its *density* and *surface-gravity* are respectively 0.22 and 0.90.

648. Albedo and Light. — The reflecting power of the planet's surface is very high, its *albedo*, according to Zöllner, being 0.64, even exceeding that of Jupiter. It is to be remembered, however, that sunlight at Uranus is only $\frac{1}{368}$ as intense as at the earth, and only about $\frac{1}{14}$ as intense as at Jupiter; so that the disc of the planet does not appear in the telescope even nearly as bright as a piece of Jupiter's disc of the same apparent size. The greenish blue tint of the planet is accounted for by the fact that its spectrum shows certain conspicuous dark bands in its lower portion, bands perhaps identical with those which are visible in the spectrum of Saturn, but much more intense. These facts probably indicate a dense atmosphere.

649. Polar Compression, Belts, and Rotation. — The disc of the planet shows a decided ellipticity — about $\frac{1}{14}$ according to the Princeton observations of 1883, which agree nearly with those of Schiaparelli, and have since been confirmed by Barnard at the Lick Observatory. There are also sometimes visible upon the planet's disc certain extremely faint bands or belts, much like the belts of Jupiter viewed with a very small telescope. What is exceedingly singular, however, is that the trend of these belts seems to indicate a *plane of rotation not coinciding with the plane of the satellites' orbits.* Nearly all the observers who have seen them at all find that they are inclined to the satellites' orbit-plane at an angle of from 15° to 40°. Now unless there is some error in Tisserand's investigations upon the motions of satellites, it is certain that the plane of these orbits must of necessity nearly coincide with the planet's equator. Probably the error lies in judging the direction of the belts, which at the best are at the very limit of visibility.

[1] See second note on page 406.

One or two observers have assigned to the planet rotation periods ranging from 9^h to 12^h; but it cannot be said that any determination of this element yet made is to be trusted.

650. Satellites. — The planet has four satellites; viz., Ariel, Umbriel, Titania, and Oberon; Ariel being the nearest to the planet. The two brightest of them, Oberon and Titania, were discovered by Sir William Herschel a few years after the discovery of the planet. He observed them sufficiently to obtain a reasonably correct determination of their distances and periods.

It is not certain that he saw either of the other two, though he *thought* he had found six satellites in all, and a few years ago a popular writer on astronomy actually credited the planet with *eight* satellites, — the four whose names have been given, and four others which Herschel supposed he had seen.

Ariel and Umbriel were first *certainly* discovered by Lassell in 1851, and have since been satisfactorily observed by numerous large telescopes. They are telescopically the smallest bodies in the solar system, and the most difficult to see. In real size, they are, of course, much larger than the satellites of Mars or many of the asteroids, very likely measuring from 200 to 500 miles in diameter; but they are ten times as far away as the asteroids, and illuminated by a sunlight not $\frac{1}{30}$ as brilliant as theirs.

651. Satellite Orbits. — The orbits of the satellites are sensibly circular, and all lie in one plane, which, as has been said, *ought* to be, and probably is, coincident with the plane of the planet's equator. They are very *close-packed* also, Oberon having a distance of only 375,000 miles, with a period of $13^d\ 11^h$, while Ariel has a period of $2^d\ 12^h$, at a distance of 120,000 miles. Titania, the largest and brightest of them, has a distance of 280,000 miles, somewhat greater than that of the moon from the earth, with a period of $8^d\ 17^h$. Under favorable circumstances this satellite can be just seen with a telescope of eight or nine inches aperture.

652. Plane of Revolution. — The most remarkable thing about this satellite system remains to be mentioned. The *plane of their orbits is inclined 82°.2* to the plane of the ecliptic, and in that plane they revolve *backwards;* or we may say, what comes to the same thing, that their orbits are inclined to the ecliptic at an angle of *97°.8*, in which case their revolution is to be considered as *direct.*

When the line of nodes of their orbit plane passes through the earth, as it did in 1840 and 1882, the orbits are seen edgewise and appear as straight lines. On the other hand, in 1861, they were seen almost *in plan*

as nearly perfect circles, and will be seen so again in 1903. The year 1882–83 was a specially favorable time for determining the inclination of the orbits and the position of the nodes, as well as for measuring the polar compression of the planet.

NEPTUNE.

653. The discovery of this planet is justly reckoned as the greatest triumph of mathematical astronomy. Uranus failed to move precisely in the path which the computers predicted for it, and was misguided by some unknown influence to an extent which a keen eye might almost see without telescopic aid. The difference between its observed place and that prescribed for it had become in 1845 nearly as much as the "intolerable" quantity of 2′ of arc.

The following illustration will show how extremely small was this discrepancy which the astronomers considered to be "intolerable."

Near the bright star Vega there are two little stars which form with it a small equilateral triangle, the sides of the triangle being about 1¾° long. The northern one of the two little stars is the beautiful double-double star ε Lyræ, and can be seen as double by a keen eye without a telescope, the two companions being about 3½′ apart. Now the distance between the computed place of Uranus and its actual position was, when at its maximum, just a little more than half of the distance between these components of ε Lyræ, that only a keen eye can separate. One would almost say that such a difference was hardly worth minding.

But just these minute discrepancies constituted the data which were found sufficient for calculating the position of a hitherto unknown planet, and bringing it to light. Leverrier wrote to Galle, in substance: "*Direct your telescope to a point on the ecliptic in the constellation of Aquarius, in longitude 326°, and you will find within a degree of that place a new planet, looking like a star of about the ninth magnitude, and having a perceptible disc.*" The planet was found at Berlin on the night of Sept. 23, 1846, in exact accordance with this prediction, within half an hour after the astronomers began looking for it, and only about 52′ distant from the precise point that Leverrier had indicated.

654. So far as the mathematical operations are concerned, the honor is to be equally divided between two then young men,—Leverrier of Paris, and Adams of Cambridge, England. Each took up the problem, and by perfectly independent and considerably different methods arrived at substantially the same solution, and each

promptly communicated the result (Adams some months earlier than Leverrier) to a practical astronomer provided with the necessary apparatus for actually detecting the planet.

Adams, who was then a graduate of three years' standing, a fellow and a tutor in his college, communicated his results to Challis, his professor of astronomy at Cambridge, in the autumn of 1845. Challis at once consulted Airy, the Astronomer Royal, but between them the matter rather lay in abeyance for some months, until a notice appeared of a preliminary paper by Leverrier, which indicated that he also had reached substantially the same conclusions as Adams. Then, at the urgent suggestion of Airy, Challis decided to begin the search at once, and to capture the planet by siege, so to speak. If he had had such star-maps as we now possess of the regions where the planet lay concealed, it would have been comparatively an easy operation; but as he had not, he decided to go over a space 10° wide by 30° long, and to go over it three times. The positions of all fixed stars would of course be the same at each of the three observations, but a planet would change its place in the meantime, and so would be surely detected.

He began his work on July 29, including in his sweep all stars down to the tenth magnitude. When, on Oct. 1, he learned of the actual discovery of the planet, he had recorded the positions of something over 3000 stars, and was preparing to map them. He had already secured, as it turned out, three observations of the planet on Aug. 4, Aug. 12, and Sept. 29, and of course it was only the question of a few weeks more or less when the discussion of the observations would have brought the planet to light.

But while this rather deliberate process was going on in England, Leverrier had revised his work, making a second approximation, and had communicated his results to Galle, at Berlin, substantially as above indicated. The Berlin astronomers had the great advantage of a new star-chart by Bremiker, covering that very region of the sky, and therefore did not need to enter upon any such tedious campaign as that begun by Challis. In less than half an hour they found a new star, not indicated on the map, and showing a sensible disc, just as Leverrier had predicted; and within twenty-four hours its motion proved it to be the planet.

655. Computed Elements Erroneous. — Both Adams and Leverrier, besides computing the planet's position in the sky had deduced elements of its orbit, and a value for its mass, which turned out to be considerably erroneous. The reason was that they had assumed *that the mean distance of the new planet from the sun would follow Bode's law*, a supposition which, as it turned out, is not even roughly true, although it was entirely warranted by the existing facts, since all the then known planets, not excepting Uranus, obey it with reasonable

exactness. This assumption of an erroneous mean distance of thirty-eight astronomical units, instead of the true distance of thirty, carried with it errors in all the other elements of the orbit; and the computed elements are so wide of the truth that great authorities have maintained that the actual Neptune was not at all the Neptune of Leverrier and Adams, but an entirely different planet; and even that the discovery was a "happy accident." It was not an accident at all, however. While the data and methods employed were not competent to determine the planet's *orbit* accurately, they *were* sufficient to determine the *direction* of the unknown body, which was the one thing needed to insure its discovery. The computers informed the searchers precisely where to point their telescopes, and could do so again were a new case of the same kind to appear.

656. Old Observations of Neptune. — After a few weeks' observation of the new planet it became possible to compute an approximate orbit; and reckoning back by means of this approximate orbit, the approximate place on any given date for many years preceding could be found. On examining the observations of stars made by different astronomers in these regions of the sky, there were found several instances in which they had observed the planet; a star of the ninth magnitude in the proper place for Neptune being recorded in their star-catalogues, while the place is now vacant. These old observations, thus recovered, were of great use in determining the planet's orbit with accuracy.

657. The Orbit of Neptune. — The planet's *mean distance* from the sun is a little more than 2800,000000 of miles, instead of being over 3600,000000, as it should be according to Bode's law. The orbit, instead of being considerably eccentric, as it appeared to be from the computation of Adams and Leverrier, is more nearly circular than any other in the system except that of Venus, its *eccentricity* being only $\frac{9}{1000}$. Even this small fraction, however, makes a variation of over 50,000000 of miles in the planet's distance from the sun at different parts of its orbit. The *inclination* of the orbit is about $1\frac{3}{4}°$. The *period* of the planet is about 164 years, instead of 217, as it should have been according to Leverrier's computed orbit. The *orbital velocity* is about $3\frac{1}{3}$ miles a second.

658. The Solar System as seen from Neptune. — At Neptune's distance the sun itself has an apparent diameter of only a little more than 1′ of arc, — only about the diameter of Venus when nearest us, and too small to be seen as a disc by the eye, if there are eyes on Neptune. The light and heat received from it are only $\frac{1}{900}$ part of

what we get at the earth. Still, we must not imagine that, as compared with starlight or even moonlight, the Neptunian sunlight is feeble.

Assuming Zöllner's estimate that sunlight at the earth is 618,000 times as intense as the light of the full moon, we find that the sun, even at Neptune, gives a light equal to 687 full moons. This is about seventy-eight times the light of a standard candle at one metre distance, or about the light of a thousand candle power electric arc at a distance of 10½ feet — abundant for all visual purposes. In fact, as seen from Neptune, the sun would look very much like a large electric arc-lamp at a distance of a few feet. We call special attention to this, because erroneous statements are not unfrequently met with that "at Neptune the sun would be only a first magnitude star." It would really give about 44,000000 times the light of such a star.

659. From Neptune the four terrestrial planets would be hopelessly invisible, unless with powerful telescopes and by carefully screening off the sunlight. Mars would never reach an elongation of 3° from the sun; the maximum elongation of the earth would be about 2°, and that of Venus about 1½°. Jupiter, attaining an elongation of about 10°, would probably be easily seen somewhat as we see Mercury. Saturn and Uranus would be conspicuous, though the latter is the only planet of the whole system that can be better seen from Neptune than it can be from the earth.

660. The Planet itself. — Neptune appears in the telescope as a small star of between the eighth and ninth magnitudes, absolutely invisible to the naked eye, though easily seen with a good opera-glass. It shows a greenish disc, having an apparent diameter of about 2″.6, which varies very little, since the entire range of variation in the planet's distance from us is only about $\frac{1}{15}$ of the whole. The real *diameter* of the planet is about 35,000[1] miles (but the probable error of this must be fully 500 miles); the *volume* is about 85 times that of the earth. Its *mass*, as determined by means of its satellite, is about 17 times that of the earth, and its *density* 0.20.

The planet's *albedo*, according to Zöllner, is about forty-six per cent, a trifle lower than that of Saturn and Venus, and considerably below that of Jupiter and Uranus. There are no visible markings upon its surface, and nothing is known as to its rotation. The spectrum of the planet appears to be precisely like that of Uranus. The light is so feeble that the ordinary lines of the solar spectrum are difficult to make out, but there are a number of heavy, dark bands, which indicate the existence of a dense atmosphere, through which the light, reflected by the cloud-covered surface of the planet, is transmitted, — an atmosphere which appears to be identically the

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[1] See second note on page 406.

same on both Uranus and Neptune, while some of its constituents are probably present in Jupiter and Saturn, as shown by the principal dark band in the red. It is not possible as yet to identify the substance which produces these bands.

It will be seen that Uranus and Neptune form a "pair of twins" very much as the earth and Venus do; being nearly alike in magnitude, density, and other characteristics.

661. Satellite. — Neptune has one satellite, discovered by Lassell within a month after the discovery of the planet itself. Its distance is 223,000 miles, and its period is $5^d\ 21^h\ 2^m.7$. Its orbit is inclined 34° 53′, and it moves *backward* in it; *i.e.*, clockwise, from east towards the west, like the satellites of Uranus. It is a very small object, appearing of about the same brightness as Oberon, the outer satellite of Uranus. From its brightness, as compared with that of Neptune itself, we estimate that its diameter is about the same as that of our own moon, though perhaps a little larger.

662. Trans-Neptunian Planet. — Perhaps the breaking down of Bode's law at Neptune may be regarded as an indication that the system terminates with him, and that there is no remoter planet. If such a planet exists, however, it is sure to be found sooner or later, either by means of its disturbing action upon Uranus and Neptune, or else by the methods of the asteroid hunters, although, of course, its slow motion will render its detection in this way difficult. Several observers have already devoted a good deal of time and labor to the search.

663. In the Appendix, we give tables containing the most accurate data of the planetary system at present available, but with renewed cautions to the student that these data are of very different degrees of accuracy.

The *distances* (in astronomical units), and the *periods* of the planets (except perhaps some of the asteroids) are known with extreme precision; probably the very last figure of the table may be trusted. The other elements of their *orbits* are also known very closely, if not quite so precisely as the distances and periods. The *masses, in terms of the sun's mass*, stand next to the orbit-elements in order of precision, with an error probably not exceeding one per cent (except, however, in the case of Mercury, the mass of which remains still very uncertain).

The ratio of the *earth's* mass to the sun's is however less accurately known, being at present subject to an uncertainty of at least one per cent. This is because its determination involves a knowledge of the solar parallax (Art. 278*), the *cube* of which appears in the formula for the ratio of the masses.

Of course all the masses of the planets *expressed in terms of the earth's mass* are subject to the same uncertainty in addition to all other possible causes of error.

When we come to the *diameters*, *volumes*, and *densities* of the planets, the data become less and less certain, as has been pointed out before. For the nearer and larger planets, say Venus, Mars, and Jupiter, they are reasonably satisfactory, for the remoter ones less so, and the figures for the density of the distant planets,—Mercury, Uranus, and Neptune, for instance,—are very likely subject to errors of ten or twenty per cent, if not more.

664. We borrow from Herschel's "Outlines of Astronomy" the following illustration of the relative magnitudes and distances of the members of our system. "Choose any well-levelled field. On it place a globe two feet in diameter. This will represent the sun; Mercury will be represented by a *grain of mustard-seed* on the circumference of a circle 164 feet in diameter for its orbit; Venus, a *pea* on a circle of 284 feet in diameter; the Earth also, a *pea* on a circle of 430 feet; Mars, a rather large *pin's-head* on a circle of 654 feet; the asteroids, *grains of sand* in orbits of 1000 to 1200 feet; Jupiter, a *moderate-sized orange* in a circle nearly half a mile across; Saturn, a *small orange* on a circle of four-fifths of a mile; Uranus, a *full-sized cherry* or *small plum* upon the circumference of a circle more than a mile and a half; and finally Neptune, a *good-sized plum* on a circle about two miles and a half in diameter. . . . To imitate the motions of the planets in the above-mentioned orbits, Mercury must describe its own diameter in 41 seconds; Venus, in 4^m 14^s; the Earth, in 7 minutes; Mars, in 4^m 48^s; Jupiter, in 2^h 56^m; Saturn, in 3^h 13^m; Uranus, in 2^h 16^m; and Neptune, in 3^h 30^m." We may add that on this scale the nearest *star* would be on the opposite side of the globe, at the antipodes, 8000 miles away.

Exercises on Chapter XVI.

1. When Jupiter is visible in the evening, do the shadows of his satellites precede or follow the satellites as they cross the planet's disc?

2. On which limb, the eastern or the western, do the satellites appear to enter upon the disc?

3. What probable effect would the great mass of Jupiter have upon the size of animals inhabiting it, if there were any?

4. How would sunlight upon Saturn compare with sunlight on the earth? How with moonlight?

5. What would be the greatest elongation of the earth from the sun as seen from Jupiter; from Saturn; from Uranus?

6. What would be the apparent angular diameter of the earth when "transiting" the sun as seen from Jupiter?

7. What is the rate in miles per hour at which a white spot on the equator of Jupiter, showing a rotation-period of $9^h\ 50^m$, would pass a dark spot indicating a period of $9^h\ 55^m$?

NOTE TO ARTS. 631 AND 643.

THE NEW SATELLITES. The sixth and seventh satellites of Jupiter were discovered in January and February, 1905, by Perrine, at the Lick Observatory, on photographs made with the Crossley reflector. They are both extremely small, — the seventh the smaller, — and probably beyond the reach of visual observation. They are far outside the region of the older satellites, — a pair of twins with orbits of nearly the same size, more than seven million miles in diameter, inclined about 30° to the plane of the planet's equator and to each other. But the data given in Table II are likely to be modified by later observations.

Phœbe, the ninth satellite of Saturn, was first announced by Professor W. H. Pickering, in 1898, as found on photographs made at Arequipa with the Bruce telescope. The discovery remained, however, without confirmation until 1904, when the satellite was again found upon a large number of later photographs, sufficient to permit a reasonably accurate determination of its orbit. The distance from the planet is about 8 000000 miles, the period 18 months, and *the orbital motion is retrograde!*

Themis, Saturn's *tenth* satellite, was found by Pickering in April, 1905, upon nine of the plates which had been used in the investigation of Phœbe. She is a little twin sister of Hyperion, but is three magnitudes fainter, and has an orbit of almost the same size and period, though more eccentric and differently tilted. The data given in Table II are to be regarded as provisional. It is worth noting that a number of the plates which were examined with reference to Themis show unexplained objects, — possibly asteroids, possibly other satellites.

NOTE TO ARTS. 647 AND 660.

Professor See at the U. S. Naval Observatory found much smaller values for the diameters of Uranus and Neptune. For Uranus he got 27,930 miles, and for the latter 27,190, reversing the hitherto received order of magnitude. This illustrates very well the uncertainty still hanging about the determination of the diameter of a small luminous disc.

CHAPTER XVII.

THE DETERMINATION OF THE SUN'S HORIZONTAL PARALLAX AND DISTANCE.—TRANSITS OF VENUS AND OPPOSITIONS OF MARS. — GRAVITATIONAL METHODS. — DETERMINATION BY MEANS OF THE VELOCITY OF LIGHT.

665. THIS problem, from some points of views, is the most fundamental of all that are encountered by the astronomer. It is true that it is possible to deal with many of the subjects that present themselves in the science without the necessity of employing any units of length and mass but those that are purely astronomical, leaving for subsequent determination the relation between these units and the more familiar ones of ordinary life: we can get, so to speak, a map of the solar system, *correct in proportion, though without a scale of miles.* But to give the map its real meaning and use, we must find the scale finally, if not at first, and until this is done we can form no true conceptions of the actual dimensions, masses, and distances of the heavenly bodies.

The problem of finding the true value of the astronomical unit is difficult, because of the great disproportion between the size of the earth and the distance of the sun. The relative smallness of the earth limits the length of our available "base line," which is less than $\frac{1}{12000}$ part of the distance to be determined by it. It is as if a person confined in an upper room with a wide prospect were set to determine the distance of objects ten miles or more away, without going outside the limits of his single window. It is hopeless to look for accurate results by *direct* methods, such as answer well enough in the moon's case, and astronomers are driven to indirect ones.

666. Historical. — Until nearly 1700 no even reasonably accurate knowledge of the sun's distance had been obtained. Aristarchus, by a very ingenious though inaccurate method, had found, as he thought, that the distance of the sun was nineteen times as great as that of the moon (it is really 390 times as great), and Hipparchus, combining this determination of Aristarchus with his own knowledge of the moon's distance, estimated the

sun's parallax at 3′, which is more than twenty times too large. This value was accepted by Ptolemy, and remained undisputed for twelve centuries, until Kepler, from Tycho's observations of Mars, satisfied himself that the sun's parallax could not exceed 1′; *i.e.*, that the sun's distance must be at least as great as twelve or fifteen millions of miles. Between 1670 and 1680 Cassini proposed to determine the solar parallax by observations of Mars; for by that time the distance of Mars from the earth at any moment could be very accurately computed in astronomical units, so that the determination of the parallax of Mars would make known that of the sun. Observations in France, compared with observations made in South America by astronomers sent out for the purpose, showed that the parallax of Mars could not exceed 25″, or that of the sun, 10″. Cassini set it at 9″.5, corresponding to a distance of 86,000000 of miles, — giving the first reasonable approach to the true dimensions of the solar system.

In 1677, and more fully in 1716, Halley explained how transits of Venus might be utilized to furnish a far more accurate determination of the solar parallax than was possible by any method before used. He died before the transits of 1761 and 1769 occurred, but they were both observed, the first not very satisfactorily, but the second with perfect success, and in the most widely separated parts of the globe. The results, however, were by no means as accordant as had been expected. Various values of the sun's parallax were deduced, ranging all the way from 8½″ to 9″, according to the observations used, and the way they were treated in the discussion. Towards the end of the century, La Place adopted and used for a while the value 8″.81, while Delambre preferred 8″.6.

667. In 1822–24 Encke collected all the transit observations that had been published, and discussed them as a whole in an extremely thorough manner, deducing as a final result from the two transits of 1761 and 1769, 8″.5776, corresponding to a distance of 95¼ millions of miles. The decimal is very imposing, and the discussion by which it was obtained was unquestionably thorough and laborious, so that his value stood unquestioned and classical for many years.

The first note of dissent was heard in 1854, when Hansen, in publishing certain researches upon the motion of the moon, announced that Encke's parallax was certainly too small; he afterwards fixed the figure at 8″.97, but the correction of certain numerical errors in his work reduced this result to 8″.92.

Three or four years later Leverrier found a value of 8″.95 from the so-called *lunar equation* of the sun's motion; and in 1862 Foucault, from a new determination of the velocity of light, combined with the constant of aberration, got the value 8″.86. A re-discussion of the old transit of Venus observations was then made by Stone, of England, who deduced from them

a value of 8″.943. The value of 8″.95 was adopted by the British Nautical Almanac, and used in it until the issue of 1882. The corresponding distance of 91⅓ millions of miles found its way into numerous text-books, and, though known to be erroneous, still holds its place in some of them.

In 1867 Newcomb made a discussion of all the data then available, and obtained the value 8″.848 (or 8″.85 practically), which value is still (1897) used in all the Nautical Almanacs except the French, which uses 8″.86. After 1900, however, it is agreed to use 8″.80 in all of them.

668. The observations of Gill on the planet Mars in 1877, and the new determinations of the velocity of light by Michelson and Newcomb in this country, as well as the investigations of Neison and others upon the so-called "parallactic inequality" of the moon, all point, however, to a somewhat smaller value. Professor Newcomb says (in 1885), "All we can say at present is that the solar parallax is probably between 8″.78 and 8″.83, or if outside these limits, that it can be very little outside." The latest investigations fully confirm this conclusion. (See note at end of the chapter.)

It was not, however, thought worth while to change the constant used in the almanacs until the final reduction of the transits of 1874 and 1882 had been made, and until certain experiments and investigations in progress have been finished. The difference between 8″.80 and 8″.85 is of no practical account for *almanac* purposes, and the change would involve alterations in a number of the tables.

Accepting Clarke's value of the earth's equatorial radius (Art. 145), *viz.*, 6378206.4^m or 3963.3 miles, we find that a solar parallax of

8″.75	corresponds to	23573	radii of the earth	= 93428000 miles.
8″.80	" "	23439	" " " "	= 92897000 "
8″.85	" "	23307	" " " "	= 92372000 "
8″.90	" "	23196	" " " "	= 91852000 "

669. Methods of finding the Solar Parallax and Distance.—We may classify them as follows:—

I. Ancient Methods.

(*A*) Method of Aristarchus [0].
(*B*) Method of Hipparchus [0].

II. Geometrical and Trigonometrical Methods, in which we attempt to find by angular measurements the parallax, either of the sun itself or of one of the nearer planets.

(*A*) The direct method [0].

(*B*) Observations of the displacement of Mars among the stars at the time of opposition.

(*a*) Declination observations from two or more stations in widely different latitudes made with meridian circles or micrometer [25].

(*b*) Observations made at a single station near the equator, by measuring the distance of the planet east or west from neighboring stars, using the heliometer [90].

(*C*) Declination observations of Venus [20].

(*D*) Observations of some of the nearer asteroids in the same way as Mars.

(*a*) Meridian observations at two stations in widely different latitudes [20].

(*b*) Heliometer observations at an equatorial station [90].

(*E*) Observations of the transits of Venus at widely separated stations.

(*a*) Observations of the contacts.

(1) Halley's method — the "method of *durations*" [40].

(2) Delisle's method — observation of *absolute times* [50].

(*b*) Heliometer measurements of the position of the planet on the sun [75].

(*c*) Photographic methods — various [20 to 75].

III. Gravitational Methods.

(*A*) By the moon's parallactic inequality [70].

(*B*) By the lunar equation of the sun's motion [40].

(*C*) By the perturbations produced by the earth on Venus and Mars [70]; (ultimately [95]).

IV. By the Velocity of Light, combined with

(*A*) The light equation [80].

(*B*) The constant of aberration [90].[1]

The figures in brackets at the right are intended to express roughly the relative value of the different methods, on the scale of 100 for a method which would insure absolute accuracy.

670. Of the Ancient Methods, that of Aristarchus is so ingenious and simple that it really deserved to be successful. When the moon is exactly at the half phase, the angle at *M* (Fig. 184) must be just

[1] For spectroscopic method, see note on page 427.

90°, and the angle AEM must equal MSE. If, then, we can find how much shorter the arc NM (from new to half moon) is than MF (from half moon to full), *half the difference will measure AM, and give the angle at S.* Aristarchus concluded that the first quarter of the month was just about *twelve hours* shorter than the second, so that AM was measured by six hours' motion of the moon, or a little less than 4°. Hence he found SE, the distance of the sun, to be about nineteen times EM — a value absurdly wrong, since SE is in fact nearly 390 times EM. The real difference between the two quarters of the month is only about half an hour, instead of twelve hours.

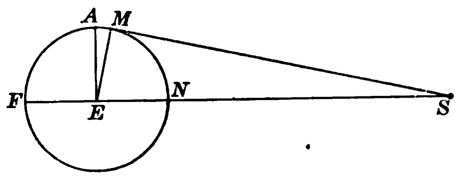

FIG. 184.

Aristarchus' Method of Determining the Sun's Distance.

The difficulty with the method is that, owing to the ragged and broken character of the lunar surface, it is impossible to observe the instant of half moon with sufficient accuracy.

671. The estimate of Hipparchus was based upon the erroneous calculation of Aristarchus that the sun's distance is 19 times the moon's, and the solar parallax, therefore, $\frac{1}{19}$ of the moon's parallax.

The "radius of the earth's shadow," where the moon cuts it at a lunar eclipse, is given, as Hipparchus knew, by the formula $\rho = P + p - S$ (Art. 372), or $P + p = \rho + S$. Assuming that $P = 19p$, we have $20p = \rho + S$. Now S, the sun's semi-diameter, is about 15′; and from the duration of lunar eclipses Hipparchus found ρ to be about 40′; hence he obtained for p, the solar parallax, a value a little less than 3′, which, as has been already mentioned, was accepted by Ptolemy, and by succeeding astronomers for more than 1500 years. (Wolf's "History of Astronomy," p. 175.)

672. Of the Geometrical Methods, *A*, the "direct method" consists in observing the sun's apparent declination with the meridian circle at two stations widely differing in latitude, just as we observe the moon when finding its parallax (Art. 239). Theoretically, observations of this sort might give the value of the sun's parallax within ½″ or so, but the method is practically worthless, because the errors of observation are large as compared with the quantity to be determined. The sun's limb is a very bad object to point on, and besides, its heat disturbs the adjustments of the instrument, thus rendering the observations still more inaccurate.

673. The first of the two methods of observing the planet **Mars** is precisely the same as this direct method of observing the sun; but the distance of Mars at a "near opposition" is only a little more than $\frac{1}{3}$ that of the sun, so that any error of observation affects the final result by only about $\frac{1}{3}$ as much; and, moreover, Mars is a very good object to observe, so that the errors of observation themselves are much lessened. The planet's distance from the earth having been found in astronomical units by the method of Art. 515, the determination of its distance in miles will fix the value of this unit, and so give us directly the sun's distance and parallax.

The method requires two observers working at a distance from each other with different instruments, which is a serious disadvantage.

For some unexplained reason, observations of this sort seem almost invariably to give too large a result for the solar parallax, averaging between 8″.90 and 8″.98. The red color of the planet may possibly have something to do with this by affecting the astronomical refraction. This method, in 1680, was the first to give a reasonable approximation to the sun's true distance, as has been mentioned before.

The planet Venus can be observed in the same way, and has been once so observed by Gillis, 1849–52, at Santiago, Chili, in co-operation with the Washington observers, but the result was not very satisfactory.

674. Heliometer Observations of Mars (Method *b*).—It is possible, however, for a *single* observer to obtain better results than can be got by two or more using the preceding method. Suppose that the orbital motion of Mars is suspended for a while at opposition, and that the planet is on or near the celestial equator;

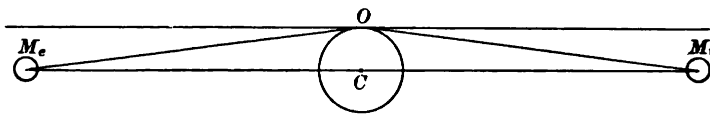

Fig. 185.—Effect of Parallax on the Right Ascension of Mars.

and also that the observer is at a station, *O*, on the earth's equator. When Mars is rising at M_e, Fig. 185, the horizontal parallax OM_eC depresses the planet; that is, he appears from *O* to be further *east* than he would if seen from *C*, the centre of the earth; so that the parallax then *increases* the planet's right ascension. Twelve hours later, when he is setting, the parallax will

throw him towards the west, *diminishing* his right ascension by the same amount. If, then, when the planet is rising, we measure carefully its distance west of a star S, which is supposed to be just east of it (the distance M_eS in Fig. 186), and then measure the distance M_wS from the same star again when it is setting, the difference will give us twice the horizontal parallax. The earth's rotation will have performed for the observer the function of a long journey in transporting him from one station to another 8000 miles away in a straight line.

675. Of course the observations are not practically limited to the moment when the planet is just rising, nor is it necessary that the star measured from should be exactly east or west of the planet. Measures from a *number* of the neighboring stars, S_1, S_2, S_3, and S_4 would fix the positions M_e and M_w with more accuracy than measures from S alone. Nor will the planet stop in its orbit to be observed, nor will it have a declination of zero, nor can the observer command a station exactly on the earth's equator. But these variations from the ideal conditions do not at all affect the principles involved; they simply complicate the calculations slightly without compromising their accuracy.

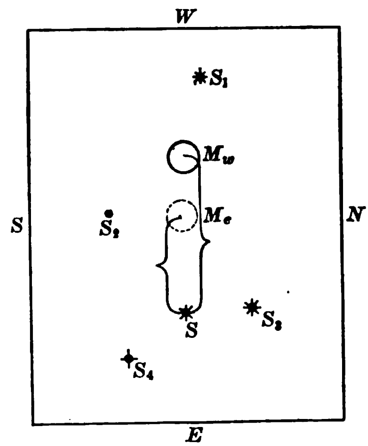

FIG. 186.

Micrometric Comparison of Mars with Neighboring Stars.

The method has the very great advantage that all the observations are made by one person, and with one instrument, so that, as far as can be seen, all errors that could affect the result are very thoroughly eliminated.

676. The most elaborate determination of the solar parallax yet made by this method is that of Mr. Gill, who was sent out for the purpose by the Royal Astronomical Society in 1877 to Ascension Island in the Atlantic Ocean. His result, from 350 sets of measurements, gives a solar parallax of $8''.783 \pm 0''.015$, — a result probably very close to the truth, though possibly a little small. In 1892 and 1894 favorable oppositions of Mars again occurred, and the observations were repeated at the Cape of Good Hope and elsewhere with confirmatory results.

Venus cannot be observed in this way, since either her rising or setting is in the daytime, when the small stars cannot be seen near her.

676*. Heliometer Observations of Asteroids. — Heliometer observations of the nearer asteroids furnish perhaps the best of all the geometrical methods. It is true that the minor planets are more distant than Mars, and that a given error in their observation entails, therefore, a greater error in the final result. But on the other hand, they can be observed with far greater precision, because they appear as mere star-like points, in brightness and color just like the stars around them which serve as points of reference.

The method has been applied several times, most recently[1] in 1889–90, when three of the asteroids, Victoria, Iris, and Sappho, were concertedly, and most carefully, observed by Dr. Gill at the Cape of Good Hope, Dr. Elkin at New Haven, and two or three observers in Europe. The observations have been thoroughly reduced, and the results are very accordant and apparently extremely accurate. Gill obtains from them for the parallax of the sun $8''.802 \pm 0''.005$.

677. The Heliometer. — The heliometer, the instrument employed in these measures, is one of the most important of the modern instruments of precision. As its name implies, it was first designed to measure the diameter of the sun, but it is now used to measure any distance ranging from a few minutes up to one or two degrees, which it does with the same accuracy as that with which the filar micrometer measures distances of a few seconds. It is a "double image" micrometer, made by dividing the object-glass of a telescope along its diameter, as shown in Fig. 187. The two halves are so mounted that they can slide by each other for a distance of three or four inches, the separation of the centres being accurately measured by a delicate scale, or by a micrometer screw operated and read by a suitable arrangement from the eye-end. The instrument is mounted equatorially with clock-work, and the tube can be turned in its cradle so as to make the line of division of the lenses lie in any desired direction. When the centres of the two halves of the object-glass coincide, the whole acts as a single lens. As soon as the centres are separated, each half of the object-glass forms its own image.

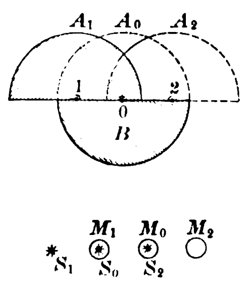

FIG. 187. — The Heliometer.

To measure the distance from Mars to a star, the telescope tube is turned so that the line of centres points in the right direction, and then the semi-lenses are separated until one of the two images of the star comes exactly in the centre of one of the images of Mars; this can be done in two positions

[1] See note on page 426.

of the semi-lens A with respect to B, as shown by the figure. We may either make S_0 (the image of the star formed by semi-lens B) coincide with M_1 formed by A, or make S_2 coincide with M_0. The whole distance from 1 to 2 then measures *twice* the distance between M and S.

678. Transit of Venus Observations. — At the time when Venus passes between us and the sun, her distance from the earth is only some 26 000000 of miles, so that her horizontal parallax is nearly four times as great as that of the sun itself. At this time her apparent displacement upon the sun's disc, due to a change of the observer's station upon the earth, is the *difference* between her own parallax due to this displacement, and that of the sun itself; and this difference is greater than the sun's parallax nearly in the ratio of 3 to 1, or, more exactly, of 723 to 277.[1] The object, then, of the observations of a transit is to obtain in some way a measure of the angular displacement of Venus on the sun's disc, corresponding to the known distance between the observer's stations upon the earth.

679. Halley's Method. — The method proposed by Halley, who in 1677 brought to notice the great advantages presented by a transit of Venus for determining the sun's parallax, was as follows: Two stations are chosen upon the earth's surface, as far separated *in latitude* as possible. From them we observe the ***duration*** of the transit; that is, the interval of time between its beginning and end, both of which must be visible at both stations. If the clock runs correctly during the few hours during which the transit lasts, this is all that is necessary. We do not need to know its error in reference to Greenwich time, nor even in respect to the local time, except roughly. This was a great advantage of the method in those days, before the era of chronometers, when the determination of the longitude of a place was a very difficult and uncertain operation.

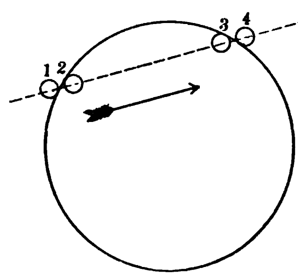

FIG. 188.
Contacts in a Transit of Venus.

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[1] Since the distance of Venus from the sun is 0.723 of the astronomical unit, her distance from the earth at time of transit is 0.277. Let p be the sun's parallax and v that of Venus: then, since the parallax of a body is inversely proportional to its distance from the earth (Art. 83), $v = p \times \frac{1000}{277}$; and $v - p = p\left(\frac{1000 - 277}{277}\right) = p \times \frac{723}{277}$, as stated.

The observation to be made is simply to note the clock time at which "*contact*" occurs, there being four of these contacts, — two exterior and two internal, at the points marked 1, 2, 3, 4, in Fig. 188. Halley depended mainly on the two internal contacts, which he supposed could be observed with an error not exceeding one second of time.

680. Computation of the Parallax. — Having the durations of the transits at the two stations, and knowing the hourly angular motion of Venus, we have at once and very accurately the length of the two chords described by Venus upon the sun, expressed in seconds of arc. We also know the sun's semi-diameter in seconds, and hence

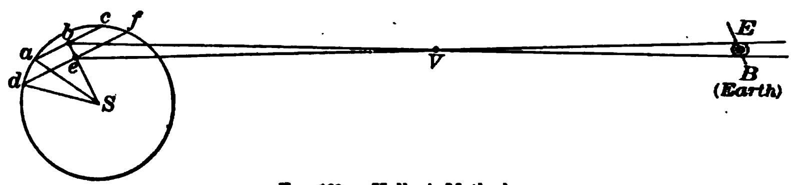

FIG. 189. — Halley's Method.

in the triangles (Fig. 189) *Sab* and *Sde*, we can compute the length (in seconds still) of *Sb* and *Se*, the difference of which, *be*, is the angular displacement, due to the distance between the stations on the earth.[1] The virtual base line is, of course, not the distance between *B* and *E* as a straight line, because that line is not perpendicular to the line of sight from the earth to Venus, nor to the plane of the planet's orbit, but the true value to be used is easily found. Calling this base line β, we have

$$p'' = (be)'' \times \left(\frac{277}{723}\right)\left(\frac{r}{\beta}\right),$$

r being the radius of the earth.

The rotation of the earth, of course, comes in to shift the places of *E* and *B* during the transit, but this can easily be allowed for; and if the stations are well situated, it increases the difference between the two durations, and increases the accuracy of the result.

[1] In order that the method may be practically successful, it is necessary that the transit track should lie near the edge of the sun's disc, for two reasons. It is desirable that the duration should not be more than three or four hours, while for a *central* transit it lasts eight hours (Art. 575). Moreover, if the two chords were near the centre of the disc, any small error in the length of either chord would produce a great error in the computed distance between them. When they lie as in the figure (which has been the case in all recent transits), the reverse is true: a considerable error in the observed length of one of the chords affects their computed distance only slightly.

681. The Black Drop. — Halley expected, as has been said, that it would be possible to observe the instant of internal contact within a single second of time, but he reckoned without his host. At the transits of 1761 and 1769, at most of the stations the planet at the time of internal contact showed a "ligament" or "black drop," like Fig. 190, instead of presenting the appearance of a round disc neatly touching the edge of the sun; and the time of real contact was thus made doubtful by 10^s or 15^s.

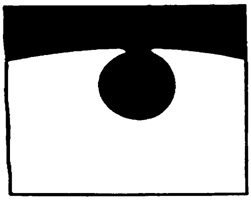

FIG. 190. — The Black Drop.

This "ligament" depends upon the fact that the optical edge of the image of a bright body is not, and in the nature of things cannot be, absolutely sharp in the eye or in the telescope. In the eye itself we have *irradiation*. In the telescope we have the difficulty that even in a *perfect* instrument the image of a luminous point or line has a certain width (which with a given magnifying power is less for a large instrument). Moreover a telescope is usually more or less imperfect, and practically adds other defects of definition, so that whenever the limbs of two objects approach each other in the field of view of a telescope we have more or less distortion due to the overlapping of the two "penumbras of imperfect definition," — the same sort of effect that is obtained by putting the thumb and finger in contact, holding them up within two or three inches of the eye and then separating them: as they separate, a "black ligament" will be seen between them.

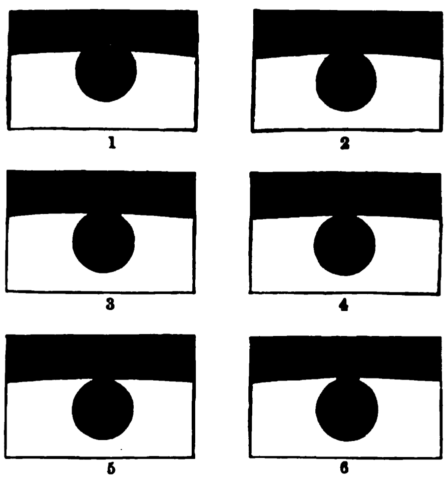

FIG. 191. — Atmosphere of Venus as seen during a Transit. (Vogel, 1882.)

With modern telescopes, and by great care in preventing the sun's image from being too bright, so as to diminish *irradiation in the eye* as far as possible, the black drop was reduced to reasonably small proportions in 1874 and 1882, and practice beforehand with an "artificial transit" enabled the observer in some degree to allow for its effect. But a new difficulty appeared, from which there seems to be absolutely no way of escape, — the *planet's atmosphere*

causes it to be surrounded by a luminous ring as it enters upon the sun's disc, and thus renders the time of the contact uncertain by at least five or six seconds. In both the transits of 1874 and 1882, differences of that amount continually appeared among the results of the best observers. Fig. 191 shows the appearances due to this cause as observed by Vogel in 1882.

682. Delisle's Method.—Halley's method requires the use of *polar* stations, uncomfortable and hard to reach, and also that the weather should permit the observer to see *both* the beginning and end of the transit.

Delisle's method, on the other hand, utilizes two stations *near the equator*, taken on a line roughly parallel to the planet's motion. It requires also that the observers *should know their longitude accurately*, so as to be able to determine the Greenwich time at any moment; but it does *not* require that they should see *both* the beginning and end of the transit; observations of *either* phase can be utilized: and this is a great advantage. Suppose, then (Fig. 192), that the observer W on one side of the earth notes the moment of internal contact in Greenwich time, the planet then being at V_1. When E notes the contact (also in Greenwich time) the planet will be at V_2, and the angle at D will be the angular diameter of the earth as seen

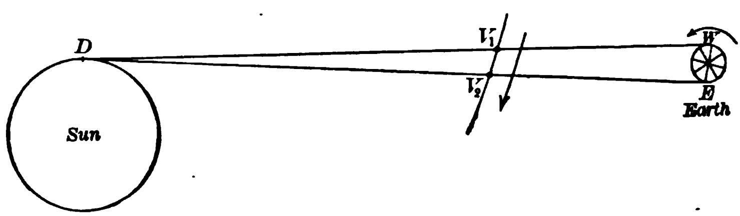

FIG. 192.—Delisle's Method.

from D; *i.e.*, *simply twice the sun's parallax.* Now the angle D is at once determined by the time occupied by Venus in passing from V_1 to V_2, since in 584 days (the synodic period) she moves completely round from the line DW to the same line again. If the time from V_1 to V_2 were twelve minutes, we should find the angle at D to be about 18″.

The observations of the internal contacts of the transits of 1874 and 1882 give results according to Newcomb ranging from 8″.72 to 8″.88, with a weighted mean according to Newcomb of 8″.794.

683. Heliometer Observations.—Instead of observing simply the times of contact, and leaving the rest of the transit unutilized, as in the two preceding methods, it is possible to make a continuous series

of measurements of the distance and direction of the planet from the nearest point of the sun's limb. These measurements are best made with the heliometer (Art. 677), and give the means of determining the planet's apparent position upon the sun's disc at any moment with extreme precision. Such sets of measurements, made at widely separated stations, will thus furnish accurate determinations of the apparent displacement of the planet on the sun's disc, corresponding to known distances on the earth, and so will give the solar parallax.

During the transit of 1882 extensive series of observations of this sort were made by the German parties, two of which were in the United States, — one at Hartford, Conn., and the other at San Antonio, Texas.

For some reason, not clearly evident, the results, while very accordant among themselves, are considerably larger than the average deduced from other methods. From the heliometer observations of 1874 the parallax came out 8″.876, and from those of 1882, 8″.879.

684. Photographic Observations. — The heliometer measurements cannot be made very rapidly. Under the most favorable circumstances a complete set requires at least fifteen minutes, so that the whole number obtainable during the seven or eight hours of the transit is quite limited. Photographs, on the other hand, can be made with great rapidity (if necessary, at the rate of two a minute), and then after the transit we can measure at leisure the position of the planet on the sun's disc as shown upon the plate. At first sight this method appears extremely promising, and in 1874 great reliance was placed upon it. Nearly all the parties, some fifty in number, were provided with elaborate photographic apparatus of various kinds. On the whole, however, the results, upon discussion, appear to be no more accordant than those obtained by other methods, so that in 1882 the method was generally abandoned, and used only by the American parties, who employed an apparatus having some peculiar advantages of its own.

685. English, German, and French Methods. — In 1874, the English parties used telescopes of six or seven inches aperture, and magnified the image of the sun formed by the object-glass by a combination of lenses applied at the eye-end. There were no special appliances for eliminating the distortion produced by the enlarging lenses, nor for ascertaining the exact orientation of the picture (that is, the direction of the image upon the plate with reference to north and south), nor for determining its scale.

The Germans and Russians employed a nearly similar apparatus, but with the important difference that at the principal focus of the object-glass they inserted a plate of glass ruled with squares. These squares are photographed upon the image of the sun, and furnish a very satisfactory means of determining the scale and distortion, if any, of the image. The object-glasses used by the English and the Germans had a focal length of seven or eight feet. The French employed object-glasses with a focal length of some fourteen feet, the telescope being horizontal, while the rays of the sun were reflected into it by a plane mirror; instead of glass plates they used the old-fashioned metallic daguerreotype plates, in order to avoid any possible "creeping" of the collodion film, which was feared in the more modern wet-plate process. The French plates furnish, however, no accurate orientation of the picture.

686. The American Apparatus. — The Americans used a similar plan, with some modifications and additions. The telescope lenses employed were five inches in diameter and forty feet in focal length, so that the image directly formed upon the plate was about 4½ inches in diameter, and needed no enlargement. The telescope was placed horizontal and in the meridian, its exact direction being determinable by a small transit instrument which was mounted in such a manner that it could look into the photograph telescope, as into a collimator, when the reflector was removed. The reflector itself was a plane mirror of unsilvered glass driven by clock-work. *Fig. 193* shows the arrangement of the apparatus. In front of the photographic plate, and close to it, was supported a glass plate ruled with squares called the "reticle plate," and in the narrow space between this and the photograph

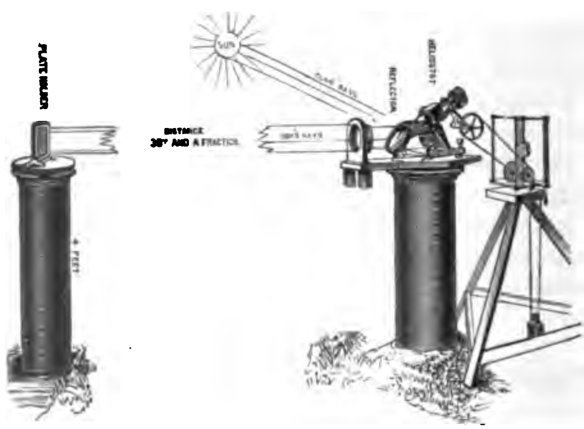

FIG. 193. — American Apparatus for Photographing the Transit of Venus.

plate was suspended a plumb-line of fine silver wire, the image of which appeared upon the plate, and gave the means of determining the orientation of the image with extreme precision. If the reflector were, and would continue to be, *perfectly plane* through the whole operation, the method could not fail to give extremely accurate results; but the measurements and discussion of the observations seem to show that this mirror was actually distorted to a considerable extent by the rays of the sun. On the whole the American plates do not appear to be much more trustworthy than those obtained by other methods. Fig. 194 is a reduced copy of one of the photographs made at Princeton during the transit of 1882. The black disc near the middle, with a bright spot in the centre, is the image of a metal disc cemented to the reticle to mark the centre lines of the reticle plate; 192 plates were taken during the transit, and at some of the stations where the weather was good the number was much greater — nearly 300 in some cases.

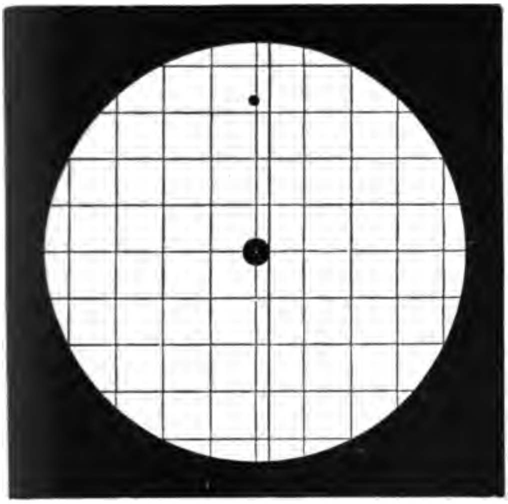

FIG. 194. — Photograph of Transit of Venus.

The difficulties to be encountered are numerous. Photographic irradiation, or the spread of the image on the plate, slight distortion of the image by the lenses or mirrors employed, irregularities of atmospheric refraction, uncertainty as to the precise scale of the picture, — all these present themselves in a very formidable manner. It is obvious why this should be so, when we recall that on a four-inch picture of the sun's disc, $\frac{1}{10000}$ of an inch corresponds to about $\frac{1}{20}$ of a second of arc, and the whole uncertainty as to the solar parallax does not amount to as much as that. An image of the sun, therefore, in which the position of Venus upon the sun's disc cannot be determined accurately without an error exceeding $\frac{1}{10000}$ of an inch, is of very little value. Imperfections that would be of no account whatever in plates taken for any other purpose make them practically worthless for this.

The mean result of the photographic measures, using the weights assigned by Newcomb, gives a parallax of 8″.834.

Newcomb in his "Astronomical Constants" combines the heliometer measures with the photographic, and as the result of all *the measures of the position of Venus upon the sun's disc during the transit* gives 8″.857 ± 0″.023; calling attention to the fact that measures

of this kind seem to be affected by some constant, systematic error.

On the whole, the outcome of the two transits of 1874 and 1882 has been to satisfy astronomers generally that other methods of determining the sun's parallax are more to be trusted.

GRAVITATIONAL METHODS.

These are too recondite to permit any full explanation here. We can only indicate briefly the principles involved.

687. (1) The first of these methods is by the *moon's parallactic inequality*, an irregularity in the moon's motion which has received this name, because by means of it the sun's parallax can be determined. It depends upon the fact that the sun's disturbing action upon the moon differs sensibly from what it would be if its distance, instead of being less than 400 times that of the moon from the earth, were *infinitely* great.

The disturbing action upon the half of the moon's orbit which lies nearest the sun is greater than on the opposite half of the orbit. The retarding action of the tangential force, therefore, during the first quarter after new moon, is perceptibly greater than the acceleration produced during the second quarter (Art. 447), so that at the first and third quarters respectively, the moon is a little more than 2′ behind and ahead of the place she would occupy if the tangential forces were equal in all four quadrants of the orbit — as they would be if the sun's distance were infinite. This puts the moon about *four minutes of time behindhand* at the first quarter, and as much *ahead* at the third; and *if the centre of the moon could be observed within a fraction of a second of arc* (as it could if she were a mere point of light like a star), the observations would give a very accurate determination of the sun's distance. The irregularities of the moon's limb, however, and the worse fact, that at the first quarter we observe the *western* limb, while at the third quarter it is the *eastern* one which alone is observable, make the result somewhat uncertain, though the method certainly ranks high.

688. (2) The "*lunar equation of the sun's motion*" is, it will be remembered, an apparent slight monthly displacement of the sun, amounting to about 6″.4, and due to the fact that both earth and moon revolve around their common centre of gravity. It is generally made use of (Art. 243) to determine the mass of the moon as compared with that of the earth, using as a

datum the assumed known distance of the sun; but if we consider the mass of the moon as known (determined by the tides, for instance), then we can find the sun's parallax[1] in terms of the lunar equation.

689. (3) The third method (*by the earth's perturbations of Venus and Mars*) is one of the most important of the whole list. It depends upon the principle that if the *mass* of the earth, as compared with that of the sun, be accurately known, then the *distance* of the sun can be found at once. The reader will remember that in Art. 278 the mass of the sun was found by comparing the distance which the earth falls towards the sun in a second (as measured by the curvature of her orbit) with the force of gravity at the earth's surface; and in the calculation the sun's distance enters as a necessary datum. Now, if we know independently *the sun's mass compared with the earth's*, the distance becomes the only unknown quantity, and can be found from the other data.

In the same way as in Art. 536 we have

$$(S + E) = \frac{4\pi^2}{G}\left(\frac{D^3}{T^2}\right),$$

in which S and E are the masses of the sun and earth, G is the "constant of gravitation," D is the mean distance of the earth from the sun, and T the number of seconds in a year. Also we have for the force of gravity at the earth's surface,

$$g = G\frac{E}{r^2}, \text{ or } E = \frac{g}{G}r^2,$$

in which r is the earth's radius.

Dividing the preceding equation by this, we get

$$\frac{S + E}{E} = \frac{4\pi^2}{gT^2}\left(\frac{D^3}{r^2}\right);$$

whence

$$D^3 = \left(\frac{S + E}{E}\right)\left(\frac{gT^2r^2}{4\pi^2}\right).$$

If we put $\frac{S}{E} = M$, this becomes

$$D^3 = \left(\frac{M + 1}{4\pi^2}\right)gT^2r^2.$$

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[1] Putting L for the maximum value of the lunar equation (about 6″.4 of arc), P for the sun's parallax, and R and r for the distance of the moon and the semi-diameter of the earth respectively, we have the equation

$$P = L\left(\frac{r}{R}\right)\left(\frac{E + m}{m}\right) = 6''.4\left(\frac{1}{60}\right)82 = 8''.75 \text{ nearly};$$

but the numbers used are only approximate.

In this equation everything in the second term is known when we have once found M, or the ratio between the masses of the sun and earth; g is found by pendulum observations on the earth, T is the length of the year in seconds, and r is the earth's radius.

Now, the disturbing force of the earth upon its next neighbors, Mars and Venus, depends directly upon its mass as compared with the sun's mass, and the ratio of the masses can be determined when the perturbations have been accurately ascertained; though the calculation is, of course, anything but simple. But the great beauty of the method lies in this, that as time goes on, and the effect of the earth upon the revolution of the nodes and apsides of the neighboring orbits accumulates, the *determination of the earth's mass in terms of the sun's becomes continually and cumulatively more precise.* Even at present the method ranks high for accuracy, — so high that Leverrier, who first developed it, would have nothing to do with the transit of Venus observations in 1874, declaring that all such old-fashioned ways of getting at the sun's parallax were relatively of no value. The method is probably the "*method of the future*," and in time will supersede all the others, — unless indeed it should appear that bodies at present unknown are interfering with the movements of our neighboring planets, or unless it should turn out that the law of gravitation is not quite so simple as it is now supposed to be.

From this method Newcomb deduces a parallax of $8''.759 \pm 0''.010$. The smallness of the value thus obtained is almost as perplexing at present as the magnitude of that derived from the measures of the transits of Venus.

THE PHYSICAL METHOD.

690. The physical, or "*photo-tachy-metrical*" method, as it has been dubbed, depends upon the fundamental assumption that light travels in interplanetary space with the same velocity as *in vacuo*. This is certainly very nearly true, and probably exactly so, though we cannot yet prove it.

By the recent experiments of Michelson and Newcomb in this country, following the general method of Foucault, the velocity of light has been ascertained with very great precision and may be taken as 299860 kilometres, or 186330 miles, with a probable error which cannot well be as great as twenty-five miles either way.

691. Sun's Distance from the Equation of Light. — (1) "*The equation of light*" is the time occupied by light in travelling between

the sun and earth, and is determined by observation of the eclipses of Jupiter's satellites (Art. 629). By simply multiplying the velocity of light by this time ($499^s \pm 2^s$) we have at once the sun's distance; and that independent of all knowledge as to the earth's dimensions. The reader will remember, however, that the determination of this "light-equation" is not yet so satisfactory as desirable on account of the indefinite nature of the eclipse observations involved.

692. From the Constant of Aberration.[1] — (2) When we know the velocity of light we can also derive the sun's distance from the "*constant of aberration*," and this constant, $20''.47$, derived from star observations (Art. 225), is known with a considerably higher percentage of accuracy than the light-equation.

Calling the constant a, we have

$$\tan a = \frac{U}{V},$$

where U is the velocity of the earth in its orbit, and V the velocity of light. Now U equals the circumference of the earth's orbit divided by the length of the year; *i.e.*,

$$U = \frac{2\pi D}{T};$$

$$\text{hence} \qquad \tan a = \frac{2\pi D}{VT}, \qquad \text{and} \qquad D = \frac{\tan a}{2\pi}(VT).$$

On the whole it seems likely at present that the value of the sun's distance thus derived is the most accurate of all. Using $a = 20''.47$ and $V = 186300$ miles, we have $D = 92\,876000$ miles, and taking the earth's equatorial radius as 3963.296 miles (Clarke, 1878), we get $8''.803$ as the sun's equatorial horizontal parallax. But it is to be noted that the *parallax* appears only as a secondary result. The method gives directly the *distance* of the sun, without demanding any knowledge of the earth's dimensions. The *velocity* of light furnishes the scale of miles.

693. The reader will notice that the *geometrical* methods give the parallax of the sun *directly*, apart from all hypothesis or assumption, except as to the accuracy of the observations themselves, and of their necessary corrections for refraction, etc.: the *gravitational* methods, on the other hand, assume the exactness of the law of

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[1] See note on page 427.

gravitation; and the *physical* method assumes that light travels in space with the same velocity as in our terrestrial experiments after allowing for the known retardation due to the refracting power of the air. The near accordance of the results obtained by the different methods shows that these assumptions must be very nearly correct. if not absolutely so.

NOTE TO ARTICLE 676*.

During the winter of 1901–2 the planet Eros came quite near the earth, and an extensive series of observations, both micrometric and photographic, was made upon it at numerous coöperating observatories for the purpose of determining the solar parallax.

It will be some time (perhaps several years) before the immense mass of data can be thoroughly discussed and the final result determined; but already partial reductions of the observations made at a few of the stations show that the correction to the assumed parallax (8″.80) will be extremely small; nor is it even certain whether the correction will be in the direction of increase or decrease.

NOTE.

Newcomb, in his "Astronomical Constants" (1896), adopts *8″.797 ± 0″.007* as the value of the solar parallax to be used in the planetary tables.

He also gives the following as the results derived by the various methods *after making allowance for probable systematic errors*, and assigns to each result the weight indicated by the number that follows it.

Motion of the Node of Venus	8″.768,	10
Gill's Observations of Mars (1877)	8 .780,	1
Pulkowa Constant of Aberration (20″.492)	8 .793,	40
Contact Observations of Transit of Venus	8 .794,	3
Heliometer Observations of Victoria and Sappho	8 .799,	5
Parallactic Inequality of the Moon	8 .794,	10
Miscellaneous Determinations of Aberration (20″.463)	8 .806,	10
Lunar Inequality of the Earth	8 .818,	1
Measures of Venus in Transit	8 .857,	1

While many would question the overwhelming weight assigned to the Pulkowa aberration, it makes very little difference in the result.

Harkness, in his "Solar Parallax and its Related Constants" (1891), obtained as his final value 8″.809 ± 0″.006.

Exercises on Chapter XVII.

1. Which of the methods of determining the distance of the sun does not presuppose a knowledge of the dimensions of the earth?

2. Why cannot the transits of Mercury be utilized for determining the solar parallax as well as the transits of Venus?

3. Can observations of Jupiter or any of the remoter planets be usefully employed in determining the sun's distance?

4. How much error in the distance of the sun follows from an error of 0″.01 in the value of the parallax?

5. How much error in the distance of the sun follows from an error of one per cent in the value of the ratio between the masses of the earth and sun as determined from planetary perturbations?

6. Could the parallax of the sun be determined within ten per cent without the use of the telescope?

Note to Arts. 174 and 802.

The spectrographic measurements of the radial velocity of stars with respect to the earth (Art. 802*) also furnish an independent determination of her orbital velocity in terms of the velocity of light, and so of our distance from the sun.

In the spectra of all stars near the ecliptic, the dark lines are found to shift regularly backward and forward during the year by an amount which indicates a range of about 37 miles a second in their radial velocity relative to the earth, or an orbital velocity of the earth equal to about 18.5 miles a second.

The first application of this method has been made by Küstner in 1905. From a series of spectroscopic determinations of the changes in the radial velocity of Arcturus in 1904–5 he deduces a solar parallax of 8″.84. As yet, however, the method cannot compete in precision with the determination by means of aberration; the result is rather to be regarded as a confirmation of Doppler's principle.

~~$D \propto cM^{1/3}$ (approx.)~~

~~$\Delta + \delta\Delta = c(1.01M)^{1/3} = (1.01)^{1/3} cM^{1/3} = (1.01)^{1/3} D = 1.00335 D$~~

~~$\delta\Delta = \sqrt[3]{(1.01)}\ 0.00335 D = .00335(92,000{,}000)$~~

~~$= 311{,}000 +$~~

$\frac{1}{3} \log M = \log \Delta$ (approx) + const.

or

$\frac{1}{3} \frac{\delta M}{M} = \frac{\delta \Delta}{D}$; $\frac{1}{3} \cdot \frac{1}{100} = \frac{\delta \Delta}{D}$

CHAPTER XVIII.

COMETS: THEIR NUMBER, MOTIONS, AND ORBITS.—THEIR CONSTITUENT PARTS AND APPEARANCE. — THEIR SPECTRA. — THEIR PHYSICAL CONSTITUTION, AND ORIGIN.

694. From time to time bodies of a very different character from the planets make their appearance in the heavens, remain visible for some weeks or months, move over a longer or shorter path among the stars, and then vanish. These are the Comets, or "*hairy stars*," as the word means, since the appearance of such as are bright enough to be visible to the naked eye is that of a star surrounded by a hazy cloud, and usually carrying with it a streaming trail of light. Some of them have been magnificent objects,—the nucleus, or central star, as brilliant as Venus and visible even by day, while the cloudy head was nearly as large as the sun itself, and the tail extended from the horizon to the zenith, — a train of shining substance long enough to reach from the earth to the sun. The majority of comets, however, are faint, and visible only with a telescope.

695. Superstitions. — In ancient times these bodies were regarded with great alarm and aversion, being considered from the astrological point of view as always ominous of evil. Their appearance was supposed to presage war, or pestilence, or the death of princes. These notions have survived until very recent times with more or less vigor, but, it is hardly necessary to say, without the least reason. The most careful research fails to show any effect upon the earth produced by a comet, even of the largest size. There is no observable change of temperature or of any meteorological condition, nor any effect upon vegetable or animal life.

696. Number of Comets. — Thus far we have on our lists nearly 700, including the different returns of the periodic comets. About 400 were recorded previous to 1600, before the invention of the telescope, and must, of course, have been fairly conspicuous. Since that time the number annually observed has increased very greatly, for only a few of these bodies, perhaps one in five, are visible without telescopic aid. Their total number must be enormous. Not unfrequently from five to eight are discovered in a

single year, and there is seldom a day when one or more is not in sight.

While telescopic comets, however, are thus numerous, brilliant ones are comparatively rare. Between 1500 and 1800 there were, according to Newcomb, 79 visible to the naked eye, or about one in three and three-fourths years. Humboldt enumerates 43 within the same period as *conspicuous;* during the first half of the present century there were 9 such, and since 1850 there have been 14. Since, and including, 1880 we have had 9, — a remarkable number for so short a time, — and two of them, the principal comet of 1881 and the great comet of 1882, were unusually fine ones. In August, 1881, for a little time two comets were conspicuously visible to the naked eye at once and near together in the sky, a thing almost if not quite unprecedented.

697. Designation of Comets. — The more remarkable ones generally bear the name of their discoverer, or of some astronomer who made important investigations relating to them, — as for instance, Halley's, Encke's, and Donati's comets. They are also designated by the year of discovery, with a Roman number indicating *the order of perihelion passage.* A third method of designation is by year and *letter*, the letters denoting the order in which the comets of a given year were discovered. Thus Donati's comet was both comet *f* and comet VI, 1858. Comet I is, however, not *necessarily* comet *a*, though it usually is so. In some cases the comet bears the name of two discoverers. Thus the Pons-Brooks comet of 1883 is a comet which was discovered by Pons in 1812, and at its return in 1883 was discovered by Brooks.

698. The Discovery of Comets. — As a rule these bodies are first seen by comet-hunters, who make a business of searching for them. For such purposes they are usually provided with a telescope known as a "comet-seeker," with an eye-piece of low power, and a large field of view. When first seen, a comet is usually a mere roundish patch of faintly luminous cloud, which, if really a comet, will reveal its true character within an hour or two by its motion. Some observers have found a great number of these bodies. Messier discovered 13 between 1760 and 1798, and Pons 27 between 1800 and 1827. In the U. S., Brooks, Barnard, and Swift have been especially successful.

699. Duration of Visibility, and Brightness. — The time during which they are visible differs very much. The great comet of 1811

was observed for seventeen months. Comet 1889 I was followed at the Lick Observatory for more than two years; and probably with our present instruments it will be possible to prolong observations far beyond limits formerly possible. When a comet is not discovered until it is receding from the sun it is sometimes observable only for a few weeks or days.

As to their *brightness* they also differ widely. The great majority can be seen only with a telescope, although a considerable number reach the limit of naked-eye vision at that part of their orbit where they are most favorably situated. A few, as has been said above, become *conspicuous;* and a *very few,* perhaps four or five in a century, are so brilliant that they can be seen by the naked eye in full sunlight, as was the case with the great comets of 1843 and 1882.

700. Their Orbits. — The ideas of the ancients as to the motions of these bodies were very vague. Aristotle and his school believed them to be nothing but earthly exhalations inflamed in the upper regions of the air, and therefore *meteorological* phenomena rather than astronomical. Ptolemy accordingly omits all notice of them in the Almagest.

Tycho Brahe was the first to show that they are more distant than the moon by comparing observations of the comet of 1577 made in different parts of Europe. Its position among the stars at any moment, as seen from his observatory at Uranienburg, was sensibly the same as that observed at Prague, more than 400 miles to the south. It followed that its distance must be much greater than that of the moon, and that its real orbit must be of enormous size, cutting through interplanetary space in a manner absolutely incompatible with the old doctrine of the crystalline spheres. He supposed the path to be circular, however, as befitted the motion of a celestial body.

Kepler supposed that comets moved in straight lines; and he seems to have been half disposed to consider them as living creatures, travelling through space with will and purpose, "like fishes in the sea."

Hevelius first, nearly a hundred years later, suggested that the orbits are probably *parabolas,* and his pupil Doerfel *proved* this to be the case in 1681 for the comet of that year. The theory of gravitation had now appeared, and Newton soon worked out and published a method by which the elements of a comet's orbit can be determined from the observations. Immediately afterwards Halley, using this method and computing the parabolic orbits of all the comets for which he could find the needed observations, ascertained that a series of brilliant comets having nearly the same orbit had appeared at intervals of about seventy-five years. He concluded that these were different appearances of one and the same comet, the orbit not being really parabolic but *elliptical,* and he predicted its return, which actually occurred in 1759 — the first of "periodic comets."

701. Determination of a Comet's Orbit. — Strictly speaking, the orbit of a comet being always a conic section, like that of a planet, requires only three perfect observations for its determination; but it seldom happens that the observations[1] can be made so accurately as to enable us to distinguish an orbit truly parabolic from one slightly hyperbolic, or from an ellipse of long period. The *plane of the orbit* and *its perihelion distance*, can be made out with reasonable accuracy from such observations as are practically obtainable, but the *eccentricity*, and the *major axis* with its corresponding *period*, can seldom be determined with much precision from the data obtained at a single appearance of a comet unless its orbit is small.

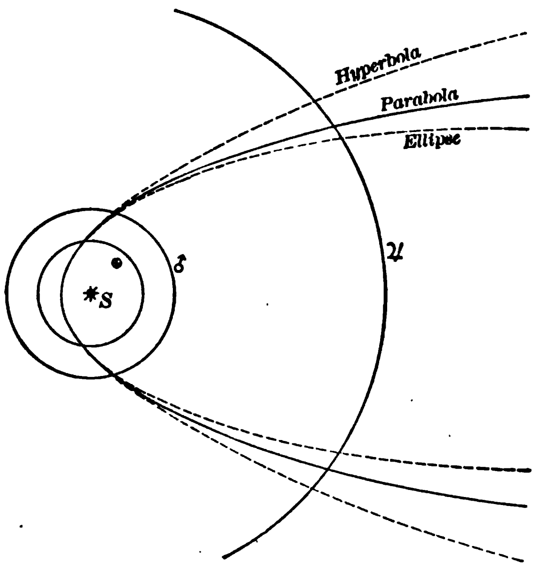

FIG. 195.

The Close Coincidence of Different Species of Cometary Orbits within the Earth's Orbit.

The reason is that a comet is visible only in that very small

[1] Observations for the determination of a comet's place are usually made with an equatorial, by measuring the apparent distance between the comet and some neighboring "comparison star" with some form of micrometer, as indicated in Art. 129. If the star's place is not already accurately known, it is afterwards specially observed with the meridian circle of some standard observatory: this observation of comparison stars forms quite an item in the regular work of such an institution.

portion of its orbit which lies near the earth and sun, and, as the figure shows (Fig. 195), in this portion of the orbit, the long ellipse, the parabola, and the hyperbola almost coincide. Moreover, from the diffuse nature of a comet it is not possible to observe it with the same accuracy as a planet.

Comets which really move in parabolas or hyperbolas visit the sun but once, and then recede, never to return; while those that move in ellipses return in regular periods, unless disturbed.

It will be understood, that in a catalogue of comets' orbits, those which are indicated as parabolic are not *strictly* so. All that can be said is that during the time while the comet was visible, its position did not deviate from the parabola given *by an amount sensible to observation*. The chances are infinity to one against a comet's moving exactly in a parabola, since the least *retardation* of its velocity would render the orbit *elliptical*, and the least *acceleration*, *hyperbolic*, according to the principles explained in Article 430.

702. Relative Numbers of Parabolic, Elliptical, and Hyperbolic Comets. — The orbits of over 400 comets have been thus far computed. Of this number over 300 are sensibly parabolic, and about *a dozen* have had orbits which were hyperbolic according to some computation or other; in no single case, however, is the hyperbolic character *certain*, though in *two* it is very probable. There are also a number of comets which, according to the best computations, appear to have orbits really elliptical, but with periods so long that their elliptical character cannot be positively asserted. About *ninety* have orbits which are certainly and distinctly oval; and *sixty-five* of these have periods which are less than one hundred years. *Eighteen* of these periodic comets have already been actually observed at more than one return (January, 1906).

As to the rest of the *sixty-five*, some of them are expected to return again within a few years, and some of them have been lost, — either in the same way as the comet of Biela, of which we shall soon speak, or by having their orbits so changed by perturbations that they no longer come near enough to the earth to be observed. See Appendix, Table III.

There are three comets with computed periods ranging between seventy and eighty years, whose returns are looked for within the next forty years. There is also one comet with a period of thirty-three years which was due to return in 1899, but failed to appear. It is known as Tempel's comet, an inconspicuous body, but of great interest from its connection with the "Leonid Meteor-Swarm."

703. Fig. 196 shows the orbits of several of the comets of short period, — from three to eight years. (It would cause confusion to insert all of them.) It will be seen that in every case the comet's orbit comes very near to the orbit of Jupiter, and when the orbit crosses that of Jupiter, one of the nodes is always near the place of apparent intersection (the node being marked on the comet's orbit by a short cross-line). If Jupiter were at that point of its orbit at the time when the comet was passing, the two bodies would really be very near to each other. The fact, as we shall see, is a very significant one, pointing to a connection between these bodies and the planet. It is true for *all* the comets whose periods are less than eight years — for those not inserted in the diagram as well as those that are. The orbits of the seventy-five-year comets are similarly related to the orbit of Neptune, and the thirty-three-year comet passes very close to the orbit of Uranus.

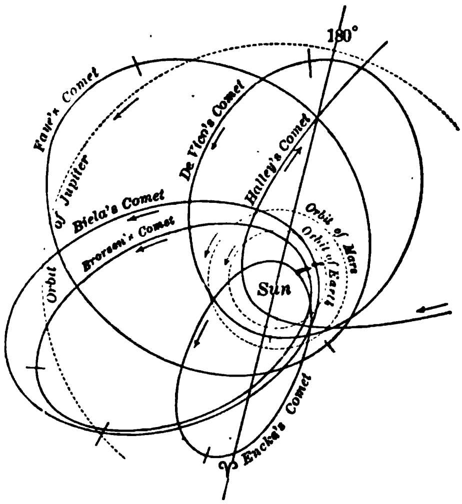

FIG. 196. — Orbits of Short-period Comets.

704. Recognition of Elliptic Comets. — Modern observations are so much more accurate than those made two centuries ago that it is now sometimes possible to determine the eccentricity and period of an elliptic comet by means of the observations made at a single appearance. Still, as a general rule, it is not safe to pronounce upon the ellipticity of a comet's orbit until it has been observed at least twice, nor always then. A comet possesses no "*personal identity*," so to speak, by which it can be recognized merely by looking at it, — no personal peculiarities like those of the planets Jupiter and Saturn. It is identifiable only by its path.

When the approximate parabolic elements of a new comet's orbit have been computed, we examine the catalogue of preceding comets to see if we can find others which resemble it; that is, which have nearly the same *incli*

nation and *longitude of the node* with the same *perihelion distance* and *perihelion longitude*. If so, it is *probable* that we have to do with the same comet in both cases. But it is not *certain*, and investigations, often very long and intricate, must be made to see whether an elliptical orbit of the necessary period can be reconciled with the observations, after taking into account the perturbations produced by planetary action. These perturbations are extremely troublesome to compute, and are often very great, since the comets not unfrequently pass near to the larger planets. In some such cases the orbit is completely altered. Even if the result of this investigation appears to show that the comets are probably identical, we are not yet absolutely safe in the conclusion, for we have what are known as —

705. Cometary Groups. — These are groups of comets *which pursue nearly the same orbits, following along one after another at a greater or smaller interval, as if they had once been united, or had come from some common source.* The existence of such groups was first pointed out by Hoek of Utrecht in 1865. The most remarkable group of this sort is the one composed of the great comets of 1668, 1843, 1880, and 1882, and there is some reason to suspect that the little comet visible on the picture of the corona of the Egyptian eclipse (Art. 328) also belongs to it. The bodies of this group have orbits very peculiar in their extremely small perihelion distance (they actually go within half a million miles of the sun's surface), and yet, although their elements are almost identical, they cannot possibly all be different appearances of one and the same comet.

So far as regards the comets of 1668 and 1843, considered alone, there is nothing absolutely forbidding the idea of their identity: perturbations might account for the differences between *their* two orbits. But the comets of 1880 and 1882 cannot possibly be one and the same; they were both observed for a considerable time and accurately, and the observations of both are absolutely inconsistent with a period of two years or anything like it. In fact, for the comet of 1882 all of the different computers found periods ranging between 600 and 900 years.

There are about half a dozen other such comet-groups now known.

706. Perihelion Distances. — These vary greatly. Twelve comets have a perihelion distance less than five millions of miles[1]; about seventy-four per cent of all that have been observed lie within the earth's orbit; about twenty-four per cent lie outside, but within twice the earth's distance from the sun; and eleven comets have been observed with a perihelion distance exceeding that limit.

[1] According to Galle's tables. Other authorities give slightly different numbers.

A single one, the comet of 1729, had a perihelion distance exceeding four astronomical units, — as great as the mean distance of the remoter asteroids. It must have been an enormous comet to be visible from such a distance. One computer made its orbit slightly *hyperbolic:* others did not.

Obviously, however, the distribution of comets as determined by observation, depends not merely on the existence of the comets themselves, but upon their visibility from the earth. Those comets which approach near the orbit of the earth have the best chance of being seen, because their conspicuousness increases as they approach us, so that we must not lay too much stress on the apparent crowding of the perihelia within the earth's orbit.

The perihelia are not distributed equally in all *directions* from the sun, but more than sixty per cent are within 45° of what is called "the sun's way"; *i.e.*, the line in space along which the sun is travelling, carrying with it its attendant systems.

707. Orbit Planes. — The *inclinations* of the comets' orbits range all the way from 0° to 90°. The *ascending nodes* are distributed all around the ecliptic, with a decided tendency, however, to cluster in two regions having a longitude of about 80° and 270°.

708. Direction of Motion. — With the two exceptions of Halley's comet, and the comet of the Leonid meteors (Art. 786), the elliptical comets which have periods less than one hundred years all move in the direction of the planets. Of the other comets, a few more move retrograde than direct, but there is no decided preponderance either way.

709. It is hardly necessary to point out that the fact that the comets move for the most part in parabolas, and that the planes of their orbits have no evident relation to the plane of the planetary motions, tends to indicate (though it falls short of demonstrating) that *they do not in any proper sense belong to the solar system itself, but are merely visitors from interstellar space.* They come towards the sun with almost precisely the velocity they would have if they had simply dropped towards it from an infinite distance, and they leave it with a velocity which, if no force but the sun's attraction operates upon them, will carry them back to an unlimited distance, or until they encounter the attraction of some other sun. With one remarkable exception, their motions appear to be just what might be expected of ponderable masses moving in empty space under the law of gravitation.

710. Acceleration of Encke's Comet. — The one exception referred to is in the case of Encke's comet which, since its first discovery in the last century (it was not, however, discovered to be a *periodic* comet until 1819), has been continually quickening its speed and shortening its period at the rate of about two hours and a half in each revolution; as if it were under the action of some resistance to its motion. No perturbation by any known body will account for such an acceleration, and thus far no reasonable explanation has been suggested as even possible, except that something encountered in its motion through interplanetary space retards the comet just as air retards a rifle-bullet. At first sight it seems almost paradoxical that a *resistance* should *accelerate* a comet's speed; but referring to Article 429 we see that since the semi-major axis of a comet's orbit is given by the equation

$$a = \frac{r}{2}\left(\frac{U^2}{U^2 - V^2}\right),$$

any diminution of V will also diminish a; and it can be shown that this reduction in the *size* of the orbit will be followed by an increase of velocity above that which the body had in the larger orbit. It gains more speed by thus falling into a smaller orbit nearer to the sun than it loses by the direct effect of the resistance.

711. Another action of such a retarding force is to diminish the eccentricity of the body's orbit, making it more nearly circular. If the action were to go on without intermission, the result would be a spiral path winding inward towards the sun, upon which the comet would ultimately fall. For many years the behavior of Encke's comet was quoted as an absolute demonstration of the existence of the "luminiferous ether." Since, however, no other comets show any such action (unless perhaps Winnecke's[1] comet — No. 5 in the table in the Appendix), and moreover, since according to the investigations of Von Asten and Backlund *the acceleration of Encke's comet itself seems suddenly to have diminished by nearly one-half in 1868*, there remains much doubt as to the theory of a resisting medium. It looks rather more probable that this acceleration is due to something else than the luminiferous ether — perhaps to some regularly recurring encounter of the comet with a cloud of meteoric matter. The fact that the *planets* show no such effect is not surprising, since, as we shall see, they are enormously more dense than any comet, so that the resistance that would bring a comet to rest within a

[1] Oppolzer, in 1880, found that according to his computations Winnecke's comet was accelerated precisely in the same way as Encke's, but by less than half the amount. His result, however, is not confirmed by the later work of Härdtl, who finds no acceleration at all.

single year would not sensibly affect a body like our earth in centuries. The "resisting medium," if it exists at all, must have much less retarding power than the residual gas in one of Crookes's best vacuum tubes.

712. Physical Characteristics of Comets. — The orbits of these bodies are now thoroughly understood, and their *motions* are calculable with as much accuracy as the nature of the observations permit; but we find in their physical constitution some of the most perplexing and baffling problems in the whole range of astronomy, — apparent paradoxes which as yet have received no satisfactory explanation. While comets are evidently subject to gravitational attraction, as shown by their orbits, they also exhibit evidence of being acted upon by powerful *repulsive* forces emanating from the sun. While they shine, in part at least, by reflected light, they are also certainly *self-luminous*, their light being developed in a way not yet satisfactorily explained. They are the *bulkiest* bodies known, in some cases thousands of times larger than the sun or stars; but they are "airy nothings," and the smallest asteroid probably rivals the largest of them in actual mass.

713. Constituent Parts of a Comet. — (*a*) The essential part of a comet — that which is always present and gives it its name — is the *coma* or nebulosity, a hazy cloud of faintly shining matter, which is usually nearly spherical or oval in shape, though not always so.

(*b*) Next we have the *nucleus*, which, however, is not found in all comets, but commonly makes its appearance as the comet approaches the sun. It is a bright, more or less star-like point near the centre of the coma, and is the object usually pointed on in determining the comet's place by observation. In some cases the nucleus is double or even multiple; that is, instead of a single nucleus there may be two or more near the centre of a comet. Perhaps three comets out of four present a nucleus during some portion of their visibility.

(*c*) The *tail* or *train*, is a streamer of light which ordinarily accompanies a bright comet, and is often found even in connection with a telescopic comet. As the comet *approaches* the sun, the tail follows it much as the smoke and steam from the locomotive trail after it. But that the tail does not really consist of matter simply *left behind* in that way, is obvious from the fact that as the comet *recedes* from the sun, the train *precedes* it instead of following. It is always *directed away from the sun*, though its precise position and form is to some extent determined by the comet's motion. There is abundant evidence that it is a material substance in an exceedingly tenuous

condition, which in some way is driven off from the comet and then repelled by some solar action. (See also Art. 736.)

(*d*) *Envelopes and Jets.* — In the case of a very brilliant comet, its head is often veined by short jets of light which appear to be con-

FIG. 197. — Naked-eye View of Donati's Comet, Oct. 4, 1858. (Bond.)

tinually emitted by the nucleus; and sometimes instead of jets the nucleus throws off a series of concentric envelopes, like hollow shells, one within the other. These phenomena, however, are not usually observed in telescopic comets to any marked extent.

714. Dimensions of Comets. — The volume of a comet is often enormous — sometimes almost beyond conception, if the tail be included in the estimate of bulk.

As a general rule the *head or coma* of a telescopic comet is from 40000 to 100000 miles in diameter. A comet less than 10000 miles in diameter is very unusual; in fact, such a comet would be almost sure to escape observation. Many, however, are much larger than 100000 miles. The head of the comet of 1811 at one time measured nearly 1200000 miles, — more than forty per cent larger than the diameter of the sun itself. The comet of 1680 had a head 600000 miles across. The head of Donati's comet of 1858 was 250000 miles in diameter. Holmes' comet of 1892, remarkable in many ways though not brilliant, had a diameter of over 700000 miles, but no visible nucleus at that time. A few weeks later it looked like a mere hazy star.

715. Contraction of a Comet's Head as it approaches the Sun. — It is a very singular fact that the head of a comet continually changes its diameter as it approaches to and recedes from the sun; and what is more singular yet, it usually *contracts when it approaches the sun*, instead of expanding, as one would naturally expect it to do under the action of the solar heat. No satisfactory explanation is known. Perhaps the one suggested by Sir John Herschel is as plausible as any, — that the change is optical rather than real; that near the sun a part of the cometary matter becomes invisible, having been *evaporated*, perhaps, by the solar heat, just as a cloud of fog might be.

The change is especially conspicuous in Encke's comet. When this body first comes into sight, at a distance of about 130 000000 miles from the sun, it has a diameter of nearly 300000 miles. When it is near the perihelion, at a distance from the sun of only 33 000000 miles, its diameter shrinks to 12000 or 14000 miles, the volume then being less than $\frac{1}{10000}$ of what it was when first seen. As it recedes it expands, and resumes its original dimensions. Other comets show a similar, but usually less striking, change.

716. Dimensions of the Nucleus. — This has a diameter ranging in different comets from 6000 or 8000 miles in diameter (Comet III, 1845) to a mere point not exceeding 100 miles. Like the head, it also undergoes considerable and rapid changes in diameter, though its changes do not appear to depend in any regular way upon the comet's

distance from the sun, but rather upon its activity at the time. They are usually associated with the development of jets and envelopes.

717. Dimensions of a Comet's Tail. — The tail of a large comet, as regards simple magnitude, is by far its most imposing feature. The length is seldom less than 10,000000 to 15,000000 miles; it frequently reaches from 30,000000 to 50,000000, and in several cases has been known to exceed 100,000000. It is usually more or less fan-shaped, so that at the outer extremity it is millions of miles across, being shaped roughly like a cone projecting behind the comet from the sun, and more or less bent like a horn. The volume of such a train as that of the comet of 1882, 100,000000 miles in length, and some 200,000 miles in diameter at the comet's head, with a diameter of 10,000000 at its extremity, exceeds the bulk of the sun itself by more than 8000 times.

718. The Mass of Comets. — While the volume of comets is enormous, their *masses* appear to be insignificant. Our knowledge in this respect is, however, thus far entirely *negative;* that is, while in many cases we are able to say positively that the mass of a particular comet *cannot have exceeded* a limit which can be named, we have never been able to fix a lower limit which we know it must have reached; *it has in no case been possible to detect any action whatever produced by a comet on the earth or any other body of the planetary system, from which we can deduce the comet's mass;* and this, although they have frequently come so near the earth and other planets that their own orbits have been entirely transformed, and if their masses had been as much as $\frac{1}{100000}$ of the earth's, they would have produced very appreciable effects upon the motion of the planet which disturbed them.

Lexell's comet of 1770, Biela's comet on more than one occasion, and several others, have come so near the earth that the length of their periods of revolution have been changed by the earth's attraction to the extent of several weeks, but in no instance has the length of the year been altered by a single second. One might be tempted to think that comets were possessed of matter without attracting power; but attraction is always *mutual,* and since the comets move according to the law of gravitation, and themselves suffer perturbation from attraction, there is no escape from the conclusion that, enormous as they are in volume, they contain very little matter. Some have gone so far as to say that a comet properly packed could be carried about in a hat-box or a man's pocket, which, of course, is an extravagant assertion. The probability is that the total amount of matter in a comet of any size, though very small as compared with its bulk, is yet to be estimated

at many millions of tons. The earth's mass (Art. 132, 4) is expressed in tons by 6 with twenty-one ciphers following (6000 millions, of millions, of millions of tons). A body, therefore, weighing only one-millionth as much as the earth would contain 6000 millions of millions of tons, which is very nearly equal to the mass of the earth's atmosphere.

719. The late Professor Peirce based his estimate of a comet's mass upon the extent of the nebulous envelope which it carries with it, assuming that this envelope is gaseous, and is held *in permanent equilibrium* by the attraction of solid matter in and near the nucleus; and on this very doubtful assumption he came to the conclusion that the matter in and near the nucleus of an average comet must be equivalent in mass to an *iron ball as much as* 100 *miles in diameter.* This would be about $\frac{1}{300000}$ of the earth's mass. While this estimate is not intrinsically improbable, it cannot, however, be relied upon. We simply do not know anything about a comet's mass, except that it is exceedingly small as compared with that of the earth.

720. Density. — This must necessarily be almost inconceivably small. If a comet 40000 miles in diameter has a mass equal to $\frac{1}{950000}$ of the earth's mass, its mean density is a little less than $\frac{1}{9000}$ of that of the air at the earth's surface, — much lower than that of the best airpump vacuum. Near the centre of the comet the density would probably be greater than the mean; but near its exterior very much less. As for the density of its tail, when such a comet has one, that, of course, must be far lower yet, and much below the density of the residual gas left in the best vacuum we can make by any means known to science.

This estimate of the density of a comet is borne out by the fact that small stars can be seen through the head of a comet 100000 miles in diameter, and even very near its nucleus, with hardly any perceptible diminution of their lustre. In such cases the writer has noticed that the image of a star is rendered a little indistinct; and recent observations of several astronomers have shown a very small apparent displacement of the star, such as might be ascribed to a slight refraction produced by the gaseous matter of the comet.

Students often find difficulty in conceiving how bodies of so infinitesimal density as comets can move in orbits like solid masses, and with such enormous velocities. They forget that *in a vacuum* a feather falls as freely and as swiftly as a stone. Interplanetary space is a vacuum far more perfect than any airpump could produce, and in it the rarest and most tenuous bodies move as freely and swiftly as the densest.

721. The reader, however, must bear in mind that, although the *mean* density of a comet (that is, the quantity of matter in a cubic mile) is small, *the density of the constituent particles of a comet need not necessarily be so.* The comet may be composed of small, heavy bodies, *widely separated,* and there is some reason for thinking that this is the case; that, in fact, the head of a comet is a swarm of meteoric stones; though whether these stones are many feet in diameter, or only a few inches, or only a few thousandths of an inch, like particles of dust, no one can say. In fact, it now seems quite likely that the greatest portion of a comet's mass is made up of such particles of solid matter, carrying with them a certain quantity of enveloping gas.

722. Light of Comets. — There has been much discussion whether these bodies shine by light reflected or intrinsic. The fact that they become less brilliant as they recede from the sun, and finally *disappear* while they are in full sight simply on account of faintness,[1] and not by becoming too small to be seen, shows that their light *is in* some way derived from the sun. The further fact that the light shows traces of polarization also indicates the presence of *reflected* sunlight. But while the light of a comet is thus in some way attributable to the sun's action, the spectroscope shows that it does not consist, to any considerable extent, of mere reflected sunlight, like that of the moon or a planet.

723. If a comet shone by mere reflected light, or by any light the intensity of which is proportional inversely to the square of the sun's distance (as would naturally be the case if the light were excited directly by the sun's radiation, and proportional to it), we should have its apparent brightness at any time equal to the quantity $\frac{1}{D^2\Delta^2}$, in which D and Δ are the comet's distances from the sun and from the earth respectively. The brightness of a comet does, in fact, generally follow this law roughly, but with many and striking exceptions. The light of a comet often *varies greatly and almost capriciously,* shining out for a few hours with a splendor seven or eight fold multiplied, and then falling back to the normal state or even below it. The Pons-Brooks comet in 1883 and Holmes' comet in 1892 furnished remarkable instances of this sort.

724. The Spectra of Comets. — The spectrum of most comets consists of a more or less faint continuous spectrum (which may be

[1] If a comet shone with its own independent light, like a star or a nebula, then, so long as it continued to show a disc of sensible diameter, the *intrinsic brightness* of this disc would remain unchanged: it would only grow *smaller* as it receded from the earth, not *fainter.*

due to reflected sunlight, though it is usually too faint to show the Fraunhofer lines) overlaid by three bright bands,—one in the yellow, one in the green, and the third in the blue. These bands are sharply defined on the lower, or less refrangible, edge, and fade out towards the blue end of the spectrum. A fourth band is sometimes visible in the violet. The green band, which is much the brightest of the three, in some cases is crossed by a number of fine, bright lines, and there are traces of similar lines in the yellow and blue bands. This spectrum is *absolutely identical with that given by the blue base of an ordinary gas or candle flame;*[1] or better, by the blue flame of a Bunsen burner consuming ordinary illuminating gas. Almost beyond question it *indicates the presence in the comet of some gaseous carbon compound* (*perhaps cyanogen*), which in some way is made to shine; either by a *general* heating to the point of luminosity (which is hardly probable), or by electric discharges within it, or by *local* heatings due to collisions between the solid masses disseminated through the gaseous envelope; or possibly to *phosphorescence* due to the action of sunlight: or none of these surmises may be correct, and we may have to seek some other explanation not yet suggested.

It is not at all certain that the temperature of the comet, considered as a whole, is very high. Nor will it do to suppose that because the spectrum reveals the presence in the comet of gaseous hydrocarbon, this substance, therefore, composes the greater part of the comet's mass. The probability is that the gaseous portion of the comet is only a small percentage of the whole.

725. Metallic Lines in Spectrum. — When a comet approaches very near to the sun, as did Wells's comet in 1882, and a few weeks later the great comet of that year, the spectrum shows bright metallic lines in addition to the hydrocarbon bands. The lines of sodium and magnesium are most easily and certainly recognizable. As for the other lines — a multitude of which were seen by Ricco (of Palermo) for a few hours, in the spectrum of the great comet of 1882—they are probably due to iron; though that is not certain, for they were not seen long enough to be studied thoroughly.

726. Anomalous Spectra. — While most comets show the hydrocarbon spectrum, occasionally a different spectrum of bands appears. Fig. 198 shows the spectra of three comets compared with the solar spectrum and with that of hydrocarbon gas.

[1] It is *not* the spectrum of carbon monoxide, CO, as has been stated by Flammarion and others, though there is some evidence of the presence of that substance as a subordinate constituent.

The first, the spectrum of Tebbutt's comet of 1881, is the usual one. The other two are unique. Brorsen's comet, at its later returns, showed the *ordinary* comet spectrum, and it might perhaps be considered possible that an error was made in fixing the position of the bands at the first observation. But the peculiar spectrum of comet C, 1877, hardly permits such an explanation. It was observed at Dunecht on the same night, by the same observers and with the same spectroscope, as another comet which

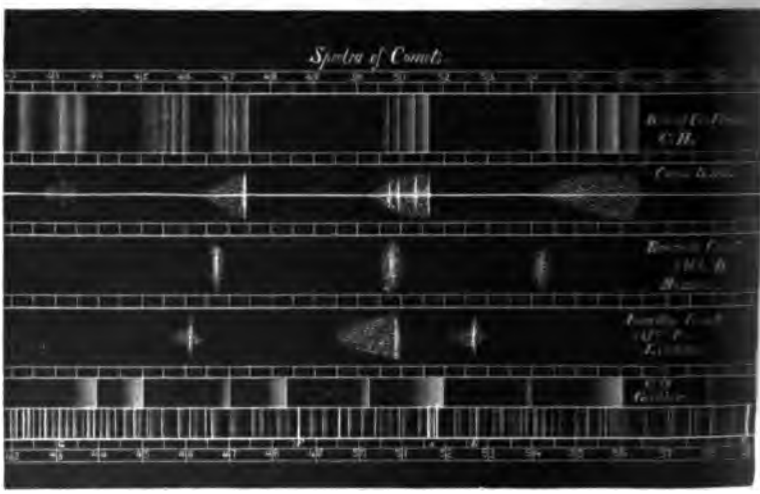

FIG. 198. — Comet Spectra.

(For convenience in engraving, the *dark* lines of the solar spectrum in the lowest strip of the figure are represented as *bright*.)

gave the usual spectrum; so that in this case it hardly seems possible that the anomalous result can be a mistake, though the spectrum itself as yet remains unidentified and unexplained.

Holmes's comet of 1892, unlike any other yet observed, gave a *simply continuous* spectrum, without perceptible markings either bright or dark.

It is maintained by Mr. Lockyer that the spectrum of a comet *changes* as it varies its distance from the sun, the bands altering in appearance and shifting their position. But the evidence of this is not yet conclusive.

It is certainly remarkable that comets, coming as they do from widely separated regions of space, show so little variety in their spectra: *à priori* we should expect difference rather than resemblance.

727. Development of Jets and Envelopes. — When a comet is first seen at a great distance from the sun it is ordinarily a mere roundish, hazy patch of faint nebulosity, a little brighter near the centre.

As the comet draws near the sun it brightens, and the central condensation becomes more conspicuous and sharply defined, or star-like.

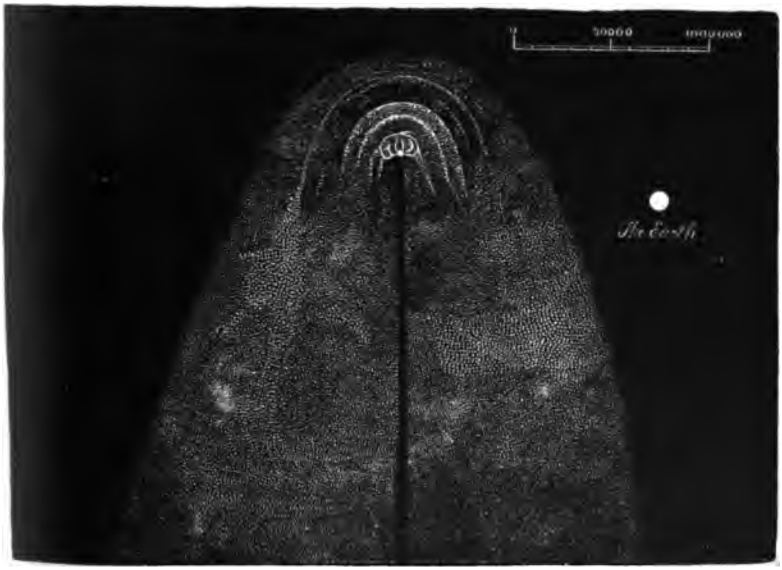

FIG. 199. — Head of Donati's Comet, Oct. 5, 1858. (Bond.)

Then, on the side next the sun, the newly formed nucleus begins to emit jets and streamers of light, or to throw off more or less symmetrical envelopes, which follow each other concentrically at intervals of some hours, expanding and growing fainter as they ascend, until they are lost in the general nebulosity which forms the head. During these processes the nucleus continually changes in brilliancy and magnitude, usually growing smaller and brighter just before the liberation of each envelope. When jets are thrown off, the nucleus seems to oscillate, moving slightly from side to side; but no evidences of a continuous rotation have ever been discovered. The two figures, 199 and 200, represent the

FIG. 200. — Tebbutt's Comet, 1881. (Common.)

heads of two comets which behaved quite differently. Fig. 199 is the head of Donati's comet as seen on Oct. 5, 1858. This comet was characterized by the quiet, orderly vigor of its action. It did very little that was anomalous or erratic, but behaved in all respects with perfect propriety. The system of envelopes in the head of this comet was probably the most symmetrical and beautiful ever seen. Fig. 200 is from a drawing by Common of the head of Tebbutt's comet in 1881. This comet, on the other hand, was always doing something *outré*, throwing off jets, breaking into fragments, and, in fact, continually exhibiting unexpected phenomena.

728. Formation of the Tail. — The material which is projected from the nucleus of the comet, as if repelled by it, is also *repelled by the sun*, and driven backward, still luminous, to form the train. (At least, this is the appearance.) Fig. 201 shows the manner in which the tail is thus supposed to be formed.[1]

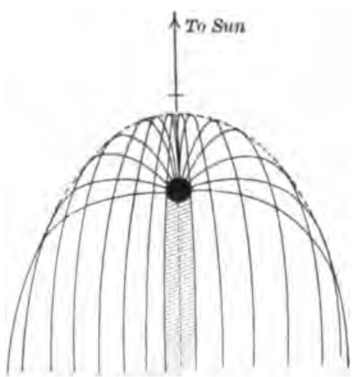

FIG. 201.
Formation of a Comet's Tail by Matter expelled from the Head.

The researches of Bessel, Norton, and especially the late investigations of the Russian Bredichin, have shown that this theory — that the tail is composed of matter repelled by both the comet and the sun — not only accounts for the phenomena in a *general* way, but for almost all the details, and agrees mathematically with the observed position and magnitude of the tail on different dates.

The repelled particles are still subject to the sun's gravitational attraction, and the *effective* force acting upon them is therefore the difference between the gravitational attraction and the electrical (?) repulsion. This *difference* may or may not be in favor of the attraction, but in any case, the sun's attracting force is, at least, lessened. The consequence is that those repelled

[1] Other theories of comets' tails have been presented, and have had a certain currency, — theories according to which the tail is a mere "luminous shadow" of the comet, so to speak, or a swarm of meteors. But all these theories break down in the details. They fail to account for the phenomena of jets, envelopes, etc., in the head of the comet, and they furnish no mathematical determination of the outlines and curvature of the tail.

particles, as soon as they get a little away from the comet, begin to move around the sun in *hyperbolic*[1] orbits which lie in the plane of the comet's orbit, or nearly so, and are perfectly amenable to calculation.

The tail is simply an assemblage of these repelled particles, and, according to theory, ought, therefore, to be a sort of flat, hollow, horn-shaped cone, as represented by Fig. 202, open at the large end, and rounded and closed at the smaller one, which contains the nucleus.

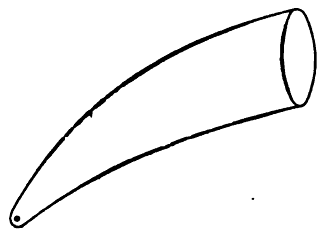

FIG. 202.
A Comet's Tail as a Hollow Cone.

729. Curvature of the Tail. — The cone is curved as shown, because the particles repelled still retain their original orbital motion, so that they will not be arranged along a straight line drawn from

FIG. 203. — A Comet's Tail at Different Points in its Orbit near Perihelion.

the sun through the comet, but along a curve convex to the direction of the comet's motion; but the stronger the repulsion, the less will be the curvature. Fig. 203 shows how the tail ought to lie as the

[1] Referring to the formula for the semi-major axis of an orbit, viz.,

$$a = \frac{r}{2}\left(\frac{U^2}{U^2 - V^2}\right), \qquad \text{(Art. 429)},$$

we see that a repulsive force acting from the sun diminishes U (which measures the sun's attraction), and the consequence is that if the unrepelled particles are describing a parabola (in which case $U^2 = V^2$), then for the *repelled* particles the denominator will become negative (U having been made smaller than V by the repulsive action), and thus a will also become negative, so that the orbit for a repelled particle will be a hyperbola.

comet rounds the perihelion of its orbit. According to this theory, the tail should be *hollow*, and in the case of comets when at their brightest it usually seems to be so, the centre being darker than the edges.

730. The Central Stripe in a Comet's Tail. — Very often, there is a peculiar straight, dark stripe through the axis of the tail as shown in Figs. 199 and 200 of the head of Donati's and Tebbutt's comets. It might be mistaken for the shadow of the nucleus if it were pointed exactly away from the sun; but it is not, usually making an angle of several degrees with the direction of a true shadow. Sometimes, however, and not very unfrequently, the tail has a *bright* centre instead of a dark one, perhaps on account of the feebleness of the comet's own repulsive action; in fact, this seems to be *usually* the case when the comet has reached a considerable distance from the sun in receding from it, and often it is so when the comet is approaching the sun, but is still remote, as in the case of Coggia's comet shown in Fig. 204. In such cases the tail is generally faint and ill-defined at the edge, with a central spine of light, and in some cases it becomes apparently a mere slender ray, of less diameter than the head of the comet itself. This, however, is unusual. The explanation of this kind of tail requires a slight modification of the theory, so far as to admit that the particles at first repelled by the front of the comet are afterwards attracted by it, though still repelled by the sun.

FIG. 204.

Bright Centred Tail of Coggia's Comet. June, 1874.

731. Tails of Three Different Types. — Bredichin has found that the tails of comets may be grouped under three types: —

1. The long, straight rays. They are formed of matter upon which the sun's repulsive action is from twelve to fifteen times as great as the gravitational attraction, so that the particles leave the comet with a relative velocity of at least four or five miles a second; and this velocity is continually increased as they recede, until at last it becomes enormous, the particles travelling several millions of miles in a day. The straight rays which are seen in the figure of the tail of Donati's comet, tangential to the tail, are streamers of this first type; as also was the enormous tail of the comet of 1861.

2. The second type is the curved, plume-like train, like the principal tail of Donati's comet. In this type the repulsive force varies from 2.2 times gravity (for the particles on the convex edge of the tail) to half that amount for those which form the inner edge. This is by far the most common type of cometary train.

3. A few comets show tails of the third type, — short, stubby brushes violently curved, and due to matter of which the repulsive force is only a fraction of gravity, — from $\frac{1}{10}$ to $\frac{1}{2}$.

FIG. 205. — Bredichin's Three Types of Cometary Tails.

732. According to Bredichin, the tails of the first type are probably composed of *hydrogen*, those of the second type of some *hydrocarbon gas*, and those of the third of *iron vapor*, with probably an admixture of sodium and other materials.

There has been no opportunity since Bredichin published this result to test the matter spectroscopically for tails of the first and third types, by looking for the lines of hydrogen and iron. The hydrogen tails are almost always very faint, and the tails of the third class are uncommon. Tails of the second type, which are brightest and most usual, do show a hydrocarbon spectrum throughout their entire length, and so far confirm his view.

The reason for this conclusion of Bredichin is that he supposes the repulsive force to be a *surface action*, the same for equal surfaces of any kind of matter; the *effective* accelerating force, therefore, measured by the velocity it would produce, would depend upon the *ratio of surface to mass* in the particles acted upon, and so, in his view, should be inversely proportional

to their molecular weights. Now the molecular weights of hydrogen, of hydrocarbon gases, and of the vapor of iron, bear to each other just about the required proportion.

733. Nature of the Repulsive Force. — As to this, our knowledge is still imperfect, but the remarkable discoveries made since 1890 indicate that repulsions must be active wherever conditions exist like those at the surface of the sun. Phenomena must necessarily there appear resembling those exhibited in our laboratories when bodies at high temperature, or under powerful electrical excitement, or composed of such peculiar substances as radium and its congeners, project into the rarefied medium around them "ions" and "corpuscles" of various kinds and velocities. Indeed, as Maxwell pointed out thirty years ago, there must also be, according to his electromagnetic theory of light (now almost universally accepted), a pressure exerted by light-waves upon all electro-conducting masses upon which they impinge: a force very minute as compared with gravity upon masses larger than pin-heads, but greatly exceeding it for particles of the size of light-waves. This light-pressure, long vainly sought by experiment, has at last (1901–2) been detected and measured by Lebedew in Moscow, and by Nichols and Hull in our own country. We are no longer at a loss to account for the sun's repulsion upon the materials of the corona and of comets' tails, but only to determine in just what manner the different forces coöperate.

A singular theory has been proposed by Zenker, that the repulsion is due to the reaction produced by rapid evaporation on the surface of the little solid and liquid particles of which he supposed a comet to consist: this evaporation would, of course, be most rapid on the side of the particles next the sun, and would cause *a recoil* in a manner analogous to that by which the so-called spheroidal state of liquids is produced on a heated surface. Ranyard has suggested that the cometary particles may consist principally of minute liquid drops or frozen "hail-stones" of certain hydrocarbons which evaporate rapidly at a very low temperature (such as rhigoline, naphtha, and their congeners).

734. State of the Matter composing the Tail. — This also is a subject of speculation rather than of knowledge. Perhaps the simplest supposition is that we have to do with gaseous matter rarefied even beyond the limits of the gas contained in Crookes's tubes, — so rarefied that since its molecules no longer suffer frequent collisions with each other, it has thus lost all the peculiar *mechanical* characteristics of a gaseous mass, and become a mere cloud of separate parti-

cles, each particle consisting, however, of but a single molecule. Spectroscopically such a cloud would still be *gaseous*, but from a mechanical point of view extremes would have met, and this most tenuous gas would have become a cloud of finely powdered solid.

735. What becomes of the Matter thrown off in Comets' Tails. — To this we have no certain answer at present; but if the theory which has been stated is true, it is clear that most of the matter so repelled from comets can never be re-gathered by the nucleus, but must be dissipated in space.

Whenever a planet meets any of the particles, it picks them up, of course, as it picks up meteors; and Newton long ago suggested, what has of late been forcibly dwelt upon by Dr. Sterry Hunt, that in this way the atmospheres of the planets may be supplied with material to take the place of the carbon which has been absorbed and fixed by the processes of crystallization and of life. Otherwise it would seem that the processes now going on upon the earth's surface must necessarily in the course of time deprive the atmosphere of all its carbonic acid.

If this view is correct, it follows that such comets as have tails lose a portion of their substance every time that they visit the sun. It is quite conceivable, also, that the processes by which light is excited in the head of a comet may use up and render unfit for future shining, a portion of its material; so that, as a periodic comet grows old, it may become both smaller and less luminous, until finally it ceases to be observable.

736. Anomalous Tails and Streamers. — It is not very unusual for comets to show tails of two different types at the same time, as, for instance, Donati's comet. But occasionally stranger things happen, and the great comet of 1744 is reported to have had six tails diverging like a fan. Winnecke's comet of 1877 threw out a tail *laterally*, making an angle of about 60° with the normal tail, and having the same length, — about 1°. Pechüle's comet of 1880 (a small one), besides the normal tail, had another of about the same dimensions directed straight *towards* the sun: streamers of considerable length so directed are not very infrequent. The great comet of 1882 presented a number of peculiarities, which will be mentioned in the more particular description of that body, which is to follow. Most of these anomalies are as yet entirely unexplained.

737. Nature of Comets. — It is obvious from what has been said that we have little certain knowledge on this subject; but perhaps on the whole the most probable hypothesis is the one which has been

hinted at repeatedly, — that a comet is, as Professor Newton expresses it, nothing but a "*sand-bank*"; *i.e.*, a swarm of solid particles of unknown size and widely separated (say pin-heads several hundred feet apart), each particle carrying with it an envelope of gas, largely hydrocarbon, in which gas light is produced, either by electric discharges between the particles, or by some other light-evolving action[1] due to the sun's influence.

This hypothesis derives its chief plausibility from the modern discovery of the close relationship between meteors and comets, to be discussed in the next chapter.

738. Origin of Periodic Comets. — It is obvious that the comets which move in parabolic orbits are, as has been said already, mere visitors to the solar system, and not citizens of it: but as to those which now move in elliptical orbits around the sun, returning as regularly as planets, it is a question whether we are to regard them as *native-born*, or only as *naturalized*. Did they originate in the system, or are they captives?

739. Planets' Families of Comets. — It is quite clear that in some way or other many of them owe their present status in the system to Jupiter, Saturn, and the other planets. In Article 703 we called attention to the fact that, without exception, all the short-period comets (*i.e.*, those having periods ranging from three to eight years), pass very near to Jupiter's orbit at some point in their paths; and they are now recognized and spoken of as Jupiter's "*family*" of comets, — twenty-seven of them are reckoned at present, their number having been considerably increased by the discoveries of the last few years.

Fourteen of them have already been observed at two or more returns, and two or three more will probably be reobserved very soon. The others have failed to be seen a second time either by pure accident or on account of unfavorable position, or they may have suffered the same mysterious fate as Biela's comet (Art. 745), and disappeared.

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[1] Some have ascribed the light to the *collisions* between the little stones of which they assume the comet to be made up, forgetting that, although the *absolute* velocity of the comet is extremely great, the *relative* velocities of its constituent masses with reference to each other must be very slight — far too small apparently to account for any considerable rise of temperature or evolution of light in that way. It is perhaps worth considering whether gases *in the mass* may not become sensibly luminous at a much lower temperature than has usually been supposed. It would seem not improbable *a priori* that at every temperature, radiations of every wave-length must be emitted in some degree; *i.e.*, that *at any temperature above the absolute zero no body is absolutely non-luminous.*

Similarly, Saturn is at present credited with two comets, one of which is Tuttle's comet, given in the catalogue of periodic comets. Uranus stands sponsor for three,—one of them Tempel's comet, which is very interesting in its relation to the November meteors, and was expected back in 1900, but failed to appear. Finally, Neptune has a family[1] of six. Halley's comet is one of them, and two of the others have been observed for a second time since 1880; the other three are not due for some years to come.

740. Origin of Comets: the "Capture" Theory.—The generally accepted theory as to the *origin* of these comet families is that the comets which compose them have been *captured* by the planets to which they now belong. This was first suggested by Laplace.

A comet entering the system from an infinite distance, and moving in a parabolic orbit, when it comes near a planet will be either accelerated or retarded. If *accelerated*, its orbit becomes *hyperbolic*, so that it never returns for a second observation. If, on the other hand, it is *retarded*, the orbit becomes *elliptical*, and the comet will return at regular intervals, moving in a path which, of course, always passes through the point where the disturbance took place.

It is true, as Mr. Proctor has pointed out, that the attraction of Jupiter, huge as is his mass, could not *at one effort* transform a parabolic orbit into an orbit so small as that, say, of Biela's comet. But it is not necessary that the thing should be done at one effort. The comet's orbit lies near to Jupiter's, and after a lapse of time, Jupiter and the comet will be sure to come alongside again: the comet may then be sent into a hyperbolic or parabolic orbit,—the chances for such a result are nearly even;—but it *may* also have its velocity *a second time diminished, and its orbit made still smaller;* and this may be done over and over again unlimitedly, until the aphelion of the comet falls at such a distance within the orbit of Jupiter that the planet is no longer able to disturb it seriously. Given time enough, and comets enough, for Jupiter to work upon, and the final result would necessarily be a comet-family such as really exists, with the aphelia of their orbits near to the orbit of Jupiter, and periods roughly half his own. But it must be frankly admitted that the extent of time, and the quantity of cometary material demanded, are enormous.

[1] *Comet-Families* must be carefully distinguished from *Comet-Groups*. The comets of a single "*group*" all have orbits nearly coincident, at least in the region near the sun. The orbits of a "*family*" have no special resemblance to each other except in period, and in near approach to the orbit of the planet to which they belong.

740*. This "capture theory" has recently received a fine illustration in the case of a little comet, 1889 V, discovered by Brooks. It was very soon found to be a member of Jupiter's "family" with a period of about 7 years, and on careful investigation Dr. S. C. Chandler, of Cambridge (U. S.), ascertained that in 1886 the comet and the planet had come very near each other, and that as a consequence the comet's orbit must have been completely transformed, the previous orbit having been much larger, with a period of about 27 years. Now the researches of Laplace, and more recently of Leverrier, had shown that Lexell's lost comet of 1770 (which, according to the observations then made, had a period of only 5½ years, but was never seen again) (Art. 718) was removed from our range of observation by a similar encounter with Jupiter in 1779, which transformed the then small orbit into one much larger. Dr. Chandler showed that, so far as could be determined from the observations then available, it appeared not only possible, but extremely probable, that Brooks's comet was identical with Lexell's; and for some time it was generally referred to as the Lexell-Brooks comet. Later researches, however, by Dr. C. L. Poor, of Baltimore, based on more extended observations, while confirming the closeness of approach to Jupiter in 1886, throw great doubt upon the absolute identity of the Brooks comet with Lexell's, and make it more probable that the two are related merely as originally members of the same "comet-group" (Art. 705); a conclusion strengthened by Swift's discovery of comet 1895 II, which, according to Schulhof, meets the conditions of identity with Lexell's even better than Brooks's.

Comet 1889 V returned again in 1896, having been found in June very near the place predicted by Poor. It was faint and not well situated, but on the whole the observations decidedly favor Poor's conclusion that it is not identical with Lexell's.

In 1889 the comet was observed by Barnard at the Lick Observatory to be *double*, and the two parts were slowly separating at a rate, which, reckoned backward, would indicate that the disruption had occurred in 1886 when the planet was close to Jupiter.[1] According to the computations of Dr. Poor, the comet then actually passed between the surface of the planet and the orbit of the first satellite.

741. The "Ejection" Theory. — Mr. Proctor has suggested, and vigorously defended, a very different theory, — *that comets are masses of matter which have been thrown off from the heavenly bodies by eruptions of some sort;* that the comets of Jupiter's family, for instance, once formed a portion of its mass, and were at some times *ejected* with a velocity sufficient to set them free in space; and that many of the parabolic comets may have been similarly ejected from our own, or from other suns. The main difficulty with this theory is that there is no evidence of the necessary eruptive energy in Jupiter, or in any of the planets. A body would have to leave the upper surface of Jupiter's atmosphere with a velocity exceeding thirty-five miles a second.

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[1] See note at end of chapter, on the disintegration of comets.

It cannot be said, however, that there is any special *mechanical* difficulty in supposing that some of the *parabolic* comets may owe their origin to eruptions from distant *suns*. Our own sun unquestionably sometimes ejects clouds of matter (in the form of the solar prominences) with enormous velocity, perhaps in some cases sufficient to send them off into space. But so far as we can make out from the spectroscopic evidence, the material of comets is entirely different from that of the prominences.

741*. The Home of the Comets. — There are difficulties connected with the theory that comets come to us from *interstellar* space, chiefly depending upon the now certain fact that the solar system is travelling at the rate of several miles a second (Art. 806), and that therefore comets composed of matter *met* by us ought to have a relative velocity so great as to produce numerous *hyperbolic* orbits, whereas we find few such, if any. Then too there ought to be a marked concentration of the axis of cometary orbits near the direction of the solar motion. While the investigations of the late Professor Newton, of New Haven, partially relieve the objections, other astronomers still feel them; and it has been suggested that "the home of the comets," as Professor Peirce called it, may be a mass or shell of nebulous matter accompanying the system in its motion, and surrounding it at a distance some thousands of times greater than the earth's distance from the sun, but still much closer than the nearest stars (for of the stars our next neighbor, α Centauri, is 275000 times remoter than the sun). A comet starting initially from such a "shell" at a distance of 10000 astronomical units would move in a long ellipse with a period of a million years and no human observation could detect its deviation from an exact parabola. Moreover, if comets came to us indiscriminately from all portions of this nebulosity, their orbits would lie indiscriminately in all directions, and in every plane, just as we find them. But as yet we have no direct evidence of any such comet-dropping envelope.

742. Remarkable Comets. — (1) *Halley's Comet.* This was the first periodic comet whose return was predicted. Halley based his prediction upon the fact that he found its orbit in 1682 to be nearly identical with those of the comets of 1607 and 1531, which had been carefully observed by Kepler and Apian; and he also found records of the appearance of great comets in 1456, in 1301, in 1145 and 1066, which would correspond as regards the time-intervals concerned, though data were wanting for an accurate calculation of their orbits. He noticed, of course, that the two intervals between 1531 and 1607,

and between 1607 and 1682 were not quite equal; but he had sagacity enough to see that the differences were no greater than might be accounted for by the attractions of Jupiter and Saturn.

The theory of perturbation was not then sufficiently developed to make it possible to compute with precision just what the effect would be upon the next return of the comet, but he saw that the action of Jupiter would *retard* it, and he accordingly fixed upon the early part of 1759 as the time at which the comet might be expected. Before that date, however, mathematics had so advanced that the necessary calculations could be made. Clairaut, as the result of a most laborious investigation, fixed upon April 13 for the perihelion passage; but in publishing his result, he remarked that it might easily be a month out of the way owing to the uncertainty as to the masses of the planets, and the possible action of undiscovered planets beyond Saturn (Uranus and Neptune were then unknown). The comet actually came to perihelion on March 13. At this return it was best seen in the southern hemisphere, and at one time had a tail nearly 50° long. At its next return, in 1835, it came to the predicted time within two days. It did not appear on this occasion as an extremely brilliant comet, but was reasonably conspicuous, with a tail of the first type (hydrogen) about 15° in length.

Its next return will occur in May, 1911, but the necessary calculations have not yet been made to determine the precise date with accuracy.

The most remarkable of its earlier appearances were in 1066 and 1456. The comet of 1066 figures on the Bayeux tapestry as a propitious omen for William the Conqueror (of England). In 1456 the comet, according to popular belief, was formally excommunicated by Pope Calixtus III. in a bull directed mainly against the Turks, who were then threatening eastern Europe. It is doubtful, however, whether such a formal bull was ever really promulgated.

743. (2) *Encke's Comet.* This is interesting as the first of the short-period comets, and also as the comet having the shortest known time of revolution, — only about three years and a half. Encke first detected its *periodicity* in 1819, but it had been frequently observed during the preceding fifty years, and has been observed at almost every return since then. It is usually visible only in the telescope, though sometimes, under very favorable circumstances, it can be seen by the naked eye, with a tail a degree or two long. It is often irregular in form, and "lumpy," and seldom shows a well-defined nucleus; nor does it exhibit very much that is interesting in the way of jets, envelopes, and other cometary freaks. We have already mentioned its remarkable contraction in volume on approaching the sun (Art. 715), and the progressive shortening of its period, which has been ascribed to a resisting medium (Art. 710).

744. (3) *Biela's Comet.* This is also, or rather *was*, a small comet with a period of 6.6 years, — the second comet of short period in order of discovery. Its history is very interesting. It was discovered in 1826 by Biela, an Austrian officer, and its periodic character was soon detected by Gambart, whose name is connected with it by many French authorities. Its orbit comes within a very few thousand miles of the earth's orbit, the nearness varying, of course, from time to time, on account of perturbations. The approach is often so close, however, that if the comet and the earth were to arrive at the nearest point at the same time there would be a collision, and the earth would pass through the outer portions of the comet's head. At the return of the comet in 1832, some one started the report that such an encounter would occur, and in consequence there was something hardly short of a panic in southern France — the first of the since numerous "comet-scares." At this time the comet passed the critical point about a month before the earth reached it, so that the two bodies were never really within 15 000000 miles of each other.

745. At the comet's next return in 1839 it failed to be observed on account of its unfavorable position in the sky; but in 1846 it duly reappeared, and did something very strange and then unprecedented.[1] It *divided into two!* When first seen on November 28, it presented the ordinary appearance of any newly discovered comet. On December 19 it had become rather pear-shaped, and ten days later it had divided, the duplication being first seen in New Haven, and soon after at Washington, some days before any European astronomer had noticed it.

The twin comets travelled along side by side for more than four months, at an almost unvarying distance of about 160000 miles, without showing the least sign of mutual attraction or disturbance; but internally both comets were intensely active, each developing a nucleus very bright for a telescopic comet, with a tail some half a degree in length, and showing curious fluctuations of light, which seemed as a general rule to *alternate.* At times the two comets were connected by a faint arc of light.

When next the comet returned in August, 1852, it was under rather unfavorable circumstances for observation, but the twins were both seen, now separated by about 1 500000 miles, and travelling quietly in their appointed orbits. Neither of them has ever been seen again, although they ought to have returned five times, and more than once under favorable conditions for visibility.

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[1] See also Arts. 740* and 751. Also note at end of chapter.

746. But the story is not yet ended, though the remainder perhaps belongs more properly to the next chapter of our book.

On the night of November 27, 1872, just as the earth was passing the old track of the lost comet, she encountered a wonderful meteoric shower. As Miss Clerke expresses it, perhaps a little too positively, "It became evident that Biela's comet was shedding over us the pulverized products of its disintegration."

The same thing happened again in November, 1885, and 1892, when the earth once more passed the comet's path.

The meteors of this so-called Bielid swarm, in their motion through the sky, all appear to come from a point in the constellation of Andromeda, and are therefore sometimes called the "Andromedes," and their motion is parallel to the comet's orbit, at the point where it intersects our own.

747. (4) *Donati's Comet of* 1858. This, on the whole, was perhaps the finest (though not the largest or the most extraordinary) of the comets of the present century, having been very favorably situated for observation in the October sky.

It was discovered at Florence as a telescopic object on June 2. It did not, however, become visible to the naked eye until near the end of August, when it began to exhibit the beautiful phenomena which have made it, so to speak, the normal and typical comet. The comet had an apparently well-defined nucleus, which varied in diameter at different times from 500 miles to 3000. For several weeks the coma exhibited in unrivalled perfection the development and structure of concentric envelopes. Its tail was of the second or hydrocarbon type, with faint tangential streamers which belong to the first or hydrogen type; it had a maximum apparent length of about 50°, and was some 5° or 6° wide at the extremity, and its real length was about 45000000 miles, with a width of 10000000. The object was kept under accurate observation for fully nine months, so that its orbit is unusually well determined as a very long ellipse, with a periodic time of nearly 2000 years. Figs. 197 and 199 show its principal features.

Our space permits us to cite in detail only one other comet: —

748. (5) *The Great Comet of* 1882, which will always be remembered, not only for its beauty, but for the great variety of unusual phenomena it presented.

Discovery and Brightness. The comet seems to have been first seen as a naked-eye object, at Auckland, New Zealand, on September 3. By the 7th or 8th it had become somewhat conspicuous, and was observed both at Cordova (South America) and at the Cape of Good

Hope, but was not seen in the north until the day when it passed its perihelion, September 17. It was then independently discovered by Common, in England, in broad daylight, within 2° of the sun; and the next day it was similarly discovered by a number of observers, especially by Thollon, at Nice, who observed its spectrum in full sunlight, and measured the displacement of the sodium lines produced by its motion. It was so bright that there was not the slightest difficulty in seeing it by simply shutting off the sun with the hand held at arm's length.

750. Member of a Comet Group. — As has been stated before (Art. 705), its orbit — at least, that portion of it within the earth's orbit — coincides almost exactly with the orbits of the comets of 1668, 1843, and 1880. The salient peculiarity of these orbits lies in the *closeness of their approach to the sun*, the perihelion distance of each of them being less than 750000 miles, so that they all passed within 300000 miles of the sun's surface, and with a velocity which at perihelion exceeded 250 miles per second, and carried them through 180° of their orbit in less than three hours. And yet, this passage through the sun's coronal regions did not disturb their motion in the least, as is shown by the fact that the orbit of the comet of 1882, deduced from the observations made before the perihelion passage, agrees exactly with that deduced from those made after it. The inference as to the extreme rarity of the sun's corona is obvious. Only one other comet — Newton's comet of 1680 — has ever approached even nearly as close to the sun as the four comets of this group.

The comet continued visible until March, and this long period of observation enabled the computers to determine the orbit with a greater degree of accuracy than is usual. They all agree in making it a very elongated ellipse, with a period ranging from 650 years to 840 years, according to different computers.

751. Telescopic Features. — When the comet first became telescopically observable in the morning sky it presented very nearly the normal appearance. The nucleus was sensibly circular, and there were a number of clearly developed, concentric envelopes in the head; the dark, shadow-like stripe behind the nucleus was also well marked. In a few days the nucleus became elongated, and finally stretched out into a lengthened, luminous streak some 50000 miles in extent, upon which there were six or eight star-like knots of condensation. The largest and brightest of these knots was the third

from the forward end of the line, and was some 5000 miles in diameter. This "string of pearls" continued to lengthen as long as

Oct. 9. Oct. 15.

FIG. 206. — The Head of the Great Comet of 1882.

the comet was visible, until at last the length exceeded 100000 miles. The engraving (Fig. 206) represents the telescopic appearance at Princeton on October 9 and 15.

752. Tail. — The comet was so situated that its tail was not seen to the best advantage, being directed nearly away from the earth, and never having an apparent length much exceeding 35°. The actual length of the tail, however, at one time exceeded 100000000 miles, — more than the distance of the earth from the sun. It was of the second or hydrocarbon type.

A unique and so far unexplained phenomenon was a faint, straight-edged beam of light, or "*sheath,*" that accompanied the comet, enveloping the head and projecting three or four degrees in front of it, as shown in the figure (Fig. 207). Besides this, at different times, three or four irregular shreds of cometary matter were detected by Schmidt, of Athens, and other observers, accompanying the comet at a distance of three or four degrees when first seen, but gradually receding from it, and at the same time growing fainter.

Fig. 208. — Rordame's Comet, 1893.
(Photographed by Hussey.)

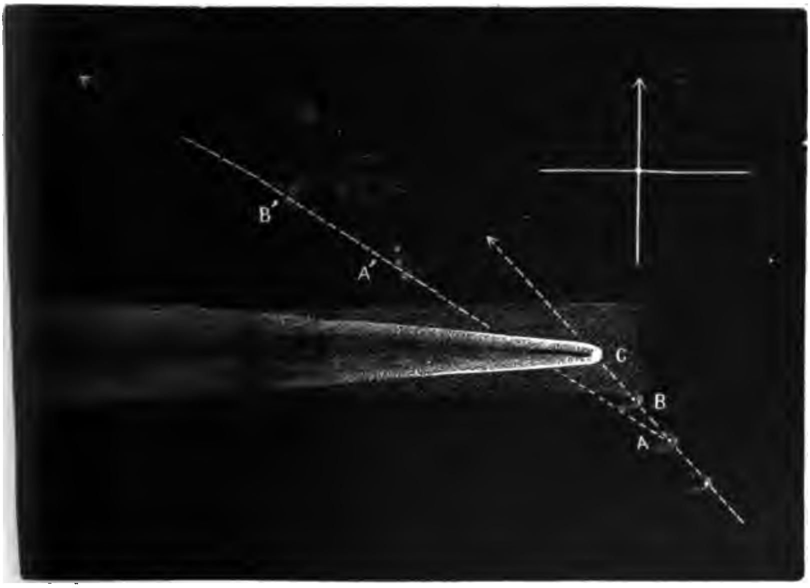

FIG. 207. — The "Sheath," and the Attendants of the Comet of 1882.

752*. Photography of Comets. — Mr. Gill at the Cape of Good Hope obtained a number of fairly good photographs of the comet of 1882, and since then the art has so improved that it is now possible to bring out with the camera peculiarities and details which are quite invisible to the eye even in powerful telescopes. This is especially the case with the comet's tail. Fig. 208 is from a photograph of Rordame's comet of 1893, for which we are indebted to the kindness of Professor Holden, of the Lick Observatory. Because the camera (strapped to a telescope tube) was of course kept pointed at the head of the comet, which was moving rapidly, the images of the stars in the field of view during the hour's exposure are drawn out into parallel streaks, the little irregularities being due to faults of the clock-work and vibrations of the telescope. The knots and streamers which characterize the comet's tail were none of them visible in the telescope, and are not the same shown upon plates, taken the day before and the day after. Other plates, made the same evening a few hours earlier and later, indicate that the "knots" were swiftly receding from the comet's head at a rate exceeding 150000 miles an hour.

In 1892 Barnard *discovered* a small comet, by the streak it left upon one of his star-plates.

We close the chapter with a few remarks upon a subject which has been much discussed.

753. The Earth's Danger from Comets. — It has been supposed that comets might do us harm in two ways, — either by actually striking the earth, or by falling into the sun, and thus producing such an increase of solar heat as to burn us up.

As regards the possibility of a collision with a comet, it is to be admitted that such an event *is* possible. In fact, if the earth lasts long enough, it is practically sure to happen; for there are several comets' orbits which pass nearer to the earth's orbit than the semi-diameter of the comet's head, and at some time the earth and comet will certainly come together. Such encounters will, however, be very rare. If we accept the estimate of Babinet, they will occur about once in 15 000000 years in the long run.

As to the consequence of such a collision it is impossible to speak with confidence, for want of sure knowledge of the state of aggregation of the matter composing a comet. If the theory presented in this chapter is true, everything depends on the size of the separate solid particles which form the main portion of the comet's mass. If they weigh *tons*, the bombardment experienced by the earth when struck by a comet would be a very serious matter; if, as seems more likely, they are for the most part smaller than pin-heads, the result would be simply a meteoric shower.

A danger of a different sort has been suggested, that if a comet were to strike the earth our atmosphere would be poisoned by the mixture with the gaseous components of the comet. Here again the probability is that on account of the low density of the cometary matter no sufficient amount of deleterious vapors would remain in the air to do any mischief at the earth's surface.

754. Effect of the Fall of a Comet into the Sun. — As regards the effect of the fall of a comet into the sun, it may be stated that, except in the case of Encke's comet, there is no evidence of any action going on that would cause a now existing *periodic* comet to strike the sun's surface; it is, however, undoubtedly possible that a comet may enter the system from without, so accurately aimed that it will hit the sun.

But, if a comet actually strikes the sun, it is not likely that the least harm will be done. If a comet having a mass equal to $\frac{1}{100000}$ of the earth's mass were to strike the sun's surface with the parabolic velocity of nearly 400 miles a second, it would generate about as much heat as the sun radiates in eight or nine hours. If this were

all instantly effective in producing increased radiation at the sun's surface (increasing it, say, eightfold, for even a single hour), mischief would follow, of course. But it is almost certain that nothing of the sort would happen. The cometary particles would pierce the photosphere, and liberate their heat mostly *below the solar surface*, simply expanding, by some slight amount, the sun's diameter, and so adding to its store of potential energy about as much as it ordinarily expends in a few hours. There might, and very likely would, be a flash of some kind at the solar surface, as the shower of cometary particles struck it, but probably nothing that the astronomer would not take delight in watching.

Note.

Callandreau has very recently published an important paper upon the disintegration of comets by the action of the sun and the planet Jupiter, showing that the limiting distance at which such an effect is possible is quite considerable, and that the breaking up of a comet ought not to be very unusual. He suggests that many of the "comet groups" may have originated in this way, and that the number of the comets in Jupiter's family has probably thus been greatly increased. The difficulty referred to in the last sentence of Art. 740 respecting the "capture theory" is thus very much relieved. (March, 1898.)

Exercises on Chapter XVIII.

1. What would be the mean density, compared with air, of the spherical head of a comet a hundred thousand miles in diameter, and having a mass one hundred-thousandth that of the earth; assuming the density of the earth as 5.55 times that of water, and the density of water as 773 times that of air? *Ans.* About $\frac{1}{17000}$.

2. What would be the diameter of such a comet if compressed to a density the same as that of the earth? *Ans.* 117 miles.

3. Can the dimensions of a comet's tail be determined with much accuracy? If not, why not?

4. How can it happen that comets whose orbits nearly coincide within a distance of a hundred million miles from the sun may have periods differing by hundreds of years? For example the comets of 1880 and 1882, of which the first has a computed period of only 33 years, and the other, of more than 600.

5. In the case of two cometary orbits very nearly parabolic, and having the same very small perihelion distance, how would the ratio of their major-axes be affected by a small difference in their perihelion velocities? (See Art. 429, remembering that, as the orbits are nearly parabolic, V^2 *must be* very nearly equal to U^2 when the comets pass perihelion.)

6. If the repulsive force of the sun upon a particle of a comet's tail were just equal to the gravitational attraction (Art. 728) what would be the path of that particle?

Ans. A straight line.

7. If the repulsive force exceeded the gravitational attraction what would be the nature of the path?

Ans. An hyperbola *convex* towards the sun, and with the sun *in the external* focus.

8. What would be the path if the repulsive force were only very small as compared with the gravitational attraction?

Ans. An orbit of slightly greater eccentricity than that of the comet itself.

9. Will a given comet (say Encke's) have precisely the same orbit on successive returns?

10. Why can we not infer with certainty that two comets which have orbits practically identical are themselves identical?

11. Can we from spectroscopic observations of a comet infer the relative proportions of the luminous and non-luminous substances present in the comet?

12. Is it probable that a comet can continue permanently in the solar system as a comet? If not, why not, and what will become of it?

CHAPTER XIX.

METEORS AND SHOOTING STARS.

755. Meteors. — Occasionally bodies fall on the earth from the sky, — masses of stone or iron which sometimes weigh several tons. During its flight through the sky such a body is called *a meteor*, and the pieces which fall from it are called *meteorites*, or *aerolites* (air-stones), or *uranoliths* (heaven-stones), or simply *meteoric stones*.

756. Circumstances of their Fall. — The circumstances which attend the fall of a meteorite are in most cases substantially as follows. If it occurs at night a ball of fire is seen, which moves with an apparent speed depending both on its real velocity and on the observer's position. If the body is coming "head on," so to speak, the motion will be comparatively slow; so also if it is very distant. The fire-ball is generally followed by a luminous train, which marks out the path of the body, and often continues visible for a long time after the meteor itself has disappeared. The motion is seldom exactly straight, but is more or less irregular, owing to the resistance of the air; and every here and there along its path the meteor seems to throw off fragments, and to change its course more or less abruptly. If the observer is near enough, the flight is accompanied by a heavy, continuous roar, accentuated by sharp detonations which accompany the visible explosions by which fragments are burst off from the principal body. The noise is sometimes tremendous, and heard for distances of forty or fifty miles, but since sound travels only about 1100 feet a second the explosions, if distant, are heard after a considerable interval, — often several minutes.

If the fall occurs by day white clouds take the place of the fire-ball and the train.

757. The Aerolites Themselves. — The mass that falls is sometimes a single piece, but more usually there are many separate fragments, as in the case of the Spanish meteors of January, 1896. Sometimes they number thousands, as in the L'Aigle meteors of 1803; then, naturally, the stones are mostly small, and sometimes they are mere grains of sand. Nearly all the aerolites that are actually seen to fall, and are found at the time, are masses of *stone;*

but a very few, perhaps three or four per cent of the whole number, consist of *nearly pure iron*, more or less alloyed with nickel. There are also a good many cases of uranoliths, which are mainly stony, but have a considerable portion of iron disseminated through the mass in grains and globules; and nearly all the stony uranoliths contain as much as twenty or thirty per cent of iron in the form of sulphides or analogous compounds.

758. The only *iron meteors which have been actually seen* to fall so far, and are represented by specimens in our museums, are the following:—

(1)	Agram, Bohemia,	1751.
(2)	Dickson Co., Tennessee,	1835.
(3)	Braunau, Bohemia,	1847.
(4)	Tabarz, Saxony,	1854.
(5)	Nejed, Arabia,	1865.
(6)	Nedagollah, India,	1870.
(7)	Maysville, California,	1873.
(8)	Rowton, Shropshire, England,	1876.
(9)	Emmett Co., Iowa,	1879.
(10)	Mazapil, Mexico,	1885.
(11)	Johnson Co., Arkansas,	1886.

The Emmett County iron was mostly in small fragments, and along with them there were many large stones with quantities of iron included. The separate fragments of pure iron which reached the earth probably came by the breaking up of the stony masses.

Besides these iron meteors which have been seen to fall, our cabinets contain a very large number of so-called meteoric irons; *i.e.*, masses of iron found under such circumstances that they cannot easily be accounted for in any way except by supposing them to be of meteoric origin.

759. The number of meteorites which have fallen since 1800 and been gathered into our cabinets[1] is about 275. The most remarkable falls in the United States have been the six following: namely, Weston, Connecticut, 1807; Bishopsville, So. Carolina, 1843; Cabarrus Co., No. Carolina, 1849; New Concord, Ohio, 1860; Amana, Iowa, 1875; and Emmett Co., Iowa, 1879. In the first case and the three last, several hundred fragments fell at the same time, ranging in size from five hundred pounds to half an ounce.

760. Twenty-five of the chemical *elements*, including *helium*, have been found in meteors, and not a single new one. The *minerals* of which

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[1] In this country the cabinets of Amherst College and Harvard and Yale Universities are especially rich in meteorites. The finest collection in the world, however, is that at Vienna. The collection of the British Museum is also noteworthy, as well as that at Paris.

meteorites are composed present a great resemblance to terrestrial minerals of volcanic origin, but many of them are peculiar, and found in meteors only. (The study of these meteoric minerals is a very curious and important branch of mineralogy, though naturally it has not many votaries.) The occasional presence of carbon is to be specially noted, and in a meteor which fell in Russia the carbon appeared to be in a crystalline form, identical with the black diamond, though in exceedingly minute particles. Fig. 209 is from a photograph of a fragment of one of the meteoric stones which fell at Amana, Iowa, in 1875. The picture is taken by the permission of the publishers from Professor Langley's "New Astronomy," where the body is designated as "part of a comet."

FIG. 209.
Fragment of one of the Amana Meteoric Stones.

761. The Crust. — The most characteristic external feature of an aerolite is the thin, black *crust* that covers it, usually, but not always, glossy like a varnish. It is formed by the fusion of the surface in the meteor's swift motion through the air, and in some cases penetrates deeply into the mass of the meteor through fissures and veins. It is largely composed of oxide of iron, and is always strongly magnetic. The crusted surface usually exhibits pits and hollows, such as would be produced by thrusting the thumb into a mass of putty. These cavities are explained by the burning out of certain more fusible substances during the meteor's flight.

762. Magnitude. — Of the meteors actually seen to fall the largest pieces found thus far weigh about 500 pounds, though the whole mass of the body when it first entered the atmosphere has sometimes been much larger, perhaps, in a few cases, amounting to two or three tons.[1]

[1] Some of the masses of iron *supposed* to be of meteoric origin, but not actually seen to fall, are very much larger. The iron mass from Otumpa in Mexico is said to weigh fully sixteen tons. As regards some of these hypothetical meteorites,

As seen from a distance of many miles, the meteoric fire-ball sometimes *appears* to have a diameter as large as the moon, which would indicate a real diameter of several hundred feet. The great apparent size, however, is an illusion, partly due to irradiation, and partly, undoubtedly, to the fact that the meteor itself is surrounded by an extensive envelope of heated air and smoke which becomes luminous throughout. Probably no single meteor ever yet investigated was a solid mass as large as ten feet in diameter.

763. Path. — When a meteor has been observed by a number of persons at *different points*, who have noted any data which will give its altitude and bearing at identified moments, the path can be computed. Observations from a single point are worthless for the purpose, since they can give no information as to the meteor's *distance*.

The meteor is generally first seen at an altitude of between eighty and 100 miles, and disappears at an altitude of between five and ten miles. The length of the path may be anywhere from 50 miles to 500, according to its inclination to the earth's surface. The velocity is rather difficult to ascertain, but is found to range from ten to forty miles per second at the moment when the meteor first becomes visible, and diminishes to one or two miles per second, at the time when it disappears. The *average* velocity with which meteors enter the atmosphere appears not to vary much from the "parabolic velocity" of twenty-six miles per second, due to the sun's attraction at the earth's distance — a fact which, of course, indicates that these bodies, whatever their origin may be, are now moving in space, like the comets, under the sun's attraction.

With possibly a very few exceptions in cases where the meteor *glances*, so to speak, on the earth's atmosphere, like a skipping-stone on water, a body which has once entered the air is sure to be brought to the ground: it is hardly possible that one meteor in a million should escape after becoming involved in the atmosphere. We mention this especially, because some authorities erroneously speak of it as a usual thing for the meteor to keep on its course, and leave the earth, after throwing off a few fragments.

764. Observation of Meteors. — The object of the observation should be to obtain accurate estimates of the altitude and azimuth of the body at moments which can be identified. At night this is best

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however, their meteoric origin is extremely questionable; such, for instance, is the case with the enormous masses of iron, one of them weighing more than seventy tons, brought from the Greenland coast by Nordenskiold and Peary.

done by noting the position of the meteor with reference to neighboring stars at the moments of its appearance and disappearance, or of the intervening explosions. In the daytime it can often be done by noting the position of the object with reference to trees or buildings. The observer should then mark the exact position where he is standing, so that by going there afterwards with proper instruments he can determine the data desired.

Of course, all such measurements must be given *in angular units.* To speak of a meteor as having an altitude of twenty *feet,* and pursuing a path 100 *feet* long, is meaningless, unless the size of the "foot" is somehow defined.

The determination of the meteor's *velocity* is more difficult, as it is seldom possible to look at a watch-face quickly enough, even in the daytime. The usual course is for the observer to repeat some familiar piece of doggerel as rapidly as possible, beginning when the object first becomes visible and stopping when it explodes or disappears, noting also the precise syllable where he stops. By repeating the same sentence over again before a clock it is possible to determine within a few tenths of a second the time occupied by the meteor's flight.

765. Explanation of the Heat and Light of a Meteor. — These are due simply to the destruction of the body's velocity; its kinetic mass-energy of motion is transformed into heat by the friction of the air. If a moving body whose mass is M kilograms, and its velocity V metres per second, is stopped, the number of calories of heat developed is given by the equation

$$Q = \frac{MV^2}{8339} \text{ (Art. 354).}$$

The quantity of heat evolved in bringing to rest a body which has a velocity of forty-two kilometres, or twenty-six miles a second, is enormous, vastly more than sufficient to fuse it even if it were made of the most refractory material, and hundreds of times more than would be produced by its combustion in oxygen if it were a mass of coal.

This heat is developed all along the meteor's course, and mostly just upon its surface. As Sir William Thomson has shown, the thermal effect of the rush through the air is the same as if the meteor *were immersed in a blow-pipe flame* having a temperature of many thousand degrees; and it is to be noted that this *temperature is indepen-*

dent of the density of the air through which the meteor may be passing. The *quantity of heat* developed in a given time is greater, of course, where the air is dense; but the *temperature* produced in the air itself, at the surface where it rubs against the moving body, is the same whether the gas be dense or rare.

When a moving body has a velocity of about 1500 metres per second. the virtual temperature of the surrounding air is about that of red heat; *i.e.*, the body becomes heated as fast as it would if it were at rest and the air about it were heated to that temperature.[1] When the velocity reaches twenty or thirty miles per second, it is acted upon as if the surrounding gas were heated to the liveliest incandescence at a temperature of several thousand degrees. The surface is fused, and the liquefied portion is continually swept off by the rush of the air, condensing as it cools into the luminous powder that forms the train. The fused surface itself is continually renewed until the velocity falls below two miles a second or thereabouts, when it solidifies and forms the characteristic crust. As a general rule, therefore, the fragments are hot if found soon after their fall; but if the stone is a large one and falls nearly vertically, so as to have but a short path through the air, the heating effect will be mainly confined to its surface; and owing to the low conducting power of stone, the *centre* may still remain intensely *cold*, retaining nearly the temperature which it had in interplanetary space. It is recorded that one of the large fragments of the Dhurmsala (India) meteorite, which fell in 1860, was found in moist earth half an hour or so after the fall, *coated with ice*.

766. Train. — One unexplained feature of meteoric trains deserves notice. They often remain luminous for a long time, sometimes as much as half an hour, and are carried by the wind like clouds. It is impossible to suppose that such a cloud of dust remains *incandescent from heat* for so long a time in the cold upper regions of the atmosphere; and the question of its enduring luminosity or phosphorescence is an interesting and puzzling one.

767. Origin. — We may at once dismiss the theories which make meteors to be the *immediate* product of volcanic eruption on the earth or on the moon. They come to us for the most part, as has been said, from the depths of space, with the velocity of planets and

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[1] This is because the gaseous molecules strike the surface of the meteor as if it were at rest, and the molecules themselves were moving with speed correspondingly increased. According to the "kinetic theory" of gases the "temperature" of a gas depends entirely upon the "mean-square velocity" of its molecules.

comets, and there is no certain reason for assuming that they originated in any manner different from the larger heavenly bodies.

At the same time, many of them so closely resemble each other as almost to compel the idea of some common source; and though lunar volcanoes are now extinct, and no terrestrial volcano, not even Krakatão, is *now* competent to send off its ejected missiles through the earth's atmosphere into space, it is not certain that this was always so. Some still maintain that these bodies may be fragments which were shot off millions of years ago when the moon's volcanoes were in full vigor and the earth was young. Since then, according to this view, these masses have been travelling around the sun in long ellipses which intersect the orbit of the earth, until at last they happen to come along at the right time and encounter her atmosphere.

As to the *iron* meteors, some maintain that they come to us from the sun or some other star, basing the opinion upon the remarkable fact that these meteoric irons are generally full of "occluded" hydrogen, helium and carbon oxides, which can be extracted by proper methods. They argue that the iron could have absorbed these gases only while immersed in a hot dense atmosphere containing them, — a condition existing, so far as known, only on the sun and stars. There is no doubt of the sun's ability to project masses to planetary distances, as shown in the case of many eruptive prominences; and it is not unreasonable to suppose that other suns can do the same.

However these bodies originated, it is quite certain that before they reach the earth they have been moving independently in space for a long time, just as planets and comets do. But a recent important research by the late Professor Newton has shown that more than 90 per cent of some 200 aerolites, for the approximate determination of whose paths we have the data, were moving before their fall in orbits, not parabolic, but analogous to those of the short-period comets; and *direct*, not retrograde.

768. Detonating Meteors, or "Bolides," of which Fragments are not known to reach the Earth. — Some writers discriminate between these meteors and aerolites, but the distinction does not seem to be well founded. The phenomena appear to be precisely the same, except that in the one case the fragments are actually found, and in the other they fall into the sea, the forest, or the desert; or sometimes when the path is nearly horizontal, and therefore long, they may be consumed and dissipated in the dust and vapor of the train, without reaching the earth's surface at all, except ultimately as impalpable dust.

769. Number. — As to the number of aerolites which strike the earth, it is difficult to make a trustworthy estimate. Since the

beginning of the century, at least two or three have been seen to fall every year, and have been added to our cabinets (see Art. 759), — in some years as many as half a dozen. This, of course, implies a vastly greater number which are not seen, or are not found. Schreibers, some years ago, estimated the number as high as 700 a year, and Reichenbach sets it still higher — not less than 3000 or 4000.

SHOOTING STARS.

770. A few minutes' watching on any clear, moonless night will be sure to reveal one or more of the swiftly moving, evanescent points of light that are known as "shooting stars." No sound is ever heard from them, nor (with a single exception to be mentioned further on) has anything ever been known to reach the earth's surface from them, not even when the sky was "as full of them as of snow-flakes," as sometimes has happened in a great meteoric shower. For this reason it is perhaps justifiable to allow the old distinction to remain between them and the bodies we have been discussing, at least provisionally. The difference *may be*, and according to opinion at present prevalent very probably is, merely one of size, like that between boulders and grains of sand. Still there are some reasons for supposing that there is also a difference of constitution, — that while the aerolite is a solid, compact mass, the shooting star is a little cloud of dust and intermingled gas, like a puff of smoke.

771. Numbers. — The number of these bodies is very great. A single watcher sees on the average from four to eight hourly. If observers enough are employed to guard the whole sky, so that none can escape unnoticed, the visible number becomes from thirty to sixty an hour. Since ordinarily only those are seen which are within two or three hundred miles of the observer, the estimated total daily number of those which enter the earth's atmosphere, and are large enough to be visible to the naked eye, rises into the millions. Professor Newton sets it at from 15 000000 to 20 000000, the average distance between them being about 250 miles.

The number too small to be seen by the naked eye is still larger. One hardly ever works many hours with a telescope carrying a low power, and having a field of view as large as 15′ in diameter, without seeing several of them flash across the field. In a few instances observers have reported *dark meteors* crossing the moon's disc while they were watching it. There may be some question, however, as to the real nature of the objects seen in such a case. Birds (?).

772. Comparative Number in Morning and Evening. — The hourly number about six o'clock in the morning is fully double the hourly number in the evening. The obvious reason is simply that in the morning we are on the *front of the earth,* as regards its orbital motion, while in the evening we are in the rear; in the evening we only see such meteors as *overtake* us; in the morning we see all that we either meet or overtake.[1] If they are really moving in all directions alike, with the parabolic velocity corresponding to the earth's distance from the sun (twenty-six miles per second), theory indicates that the relative hourly numbers for morning and evening ought to be in just the observed proportion.

773. Brightness. — For the most part these bodies are much like the stars in brightness, — a few are as brilliant as Venus or Jupiter; more are like stars of the first magnitude; and the majority are like the smaller stars. The bright ones not unfrequently show trains which sometimes last from five to ten minutes, when they are folded up and wafted away by the winds of the upper air.[2]

774. Elevation, Path, and Velocity. — By observations made by two or more observers thirty or forty miles apart, it is possible to determine the height, path, and velocity of these bodies. It is found as the result of a great number of such observations that they first appear at an average elevation of about *seventy-four miles,* and disappear at an average height of about *fifty miles,* after traversing a distance of *forty or fifty miles,* with an average velocity of about *twenty-five miles* per second. They do not begin to be visible at so great an elevation as the aerolitic meteors, and they vanish before they penetrate so deeply into the atmosphere.

775. Materials. — Occasionally it has been possible to catch a glimpse of the spectrum of one of the brighter shooting stars, and the lines of sodium and magnesium (probably) are quite conspicuous, along with some other lines which cannot be securely identified.

As these bodies are completely burned up before they reach the earth, all we can ever hope to get of their material is the product of the combustion. In most places the collection and identification of this meteoric ashes is, of course, hopeless: but Norden-

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[1] The earth's orbital motion is always directed nearly towards the point on the ecliptic 90° *west* of the sun.

[2] These air currents, at an elevation of forty miles above the earth's surface, are thus observed to have, *ordinarily,* velocities of from 50 to 75 miles an hour.

skiold has thought he might find it in polar snows, and others have thought it might be found in the material dredged up from the bottom of the ocean. In fact, the Swedish naturalist, by melting several tons of Spitzbergen snow and filtering the water, *did* find in it a sediment containing minute globules of oxide and sulphide of iron: similar globules are also found in the products of deep sea dredging. These *may be* meteoric ashes; but it is quite possible that the suspected material is purely terrestrial in its origin.

776. Probable Mass of Shooting Stars. — We have no very certain means of getting at this. We can, however, fix a provisional value *by means of the amount of light they give.* Photometric comparisons between a standard star and a meteor, when we know the meteor's distance and the duration of its flight, enable us to ascertain how the total amount of light emitted by it compares with that given by a standard candle shining for one minute. Now, according to determinations made about 1860 by Thomsen at Copenhagen (which ought to be repeated), the *light given by a standard candle in a minute is equivalent to about twelve foot-pounds of energy.* This excludes all the energy of the dark, invisible radiation of the candle. Our observations of the meteor give us, therefore, its total *luminous energy* in foot-pounds; and if the whole of the meteor's energy appeared as light, then, since $Energy = \frac{1}{2}MV^2$, we could at once get its mass by dividing twice this luminous energy by the square of the meteor's velocity. Since, however, only a small portion of the meteor's whole energy is transformed into light, the mass obtained in this way would be too small, and must be multiplied by a factor which expresses the ratio between the *total* energy and that which is *purely luminous.* It is not likely that this factor exceeds *one hundred*, or is less than *ten.* Assuming the largest value, however, the photometric observations made in 1866 and 1867 by Professors Newcomb and Harkness (stationed respectively at Washington and Richmond), showed that the majority of the meteors of those star-showers weighed *less than a single grain.* The largest of them did not reach 100 grains, or about a quarter of an ounce. A similar result follows on the assumption that the "luminous efficiency" of a meteor is about the same as that of an electric incandescent lamp.

777. Growth of the Earth. — Since the earth (in fact, every planet) is thus continually receiving meteoric matter, and sending nothing away from it, *it must be constantly growing larger;* but this growth is extremely

insignificant. The meteoric matter received daily by the earth, if we accept one grain as the average weight of a shooting star, would be only about a ton, after making a reasonable addition for occasional aerolites. If we multiply this estimate by one hundred, it certainly will be exceedingly liberal, and at that rate the amount received by the earth in a year would amount to the very respectable figure of 36500 tons; and yet, even at this rate, assuming the specific gravity of the average meteor as three times that of water, it would take about 1000 000000 *years to accumulate a layer one inch thick over the earth's surface.*

778. Effect on the Earth's Orbit. — Theoretically, the encounter of the earth with meteors must *shorten the year* in three distinct ways: —

First. By acting as a resisting medium, and so diminishing the size of the earth's orbit, and indirectly accelerating its motion, in the same manner as is supposed to happen with Encke's comet.

Second. By increasing the attraction between the earth and the sun through the increase of their masses.

Third. By lengthening the day — the earth's rotation being made slower by the increase of its diameter, so that the year will contain a smaller number of days.

The whole effect, however, of the three causes combined, does not amount to $\frac{1}{1000}$ of a second in a million of years. The diminution of the earth's distance from the sun, assuming that one hundred tons of meteoric matter fall daily, and also assuming that the meteors are moving equally in all directions with the parabolic velocity of twenty-six miles per second, comes out about $\frac{1}{20000}$ of an inch *per annum.*

Theoretically, also, the same meteoric action should produce a shortening of the *month*, and Oppolzer investigated the subject a few years ago, to see what amount of meteoric matter would account for the observed *lunar acceleration* (Art. 459). He found that it would require an amount immensely greater than really falls.

779. Meteoric Heat: Effect of Meteors on the Transparency of Space. — Of course each meteor brings to the earth a certain amount of *heat* developed in the destruction of its motion; and at one time it was thought that a very considerable percentage of the total heat received by the earth might be derived from this source (see Art. 355 [2]). Assuming, however, as before, the fall of one hundred tons of meteoric matter daily with an average velocity of twenty miles per second relative to the earth, the whole amount of heat comes out about $\frac{1}{20}$ calorie *per annum* for each square metre of the earth's surface — as much *in a year* as the sun imparts to the same surface in about *one-tenth of a second.*

One other effect of meteoric matter in space should be alluded to. It must necessarily render space imperfectly transparent, like a thin haze. Less light reaches us from a remote star than if the meteors were absent.

780. Meteoric Showers. — At certain times the shooting stars, instead of appearing here and there in the sky at intervals of several minutes, and moving in all directions, appear by thousands, and even hundreds of thousands, for a few hours.

The Radiant. — At such times they do not move without system; but they all appear to diverge or "*radiate*" from one point in the sky; that is, their paths produced backward all intersect at a common point (or nearly so), which is called "*the radiant.*" As an old lady expressed it, in speaking of the meteoric shower of 1833, "The sky looked like a great umbrella." The meteors which appear near the *radiant* are stationary, or have paths extremely short, while those which appear at a distance from it have long courses. The radiant

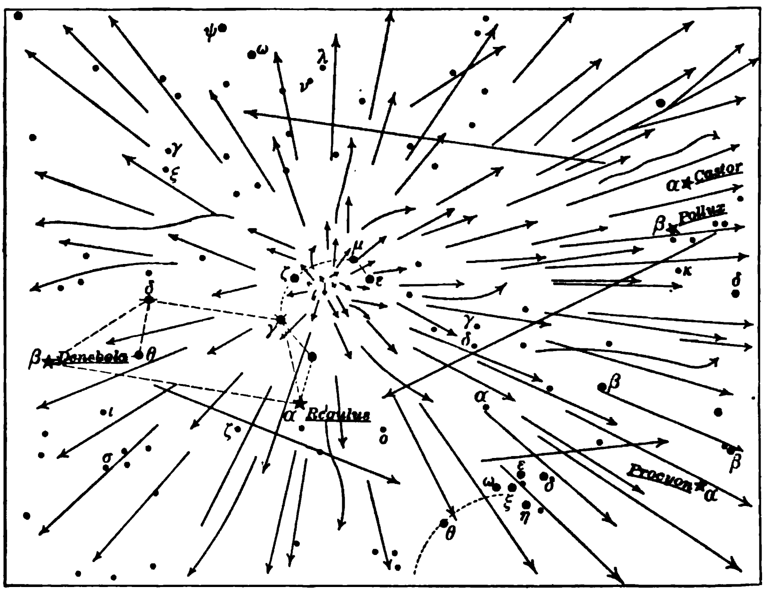

Fig. 210. — The Meteoric Radiant in Leo, Nov. 13, 1866.

keeps its place among the stars unchanged, during the whole continuance of the shower, and the shower is named accordingly. Thus we have the meteor shower of the "*Leonids*," whose radiant is in the constellation of Leo; similarly the "*Andromedes*" (or Bielids), the "*Perseids*," the "*Geminids*," the "*Lyrids*," etc. Fig. 210 is a chart of the tracks of meteors observed on the night of Nov. 13, 1866, showing the radiant near ζ Leonis.

The simple explanation is that the radiant is purely an effect of

perspective. The meteors are really moving, relatively to the observer, in lines which are sensibly straight and parallel, as are also the tracks of light which they leave in the air. Hence they all seem to diverge from one and the same perspective "vanishing point." The position of the radiant depends entirely upon the *direction* of the meteor's motion relative to the earth.

On account of the irregular form of the meteoric particles, they are deflected a little one way or the other by the air; neither is it likely that before they enter the air their paths are *exactly* parallel. The consequence is that the radiant, instead of being a point, is an *area* of some little size, usually less than 2° in diameter. (For note on "Stationary Radiants," see Art. 787*.)

781. Probably the most remarkable of all meteoric showers that ever occurred was that which appeared in the United States on Nov. 12, 1833, in the early morning — a shower of Leonids. The number that fell in the five or six hours during which the shower lasted was estimated at Boston as fully 250,000. A competent observer declared that "he never saw snow-flakes thicker in a storm than were the meteors in the sky at some moments." No sound was heard, nor was any particle known to reach the earth.

782. Dates of Showers. — Since the meteor-swarm pursues a regular orbit around the sun, the earth can only encounter it when she is at the point where her orbit cuts the path of the meteors; and this, of course, must always be on the same day of the year, except as, in the process of time, the meteors' orbits slowly shift their positions on account of perturbations. The Leonid showers, therefore, always appear on the 15th of November (within a day or two); the Andromedes on the 27th or 28th of the same month; and the Perseids early in August.

783. Meteoric Rings and Swarms. — If the meteors are scattered nearly uniformly around their whole orbit, so as to form *a ring*, the shower will recur *every year;* but if the flock is concentrated, it will occur only when the meteor group is at the meeting-place at the same time as the earth. The latter is the case with the Leonids and Andromedes. The great star-showers from these groups occur only rarely, — for the Leonids once in thirty-three years, and for the Andromedes (otherwise known as the Bielids) about once in thirteen. The Perseids are much more equally and widely distributed, so that they appear in considerable numbers every year, and are not sharply limited to a

particular date, but are more or less abundant for a fortnight in the latter part of July and the first of August.

The meteors which belong to the same group all have a resemblance to each other. The Perseids are yellowish, and move with medium velocity. The Leonids are very swift, for we meet them almost directly, and they are characterized by a greenish or bluish tint, with vivid and persistent trains. The Andromedes are sluggish in their movements, because they simply overtake the earth, instead of meeting it. They are usually decidedly red in color and have only small trains. About 100 "meteoric radiants" are now recognized and catalogued. The most conspicuous (except those already named) are the following: — the *Draconids*, January 2; *Lyrids*, April 20; *Aquariids I*, May 6; *Aquariids II*, July 28; *Orionids*, October 28 (see Art. 787*); *Geminids*, December 10.

784. The Mazapil Meteorite. — As has been said, during these showers no sound is heard, no sensible heat perceived, nor do any masses reach the ground; with the one exception, however, that on November 27, 1885, a piece of meteoric iron, mentioned in the list given in Article 758, fell at Mazapil in Northern Mexico during the shower of Andromedes which occurred that evening. Whether the coincidence is accidental or not, it is interesting. Many high authorities speak confidently of this particular iron meteor as being really a piece of Biela's comet itself.

785. The Connection between Comets and Meteors. — At the time of the great meteoric shower of 1833, Professors Olmsted and Twining, of New Haven, recognized the fact and meaning of the radiant as pointing to the existence of *swarms* of meteoric particles revolving in regular orbits around the sun; and Olmsted at the time went so far as even to call the body or swarm a "comet." In some respects, however, his views were seriously wrong, and soon received modification and correction from other astronomers. Erman especially pointed out that in some cases, at least, it would be necessary to suppose that the meteors were distributed in *rings*, and he also developed methods by which the meteoric orbits could be computed if the necessary data could be secured. Olmsted and Twining, however, were the first to show that the meteors are not terrestrial and atmospheric, but bodies truly cosmical.

The subject was taken up later by Professor Newton, of New Haven, who in 1864 showed by an examination of old records that there had been a number of great autumnal meteoric star-showers at intervals of just about thirty-three years, and he predicted confidently a shower for Nov. 13–14, 1866. As to the orbit of the meteoric body (or ring, according to Erman's view), he found that it might,

consistently with what had been so far observed, have either of *five* different orbits; one with a period of 33¼ years, two with periods of one year ± 11 days, and two with periods of half a year ± 5½ days. He considered rather most probable the period of 354 days; but he pointed out that the slow change that had taken place in the annual date of the shower[1] would furnish the means of determining which of the orbits was the true one.

This change of date indicates a slow motion of the nodes of the orbit of the meteoric body at the rate of about 52″ a year. Adams, of Neptunian fame, made the laborious calculation of the effect of planetary perturbations upon each of the five different orbits suggested by Professor Newton, and showed that the true orbit must be the largest one which has a period of 33¼ years.

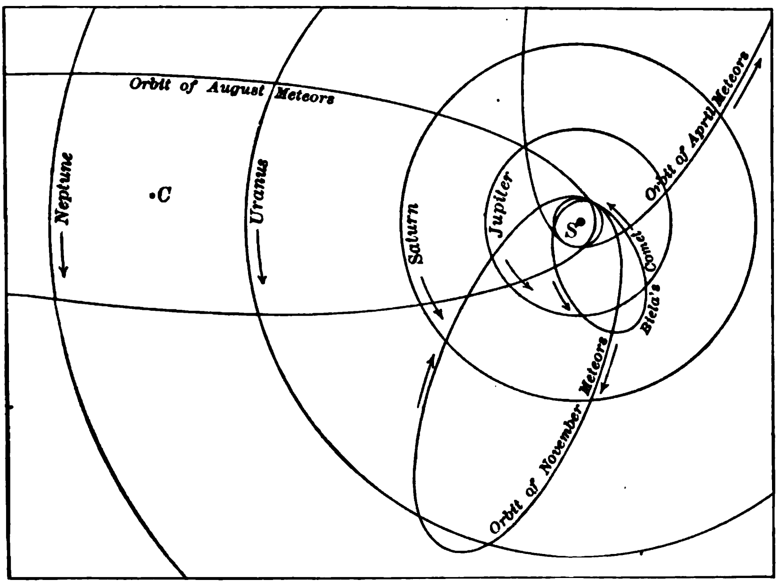

FIG. 211. — Orbits of Meteoric Swarms which are known to be associated with Comets.

The meteoric shower occurred in 1866 as predicted,[2] and was repeated in 1867, the meteor-swarm being stretched out along its orbit for such a distance that the procession is nearly three years in passing any given point.

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[1] In A.D. 902 (the "year of the stars" in the old Arab chronicles), the date was what would be Oct. 19, in our "new style" reckoning. In 1202 the shower occurred five days later, and in 1833 the date was Nov. 12.

[2] See note on page 482.

786. Identification of Cometary and Meteoric Orbits. — The researches of Newton and Adams had awakened lively interest in the subject, and Schiaparelli, of Milan, a few weeks after the Leonid shower, published a paper upon the Perseids, or August meteors, in which he brought out the remarkable fact that they were *moving in the same path as that of the bright comet of* 1862, *known as Tuttle's Comet*. Shortly after this Leverrier published his orbit of the Leonid meteors, derived from the observed position of the radiant in connection with the periodie time assigned by Adams; and almost simultaneously, but without any idea of a connection between them, Oppolzer published his orbit of Tempel's comet of 1866; and the two orbits were at once seen to be *practically identical*. Now a *single* case of such a coincidence as that pointed out by Schiaparelli, might

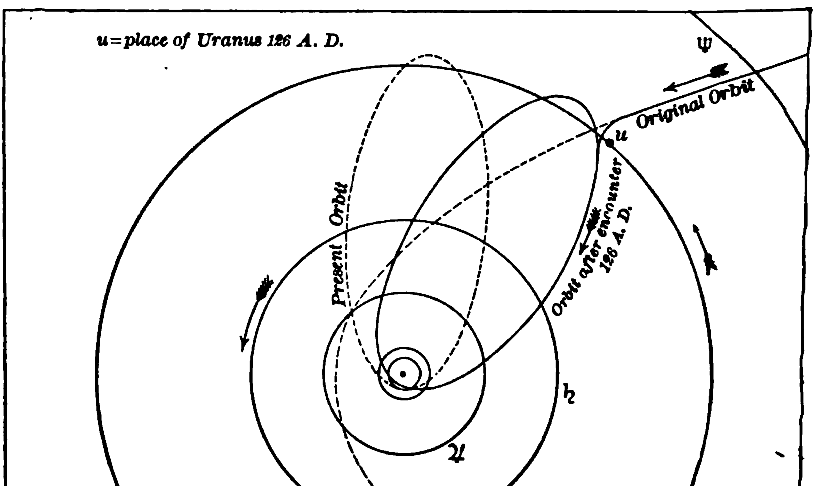

FIG. 212. — Transformation of the Orbit of the Leonids by the Encounter with Uranus, A.D. 126.

possibly be accidental, but hardly *two*. Then five years later, in 1872, came the meteoric shower of the Andromedes, following in the track of Biela's comet; and among the more than one hundred distinct meteor-swarms now recognized, Professor Alexander Herschel finds four or five others which have a "comet annexed," so to speak. Fig. 211 represents the orbits of four of the meteoric swarms which are known to be associated with comets.

787. In the cases of the Leonids and Andromedes the meteor-swarm *follows* the comet. Many believe, however, that the comet itself is simply the thickest part of the swarm. Kirkwood and Schiaparelli have both pointed out that a body constituted as a comet is supposed to be, must almost necessarily break up in consequence of the "tide-producing" perturbations of the sun, independent of any repulsive action such as is supposed to be the cause of a comet's tail. They hold that these meteor-swarms are therefore merely the *product of a comet's disintegration.*

The longer the comet has been in the system, the more widely scattered will be its particles. The Perseids are supposed, therefore, to be old inhabitants of the solar system, while the Leonids and Andromedes are comparatively new-comers. Leverrier has shown that in the year A.D. 126 Tempel's comet must have been very near to Uranus, and a natural inference is that it was introduced into the solar system at that time. Fig. 212 illustrates his hypothesis. However these things may be, it is now certain that the connection between comets and meteors is a very close one, though it can hardly be considered certain as yet that every scattered group of meteors is the result of cometary *disintegration.* We are not sure that when a cometary mass first enters the solar system from outer space, it comes in as a close-packed swarm.

787*. Stationary Radiants. — When a meteoric shower persists for days and even weeks, as is the case with the Perseids, for instance, the radiant as a rule gradually shifts its position among the stars on account of the change in the direction of the earth's motion during the time — as it ought to, since the place of the radiant depends upon the combination of the earth's motion with that of the meteors.

But Mr. Denning, of Bristol (England), for many years an assiduous observer of meteors, claims to have discovered numerous cases in which the radiant of a long-continued shower *remains stationary;* and he presents as typical the Orionids, which scatter along from about October 10 to the 24th, all the time, according to his observations, keeping their radiant close to the star ν Orionis. Only doubtful explanations of such fixity have yet appeared, and though Mr. Denning is perfectly confident of the genuineness of his discovery, and though it is very generally accepted as a fact, some very high authorities, Tisserand, for instance, have questioned it, as being "incredible and unaccountable."

Exercises on Chapter XIX.

1. If a compact swarm of meteors were now to enter the system and be deflected by the attraction of some planet into an elliptical orbit around the sun, would the swarm continue to be compact? If not, what would be the ultimate distribution of the meteors?

2. What is the probable relative age of meteoric *swarms* and meteoric *rings* as members of the solar system?

3. Assuming that the earth encounters twenty million meteors every twenty-four hours, what is the average number in a cubic space of a thousand million cubic miles, (*i.e.*, a cube a thousand miles on each edge)?

Ans. About 250.

4. If space were occupied by meteors uniformly distributed a hundred miles apart on three sets of lines perpendicular to each other, how many would be encountered by the earth in a day?

Ans. 78 700000.

Note.—In this cubical arrangement the *average* distance between the meteors much exceeds 100 miles. If they were packed as closely as possible, consistently with the condition that the distance between two neighbors should nowhere be less than 100 miles, the number would be increased by nearly forty per cent.

Note to Art. 785.

The shower of Leonids, which was expected in November, 1899, failed to appear. The preceding year Leonids were observed in considerable numbers, indicating that the meteoric swarm was nearing us, but in 1899 very few were seen, though carefully watched for. The cause of the failure is not yet quite certain; it was, however, probably due to considerable perturbations of the meteoric orbit during the past thirty-three years, caused by the action of the outer planets, especially Saturn. It seems not unlikely that the effect has been to shift the plane of this orbit so as to abolish its former "grade-crossing" with the orbit of the earth, causing the meteors to pass above or below our level.

But the mathematical difficulties of the problem are enormous, owing to the great extent of the meteoric flock, and the results of calculation are therefore somewhat uncertain. It is quite possible that we may yet run into the Leonids a year or two later.

1904.—The possibility indicated above was realized on the mornings of November 14 to 15, both in 1901 and 1902, and also to some extent in 1903. The Leonids appeared in considerable numbers, though no one of the displays was at all comparable with the showers of 1866–7, not to speak of 1833. In 1901 the meteors were visible for the most part only west of the Mississippi; in 1902 and 1903, in Europe. Tempel's comet, if it returned in 1900, escaped observation.

CHAPTER XX.

THE STARS: THEIR NATURE AND NUMBER. — THE CONSTELLATIONS. — STAR-CATALOGUES. — DESIGNATION AND NOMENCLATURE. — PROPER MOTIONS AND THE MOTION OF THE SUN IN SPACE. — STELLAR PARALLAX AND DISTANCE.

788. We enter now upon a vaster subject. Leaving the confines of the solar system we cross the void that makes an island[1] of the sun's domains, and enter the universe of the stars. The nearest star, so far as we have yet been able to ascertain, is one whose distance is more than 250000 times the radius of the earth's annual orbit; so remote that, seen from that star, the sun itself would appear only about as bright as the pole star, and from it no telescope ever yet constructed could render visible a single one of all the retinue of planets and comets that make up the solar system.

789. Nature of the Stars. — As shown by their spectra the stars are *suns;* that is, they are bodies comparable in magnitude and in physical condition with our own sun, shining by their own light as the sun does, and emitting a radiance which in many cases could not be distinguished from sunlight by any of its spectroscopic characteristics. Some of them are vastly larger and hotter than our sun, others smaller and cooler, for, as we shall see, they differ enormously among themselves.

790. Number of the Stars. — The impression on a dark night is of absolute countlessness; but, in fact, the number visible to the naked eye is very limited, as one can easily discover by taking some definite area in the sky, say the "bowl of the dipper," and counting the stars which he can see within it. He will find that the number which he

[1] That the solar system is thus isolated by a surrounding void is proved by the almost undisturbed movements of Uranus and Neptune; for their perturbations would betray the presence of any body, at all comparable with the sun in magnitude, within a distance a thousand times as great as that between the earth and sun.

can fairly count is surprisingly small, though by averted vision he will get uncertain glimpses of many more. In the whole celestial sphere the number bright enough to be visible to the naked eye is only from 6000 to 7000 in a clear, moonless sky. A little haze or moonlight cuts out fully half of them, and of course there is a great difference in eyes. But the sharpest eyes could probably never fairly see more than 2000 or 3000 at one time, since near the horizon the smaller stars are invisible, and they are immensely the most numerous, fully half of the whole number being those which are just on the verge of visibility. *The total number that can be seen well enough for observation with such instruments as were used before the invention of the telescope is not quite* 1100.

With even a small telescope the number is enormously increased. A mere opera-glass an inch and a half in diameter brings out at least 100,000. The telescope with which Argelander made his *Durchmusterung* of more than 300,000 stars — all north of the celestial equator — had a diameter of only two inches and a half. The number visible in the great Lick [1] telescope of three feet diameter is probably nearly 100,000000.

791. Constellations. — In ancient times the stars were grouped by "constellations," or "asterisms," partly as a matter of convenient reference and partly as superstition. Many of the constellations now recognized, — all of those in the zodiac and those about the northern pole, — are of prehistoric antiquity. To these groups were given fanciful names, mostly of persons or objects conspicuous in the mythological records of antiquity; a great number of them are connected in some way or other with the Argonautic expedition.

In some cases the eye can trace in the arrangement of the stars a vague resemblance to the object which gives name to the constellation; but generally no reason can be assigned why the constellation should be so named or so bounded. Of the sixty-seven constellations now usually recognized on celestial globes, forty-eight have come down from Ptolemy. The others have been formed by Hevelius, Bayer, Royer, and one or two other astronomers, to embrace stars not included in Ptolemy's constellations, and especially to furnish a nomenclature for the stars never seen by Ptolemy on account of their nearness to the southern pole. A considerable number of

[1] Neglecting the loss of light in the lenses, the Lick telescope ought, theoretically, to show stars so faint that it would take more than 30,000 of them to make a star equal to the faintest that can be seen with the naked eye. (See Art. 38.)

other constellations, which have been tentatively established at various times, and are sometimes found on globes and star-maps, have been given up as useless and impertinent.

792. We present a list of the constellations, omitting, however, some of the modern ones which are now not usually recognized by astronomers. The constellations are arranged both vertically and horizontally. The order in the vertical columns is determined by right ascension, as indicated by the Roman numbers at the left. Horizontally the arrangement is according to distance from the north pole, as shown by the headings of the columns. The number appended to each constellation gives the number of stars it contains, down to and including the 6th magnitude. The zodiacal constellations are italicized, and the modern constellations are marked by an asterisk.

The different groups of constellations are found near the meridian at half-past eight o'clock, P.M., on the dates indicated below.

Group (I., II.), Dec. 1. These constellations contain no first-magnitude stars, but Cassiopeia, Andromeda, Aries, and Cetus include enough stars of the second and third magnitude to be fairly conspicuous.

Group (III., IV.), Jan. 1. Perseus north of the zenith, and the Pleiades and Aldebaran in Taurus, are characteristic.

Group (V., VI.), Feb. 1. On the whole this is the most brilliant region of the sky and Orion the finest constellation.

Group (VII., VIII.), March 1. Characterized by Procyon and Sirius, the latter incomparably the brightest of all the fixed stars.

Group (IX., X.), April 1. Leo is the only conspicuous constellation.

Group (XI., XII.), May 1. A barren region, except for Ursa Major north of the zenith.

Group (XIII., XIV.), June 1. Marked by Arcturus, the brightest of the northern stars, with the paler Spica south of the equator.

Group (XV., XVI.), July 1. The Northern Crown and Hercules are the most characteristic configurations.

Group (XVII., XVIII.), Aug. 1. Vega is nearly overhead, and the red Antares low down in the south, with Altair near the equator, just east of Ophiuchus.

Group (XIX., XX.), Sept. 1. Cygnus is in the zenith, and Sagittarius low down, while the brightest part of the Milky Way lies athwart the meridian.

Group (XXI., XXII.), Oct. 1. A barren region, relieved only by the bright star Fomalhaut of the Southern Fish near the southern horizon.

Group (XXIII., XXIV.), Nov. 1. This region also is rather barren, though the "great square" of Pegasus is a notable configuration of stars.

LIST OF CONSTELLATIONS, SHOWING THEIR POSITION IN THE HEAVENS.

R. A. / Decl.	+90° to +50°.		+50° to +25°.		+25° to 0°.		0° to −25°.		−25° to −50°.		−50° to −90°.	
h. I, h. II,	Cassiopeia,	46	Andromeda, Triangulum,	18 5	*Pisces,* *Aries,*	18 17	Cetus,	32	Phœnix, *App. Sculp.	32 13	Phœnix, *bis.* Hydrus,	 18
III, IV,	–	–	Perseus,	40	*Taurus,*	58	Eridanus,	64	(Eridanus, *bis.*)		*Horologium, *Reticulum,	11 9
V, VI,	*Camelopardus,	36	Auriga,	35	Orion, *Gemini,*	37 28	Lepus,	18	*Columba,	15	*Dorado, *Pictor, *Mons Mensæ,	16 14 12
VII, VIII,	–	–	*Lynx,	28	Canis Minor, *Cancer,*	6 15	Canis Major, *Monoceros,	27 12	Argo-Navis,	133	Argo-Navis, *bis.* (Puppis) *Piscis Volans,	 9
IX, X,	–	–	*Leo Minor,	15	*Leo,*	47	Hydra, *Sextans,	40 3	–	–	Argo-Navis (Vela)	
XI, XII,	Ursa Major,	53	–	–	*Coma Ber.	20	Crater, Corvus,	9 8	Centaurus,	54	Argo Navis (Carina) *Chameleon,	 13
XIII, XIV,	–	–	*Canes Venat. Boötes,	15 35	–	–	*Virgo,*	39	Lupus,	34	Centaurus, *bis.* *Crux, *Musca,	 13 15
XV, XVI,	Ursa Minor,	23	Corona Bor. Hercules,	19 65	Serpens,	23	*Libra,*	23	*Norma,	14	*Circinus,	10
XVII, XVIII,	Draco,	80	Lyra,	18	Aquila, Sagitta,	37 5	*Scorpio,* Ophiuchus,	34 46	Ara,	15	*Triangul. Aust. *Apus,	11 15
XIX, XX,	–	–	Cygnus,	67	*Vulpecula, Delphinus,	23 10	*Sagittarius,*	38	Corona Austr.	7	*Telescopium, Pavo, *Octans,	16 37 22
XXI, XXII,	Cepheus,	44	*Lacerta,	13	Equuleus,	5	*Capricornus,*	22	Piscis Austr.	16	*Indus, *Octans	15
XXIII, XXIV,	–	–	–	–	Pegasus,	43	*Aquarius,*	25	*Grus,	30	*Toucana, *Octans	22

793. A thorough knowledge of these artificial groups, and of the names and locations of the stars in them, is not at all essential, even to an accomplished astronomer; but it is a matter of very great convenience to know the principal constellations, and perhaps a hundred of the brightest stars, well enough to be able to recognize them readily and to use them as points of reference. This amount of knowledge is easily acquired by three or four evenings' study of the sky in connection with a good star-map or celestial globe, taking care to observe on evenings at different seasons of the year, so as to command the whole sky.

At present the best star-atlas for reference is probably that of Klein. The maps of Argelander's "Uranometria Nova" and Heis's atlas (both in German) are handsomer, and for some purposes more convenient. There are many others, also, which are excellent. The smaller maps which are found in the text-books on astronomy are not on a scale sufficiently large to be of much scientific use, though they answer well enough the purpose of introducing the student to the principal star-groups. A new and excellent atlas by Upton has recently been published by Ginn & Co.

794. Designation of Bright Stars. — (*a*) *Names.* Some fifty or sixty of the brighter stars have names of their own in common use. A majority of the names belonging to stars of the first magnitude are of Greek or Latin origin, and significant, as, for instance, Arcturus, Sirius, Procyon, Regulus, etc. Some of the brightest stars, however, have Arabic names, as Aldebaran, Vega, and Betelgeuse, and the names of most of the smaller stars are Arabic, when they have names at all.

(*b*) *Place in Constellation.* Spica is the star in the handful of wheat carried by Virgo; Cynosure signifies the star at the end of the Dog's Tail (in ancient times the constellation we now call Ursa Minor seems to have been a dog); Capella is the goat which Auriga, the charioteer, carries in his arms. Hipparchus, Ptolemy, and, in fact, all the older astronomers, including Tycho Brahe, used this clumsy method almost entirely in designating particular stars; speaking, for instance, of the star in the "head of Hercules," or in the "right knee of Boötes," and so on.

(*c*) *Constellation and Letters.* In 1603 Bayer, in publishing a new star-map, adopted the excellent plan, ever since in vogue, of designating the stars in the different constellations by the letters of the Greek alphabet, assigned usually in order of brightness. Thus Aldebaran is

α Tauri, the next brightest star in the constellation is β Tauri, and so on, as long as the Greek letters hold out; then the Roman letters are used as long as they last; and finally, whenever it is found necessary, we use the numbers which Flamsteed assigned a century later. At present every naked-eye star can be referred to and identified by some letter or number in the constellation to which it belongs.

(*d*) *Current Number in a Star-Catalogue.* Of course all the above methods fail for the hundreds of thousands of smaller stars. In their case it is usual to refer to them as number so-and-so of some *well-*known star-catalogue; as, for instance, 22,500 Ll. (Lalande), or 2573 B. A. C. (British Association Catalogue). At present our various star-catalogues contain from 600,000 to 800,000 stars, so that, except in the Milky Way, almost any star visible in a telescope of two or three inches' aperture can be identified and referred to by means of some star-catalogue or other.

Synonyms. Of course all the brighter stars which have *names* have also letters, and are sure to be included in every star-catalogue which covers their part of the sky. A given star, therefore, has often a large number of aliases, and in dealing with the smaller stars great pains must be taken to avoid mistakes arising from this cause.

STAR-CATALOGUES.

795. These are lists of stars arranged in regular order (at present usually in order of right ascension), and giving the places of the stars at some given epoch, either by means of their right ascensions and declinations, or by their (celestial) latitudes and longitudes. The so-called "magnitude," or brightness of the star, is also ordinarily indicated. The first of these star-catalogues was that of Hipparchus, containing 1080 stars (all that are *easily* visible and measurable by naked-eye instruments), and giving their longitudes and latitudes for the epoch of 125 B.C.

This catalogue has been preserved for us by Ptolemy in the Almagest, and from it he formed his own catalogue, reducing the positions of the stars (*i.e.*, correcting for precession the positions given by Hipparchus) to his own epoch, about 150 A.D. The next of the old catalogues of any value is that of Ulugh Beigh made at Samarcand about 1450 A.D. This appears to have been formed from independent observations. It was followed in 1580 by the catalogue of Tycho Brahe containing 1005 stars, the last which was constructed before the invention of the telescope.

The modern catalogues are numerous. Some give the places of a great number of stars rather roughly, merely as a means of *identifying* them when

used for cometary observations or other similar purposes. To this class belongs Argelander's *Durchmusterung* of the northern heavens, which contains over 324000 stars, — the largest number in any one catalogue thus far published. This has since been supplemented by Schoenfeld's southern *Durchmusterung* on a similar plan. Then there are the "*catalogues of precision*," like the Pulkowa and Greenwich catalogues, which give the places of a few hundred stars as accurately as possible in order to furnish "fundamental stars," or reference points in the sky. The so-called "*Zones*" of Bessel, Argelander, Gould and many others, are catalogues covering limited portions of the heavens, containing stars arranged in zones about a degree wide in declination, and running some hours in right ascension. An immense coöperative catalogue is now in process of publication under the auspices of the German Astronomische Gesellschaft, and will contain accurate places of all stars above the 9th magnitude north of 15° south declination. The observations are practically completed, and most of the "parts" have already appeared. (See also Art. 798.)

796. Determination of Star-Places. — The observations from which a star-catalogue is constructed are usually made with the meridian circle (Art. 63). For the catalogues of precision, comparatively few stars are observed, but all with the utmost care and during several years, taking all possible means to eliminate instrumental and observational errors of every sort.

In the more extensive catalogues most of the stars are observed only once or twice, and everything is made to depend upon the accuracy of the places of the fundamental stars, which are assumed as correct. The instrument in this case is used only "differentially" to measure the comparatively small differences between the right ascension and declination of the fundamental stars and those of the stars to be catalogued.

797. Method of using a Catalogue. — The catalogue contains the *mean* right ascension and declination of its stars for the beginning of some given year; *i.e.*, the right ascension and declination the star *would have* at that time if there were no aberration of light and no irregular motion of the celestial pole to affect the position of the equator and equinox. To determine the actual *apparent* right ascension and declination of a star for a given date (which is what we want in practice), the catalogue place must be "*reduced*" to the date in question; *i.e.*, it must be corrected for precession, nutation, and aberration.

The operation with modern tables and formulæ is not a very tedious one, involving perhaps five minutes' work, but without it the catalogue places are

FIG. 213. — The Photographic Telescope of the Henry Brothers, Paris.

useless for most purposes. *Vice versa*, the observations of a fixed star with the meridian circle do not give its *mean* right ascension and declination ready to go into the catalogue, but the observations must be reduced from *apparent* place to *mean* before they can be tabulated.

798. Star-Charts. — For many purposes *charts* of the stars are more convenient than a catalogue, as, for instance, in searching for new planets. The old-fashioned way of making such charts was by plotting the results of zone observations. The modern way, introduced within the last few years, is to do it by photography, the cardinal advantages being two: first, that in this way a great number of stars can be automatically and permanently registered at one operation, and afterwards studied and measured at leisure; second, that by a sufficient prolongation of the exposure stars far too faint to be *seen* by the telescope used can be made to impress themselves upon the plate. The plan decided upon at the Paris Astronomical Congress in 1887 contemplates the photographing of the whole sky upon glass plates about eight inches square, each covering an area of 2° square (four square degrees), showing all stars down to the fourteenth magnitude. The enterprise is now (1904) well advanced, fully three-quarters of the negatives having been already made. Fourteen different observatories have coöperated in the work, only one of them, however, in America, — the Mexican National Observatory at Chepultepec.

The figure (Fig. 213) is a representation of the Paris instrument of the Henry Brothers, which was adopted as the typical instrument for the operation. It has an aperture of about fourteen inches, and a length of about eleven feet, the object-glass being specially corrected for the photographic rays. A 9-inch visual telescope is enclosed in the same tube so that the observer can watch the position of the instrument during the whole operation.

The other instruments differ in mechanical arrangements, but all have lenses of the same aperture and focal length, the scale of all the negatives being 1′ to a millimetre, — the same as that of Argelander's charts.

It was originally planned to give each plate 20 minutes' exposure, but improvements in the photographic plates since the meeting of the Congress now make it possible to cut down the time very materially. It will require about 11000 plates of the size named to cover the whole sky, and as each star is to appear on two plates at least, the whole number of plates, allowing for overlaps, will be about 22000. As every plate will contain upon it a number of well-determined catalogue stars, it will furnish the means of determining accurately, whenever needed, the place of any other star which appears upon the same plate.

The places of all stars above the twelfth magnitude are to be carefully measured upon the plates, and will furnish an enormous *catalogue*, containing at least 2 000000 stars.

798*. Several other very large photographic telescopes have recently been constructed. The Bruce telescope (presented to the Harvard College Observatory by Miss Bruce of New York) has a four-lens objective *two feet* in diameter, but with a focal length of only 11 feet, the same as those mentioned above; so that its negatives will be on the same scale.

It has been sent to Arequipa (Peru), where it will be employed in the photography (and spectroscopy) of the southern heavens. The new photographic telescope at Greenwich has the same aperture, but is much longer; and an instrument similar to it is nearly ready for mounting at the Cape of Good Hope. Both have visual "finders" 18 inches in diameter. The enormous instrument at Meudon (near Paris) has also two telescopes combined, — a visual telescope of 32 inches aperture, and a photographic of 25 inches, each 55 feet focal length. But these long-focus instruments will be used mainly for other purposes than charting. A huge instrument of 31½ inches aperture has still more recently (1899) been mounted at Potsdam.

STAR MOTIONS.

799. The stars are ordinarily called "*fixed*," in distinction from the planets or "*wanderers*," because as compared with the sun and moon and planets they have no evident motion, but keep their relative positions and configurations unchanged. Observations made at sufficiently wide intervals of time, and observations with the spectroscope, show, however, that they are really moving, and that with velocities which are comparable to the motion of the earth in her orbit.

If we compare the right ascension and declination of a star determined to-day with that determined a hundred years ago, they will be found different. The difference is *mainly* due to precession and nutation, which are not motions of the stars at all, but simply changes in the position of the reference circles used, and due to alterations in the direction of the earth's axis (Arts. 205 and 214). Aberration also comes in, and this also is not a real motion of the stars, but only an apparent one.

800. Proper Motions. — But after allowing for all these *apparent* and *common* motions, which depend upon the stars' places in the sky, and are sensibly the same for all stars in the same telescopic

field of view, whatever may be their real distance from us, we find that most of the larger stars have a *"proper motion"* of their own ("proper" as opposed to "common"), which displaces them slightly with reference to the stars about them. There are only a few stars for which this proper motion amounts to as much as 1″ a year; perhaps 150 such stars are now known, but the number is constantly increasing, as more and more of the smaller stars come to be accurately observed.

The maximum proper motion at present known (detected in 1898) is that of the 8th magnitude star No. 243 of the "Fifth hour" in the Cordoba Zone-Catalogue. It has an apparent drift of 8″.7 annually, — enough to carry it 360° in 149000 years. The next largest known proper motions are the following: —

1830, Groombridge,	7th	mag.,	7″.0	ϵ Indi,	5th	mag.,	4″.5
9352, Lacaille,	7th	"	6″.9	Lalande 21258	8th	"	4″.4
32416, Gould,	9th	"	6″.2	o_2 Eridani	6th	"	4″.4
61 Cygni,	6th	"	5″.2	μ Cassiopeiæ,	5th	"	3″.8
Lalande 21185	7th	"	4″.7	α Centauri,	1st	"	3″.7

The proper motions of Arcturus (2″.1), and of Sirius (1″.2), are considered "large," but are exceeded by a considerable number of stars besides those given above. Since the time of Ptolemy, Arcturus has moved more than a degree, and Sirius about half as much. These motions were first detected by Halley in 1718.

It is found, as might be expected, that the brighter stars, which as a class are presumably nearer than the fainter ones, have on the average a greater proper motion; on the *average* only, however, as is evident from the list given above. Many smaller stars have larger proper motions than any bright one, for there are more of them.

The *average* proper motion of the first magnitude stars is about $\frac{1}{4}''$ annually, and that of the sixth magnitude stars, — the smallest visible to the naked eye, — is about $\frac{1}{25}''$.

801. Real Motions of Stars. — The proper motion of a star gives comparatively little information as to its real motion until we know the distance of the star and the true direction of the motion, since this "proper motion" *as determined from the star-catalogues* is only the angular value of that part or component of the star's whole motion *which is perpendicular to*

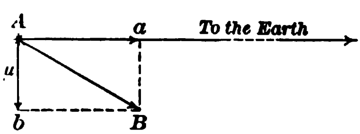

FIG. 214.

Components of a Star's Proper Motion.

the line of sight, as is clear from the figure. When the star really moves from A to B (Fig. 214), it will appear, as seen from the earth, to have moved from A to b. The angular value of Ab as seen from the earth is the "proper motion" (usually denoted by μ). Expressed in seconds of arc, we have

$$\mu'' = 206265 \left(\frac{Ab}{\text{distance}}\right).$$

A body moving directly towards or from the earth has no "proper motion" at all, speaking technically, — none that can be obtained from the comparison of star-catalogues.

$$\text{Since } Ab \text{ in miles} = \frac{\mu'' \times \text{distance}}{206265},$$

the proper motion cannot be translated into miles without a knowledge of the star's distance, and at present we know the distance in only a very few cases; nor can the true motion AB be found until we also know either the angle BAE or else Aa, the "*motion in the line of sight*," or "radial motion," which, as we shall see in the next article, is determined by spectroscopic observations.

But since AB is necessarily *greater* than Ab, it is possible in some cases to determine a *minor* limit of velocity, which must certainly be exceeded by the star. In the case of 1830, Groombridge, for instance, we have certain knowledge that its distance is not *less* than 2000000 times the earth's distance from the sun. It may be vastly greater; but it cannot be less. Now at that distance the observed proper motion of 7″ a year would correspond to an actual velocity[1] along the line Ab of more than 200 miles a second, and this is not its *whole* motion.

In the case of 61 Cygni we know the distance to be just about 500000 times that of the earth from the sun, and its proper motion of 5″.2 annually corresponds therefore to a distance Ab of about 1200 million miles, and a velocity of about 38 miles a second, — not quite twice the orbital velocity of the earth. We shall see in the next articles how the velocity Aa can be determined. For 61 Cygni it has recently been measured by Belopolsky and is found to be about 34.5 miles towards us. The entire velocity, therefore, along AB is about 51 miles (referred to the sun as the origin of measures). If we accept the parallax of the star C. Z., V., 243, as 0″.30, according to the determination of Gill, we find that its annual drift of 8″.7 corresponds to a velocity of about 85 miles a second, which is much less than that of 1830 Groombridge.

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[1] When the "parallax" of a star (Art. 808) is known, this "thwartwise velocity," or "cross-motion," is given in miles per second by the formula $\otimes = 2.944 \times \frac{\mu}{p}$, μ and p being respectively the proper motion and parallax.

802. "Motion in the Line of Sight" or "Radial Velocity."[1] — The comparison of star-catalogues furnishes no information as to the motion of stars towards or from us, but when a star is bright enough to give an observable spectrum we can ascertain the rate of its approach or recession (*Aa* in the figure) by means of the spectroscope. If its distance is increasing, then (Art. 321) its lines will be shifted towards the red, and towards the blue if it is coming nearer. The shift is ascertained by arranging the telespectroscope (Art. 313) so that in some way, by a "comparison-prism" or otherwise, the observer shall have close together, or superposed, the spectrum of the star he is dealing with and also that of some substance (say hydrogen or iron) whose lines are present in the star-spectrum: he can then appreciate and measure any displacement of the stellar lines. Sir William Huggins in 1867 was the first to apply this method, and obtained some very interesting results, quite sufficient to establish its feasibility, although from the insufficient power of his instruments they can now be regarded only as first approximations. The work has since been followed up for several years at Greenwich and some other places; but so long as visual observations were depended upon the results were not very satisfactory. Observations of this kind are extremely difficult. The star-spectra are faint at best, the displacements of the lines very minute, and the lines themselves often broad and hazy, and ill adapted for accurate measurement; so that the individual results for a single star are apt to be mournfully at variance with each other. In the case of the nebulæ, however, which give spectra containing sharp *bright* lines, the Lick observers have made visual observations which fairly compete with photographic in accuracy.

802*. Spectrographic Determination of Radial Velocity. — The unsatisfactory results of visual observations led Vogel in 1888 to apply photography, and with great success. In this case the difficulties arising from the faintness of the star-spectra can be largely overcome by prolonged exposure, and the necessary measurements can be made upon the negatives at leisure. The little cut (Fig. 215) shows how on a negative of the spectrum of β Orionis (Rigel) the recession of the star is shown by the displacement of a line in its spectrum towards the red, when compared with the corresponding

[1] We shall hereafter follow the French usage in employing the term "Radial Velocity" (*Vitesse Radiale*) to denote the rate at which a body is changing its distance from the observer, *i.e.*, advancing or receding. The equivalent expression, "Motion in Line of Sight," is rather clumsy.

bright line (black in the *negative*) of hydrogen. Fig. 215* (borrowed by permission from Frost's translation of Scheiner's Astronomical Spectroscopy) shows the actual appearance of part of the spectrum of α Aurigæ and the corresponding part of the solar spectrum, as seen under the microscope with which the measurements are made. The solar spectrum is of course on a separate plate, but this plate and the star-negative are clamped together so as to make the lines correspond, and facilitate the identification of the star-lines. The sharp black line that crosses the narrow star-spectrum is the Hydrogen γ line in the spectrum of a Geissler tube placed in the cone of rays some two feet above the slit-plate, and illuminated by electricity for

FIG. 215.

Displacement of $H\gamma$ Line in the Spectrum β Orionis.

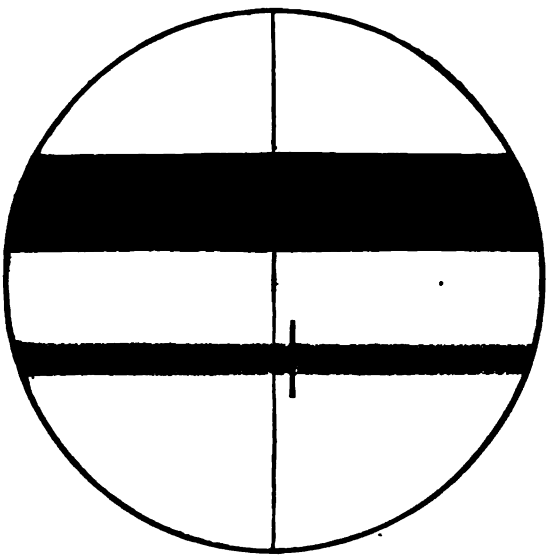

FIG. 215*.

a few seconds at various times during the long exposure (an hour or so) which is required for the star-spectrum. One sees easily that in this case the star-line is shifted a little to the right; but the line appears to be so poorly defined that accurate measurement would be difficult; for the methods by which the difficulty is overcome, and for the corrections required on account of the orbital motion of the earth and other causes, the reader is referred to the book from which the figure is taken. It is found that the probable error of the

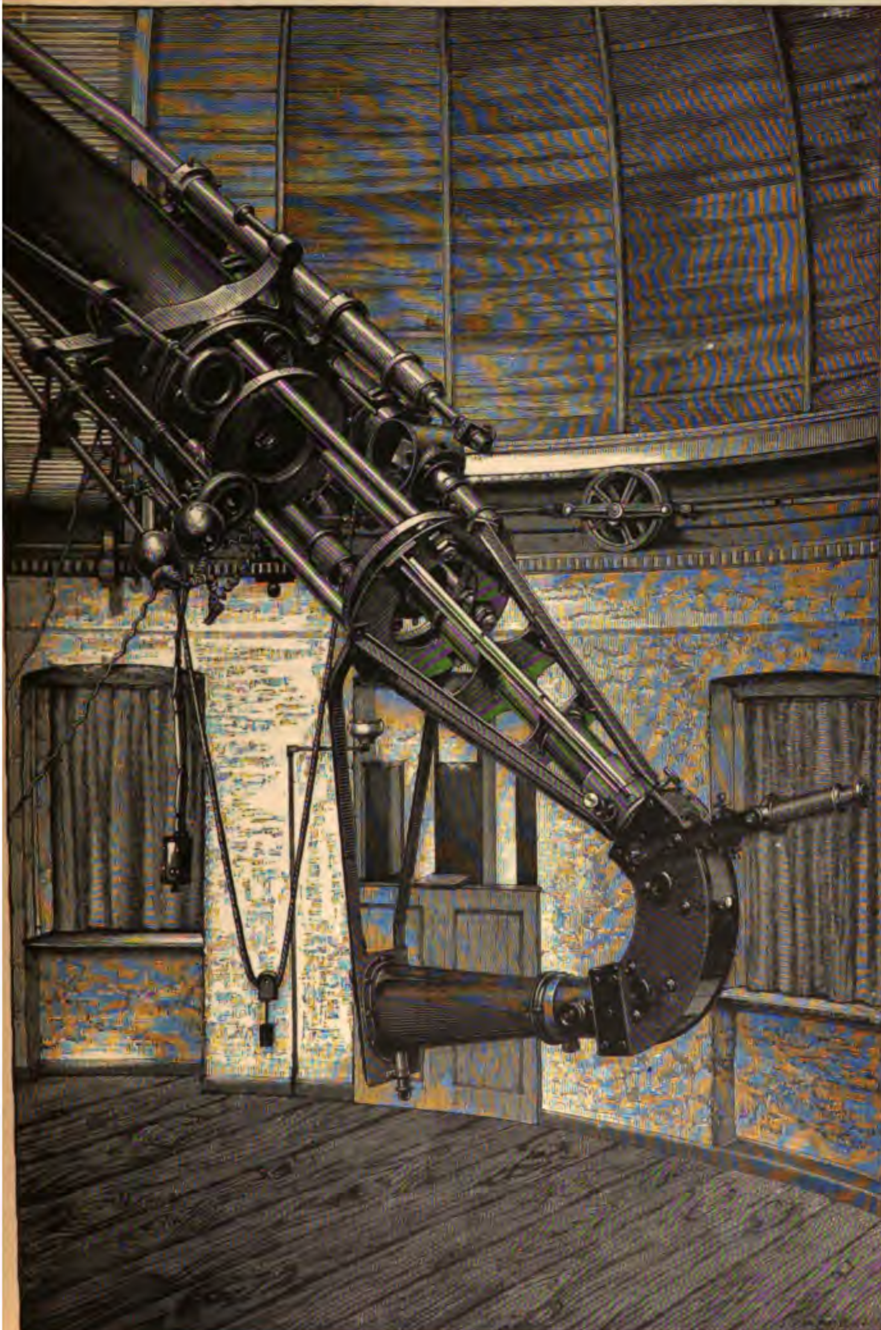

FIG. 216.

radial velocity of a star as deduced from the Potsdam photographs seldom exceeds a mile a second, and is usually much less.

The large engraving, Fig. 216, is from a photograph of the Potsdam "Spectrograph," as the photographic spectroscope is called. It is taken from the same source as the preceding figure.

Observations made by Humphreys and Mohler of Baltimore in 1895 appear to show that under heavy pressures the spectrum-lines of many elements *shift slightly towards the red*, just as if the luminous object were receding. The shift is different for different substances, but is always minute, never, even under pressures of ten or twelve atmospheres, exceeding the displacement that would be due to a receding velocity of one or two miles a second. Still, it is quite sufficient to require to be examined and taken into account in all applications of Doppler's principle.

Table VII. of the Appendix presents Vogel's results for the 51 stars that he had been able to deal with up to 1892. His telescope (since replaced by a much larger one) had an aperture of only eleven inches, which limited him to the brightest stars. The maximum velocity indicated by his observations is that of α Tauri, 30.1 miles a second, *receding*. The next in order is that of γ Leonis, 24.1 miles, *approaching*. Bélopolsky, at Pulkowa, has since found for ζ Herculis the still higher velocity of 43.8 miles, — also *approaching*.[1] Investigations of this nature are now (1904) successfully prosecuted by several other observers, prominent among whom are Bélopolsky at St. Petersburg, and Campbell and Frost at the Lick and Yerkes observatories. The number of stars whose radial velocity has been thus determined is at least 100, and the "probable error" has been reduced to about a quarter of a mile.

803. Star-Groups. — Star-atlases have been constructed by Proctor and Flammarion, which show by arrows the direction and rate of the angular proper motion of the stars as far as now known. A

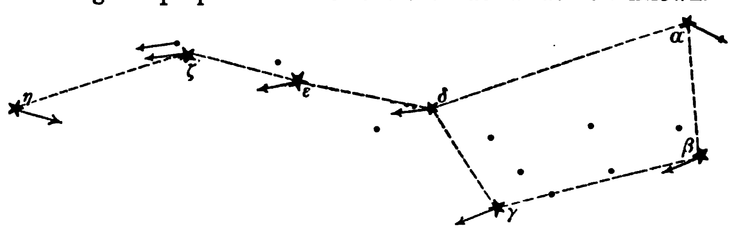

FIG. 217. — Common Proper Motions of Stars in the "Dipper" of Ursa Major.

moment's inspection shows that in many cases stars in the same neighborhood have a proper motion nearly the same in direction and in amount.

[1] Campbell finds for η Cephei a radial velocity of 54 miles, and for μ Cassiopeiæ, 61, both approaching.

Thus, Flammarion has pointed out that the stars in the "dipper" of *Ursa Major* have such a community of motion, except α and η, — the brighter of the pointers and the star in the end of the handle, — which are moving in entirely different directions, and refuse to be counted as belonging to the same group. Fig. 217 shows the proper motions of the stars which compose this group. The Potsdam observations show that all of them, excepting δ, which is too faint to be included, are approaching the sun with velocities between 12 and 20 miles a second.

The brighter stars of the Pleiades are found in the same way to have a common motion.

In fact, it appears to be the rule rather than the exception that stars apparently near each other are really connected as comrades, travelling together in groups of twos and threes, dozens or hundreds. They show, as Miss Clerke graphically expresses it, a distinctly "*gregarious* tendency."

804. The "Sun's Way." — The proper motions of the stars are due partly to their own real motion, and partly also to the motion of our sun, which is moving swiftly through space, taking with it the earth and the planets. Sir William Herschel was the first to investigate and determine the direction of this motion a little more than 100 years ago. The principle involved is this: that the apparent motion of each star is made up of its own motion combined with the motion of the sun *reversed* (Art. 492). The effect must be that, *on the whole*, the stars in that part of the sky towards which the sun is moving are separating from each other, — the *intervals between them widening out*, — while in the opposite part of the heavens *they are closing up;* and in the intermediate part of the sky the general drift must be *backward* with reference to the sun's (and earth's) real motion. Just as one walking in a park filled with people moving indiscriminately in different directions would, on the whole, find that those in front of him appeared to grow larger,[1] and the spaces between them to open out, while at the sides they would drift backwards, and in the rear close up.

Spectroscopic observations ought also to give evidence as to this solar motion; and they do so. On the whole, in the quarter of the heavens towards which the sun is moving, the stellar spectra indicate *approach*, and *vice versa* in the opposite quarter. As yet, however, the stars whose radial motion is known are too few to yield a very

[1] Theoretically, of course, the stars towards which we are moving must appear to *grow brighter* as well as to drift apart; but this change of brightness, though real, is entirely imperceptible within a human lifetime.

good determination of the *direction* of the solar motion, though they give a better determination of its *velocity* than is furnished by the other method.

805. About twenty different determinations of the point in the sky towards which this motion of the sun is directed have been worked out by various astronomers, using in their discussions the angular proper motions of from twenty to twenty-five hundred stars. All the investigations present a reasonable accordance of results, and show *that the sun is moving towards a point near the eastern edge of the constellation of Hercules, having a right ascension of about 277°.5 and a declination of about 35°.* (According to Newcomb.)

Several of the later results deduced from the proper motions of the fainter stars indicate a point farther north and east, near α Lyræ. The spectroscopic result, on the contrary, puts it farther *west* and north; but is not entitled to much weight.

This point towards which the sun's motion is directed is called "*The Apex of the Sun's Way,*" or often, and more simply, "*The Solar Apex.*"

806. Velocity of the Sun's Motion in Space. — From the discussion of the "proper motions" it appears that the sun's velocity is such as would carry it, and its system with it, about 5″ in a century as seen from the average sixth magnitude star, the sixth magnitude being the smallest easily visible to the naked eye. We could at once convert the expression into *miles* if we had any accurate knowledge of the distance of this class of stars, but as to that we can do little more than to make a reasonable conjecture. If we accept the estimate of Ludwig Struve (who has made one of the most careful investigations of the subject of "proper motions"), and assume this distance to be about 20 000000 astronomical units, the rate of the solar motion comes out about *fifteen miles a second.*

The spectroscopic observations of "motion in the line of sight," on the other hand, indicate a velocity of about *eleven miles a second;* and although these data are not yet sufficient to furnish a determination that can be considered final, this value is probably more authoritative than that deduced from the "proper motions," because it is not in any way dependent on our uncertain knowledge of stellar distances.

It is to be noted that this swift motion of the solar system, while of course it affects the real motion of the planets *in space*, converting them

into a sort of corkscrew spiral like the figure (Fig. 218), does not in the least affect the *relative* motion of sun and planets, as some paradoxers have supposed it must.

807. The Central Sun. — We mention this subject simply to say that there is no real foundation for the belief in the existence of such a body. The idea that the motion of our sun and of the other stars is a revolution around some great central sun is a very fascinating one to certain minds, and one that has been frequently suggested. It was seriously advocated some fifty years ago by Mädler, who placed this centre of the stellar universe at Alcyone, the principal star in the Pleiades.

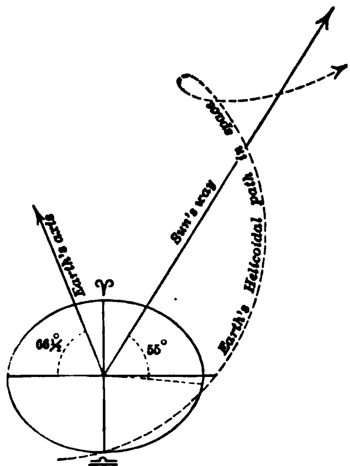

Fig. 218.

The Earth's Motion in Space as affected by the Sun's Drift.

It is certainly within bounds to deny that any such motion has been demonstrated, and it is still less probable that the star Alcyone is the centre of such a motion, if the motion exists. So far as we can judge at present it is most likely that the stars are moving, not in regular closed orbits around any centre whatever, but rather as bees do in a swarm, each for itself, under the action of the predominant attraction of its nearest neighbors. The *solar* system is an absolute monarchy with the sun supreme. The great *stellar* system appears to be a republic, without any such central, unique, and dominant authority.

THE PARALLAX AND DISTANCE OF THE STARS.

808. When we speak of the "parallax" of the moon, the sun, or a planet, we always mean the *diurnal* parallax, *i.e.*, *the angular semi-diameter of the earth* as seen from the body in question. In the case of the stars, this kind of parallax is hopelessly insensible, never reaching an amount of $\frac{1}{20000}$ of a second of arc.

The expression "parallax of a *star*" always means its *annual* parallax, that is, the semi-diameter of the earth's *orbit* as seen from the star. Even this in the case of all stars but a very few is a mere frac-

tion of a second of arc, too small to be measured. In a few instances it rises to about half a second, and in the one case of our nearest neighbor (so far as known at present), the star α Centauri, it appears to be about 0″.9, according to the earlier observers, or about 0″.75, according to the latest determination of Gill and Elkin. In Fig. 219 the angle at the star is the star's parallax.

In accordance with the principle of relative motion (Art. 492), every star has, superposed upon its own motion and combined with it, an *apparent* motion equal to that of the earth but reversed. If the star is really at rest it must seem to travel around each year in a little orbit 186 000000 miles in diameter, the precise counterpart of the earth's orbit in size and form, and having its plane parallel to the ecliptic.

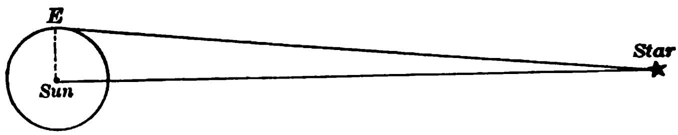

FIG. 219. — The Annual Parallax of a Star.

If the star is near the pole of the ecliptic this apparent "parallactic" orbit will be viewed perpendicularly and appear as a circle; if the star is on the ecliptic it will be seen edgewise as a short, straight line, while in the intermediate latitudes the parallactic orbit will appear as an ellipse. In this respect it is just like the "aberrational" orbit of a star (Art. 226); but while the aberrational orbit is of the same size for every star, having always a semi-major axis of 20″.47, the size of the parallactic orbit depends upon the distance of the star. Moreover, in the parallactic orbit the star is always opposite to the earth, while in the aberrational orbit it keeps just 90° ahead of her.

809. If we can find a way of measuring this parallactic orbit, the star's distance is at once determined from the equation

$$\text{Distance} = \frac{206265 \times R}{p''},$$

in which p'' is the parallax in seconds of arc (the apparent semi-major axis of the parallactic orbit), and R is the earth's distance from the sun.

The determination of stellar parallax had been attempted over and over again from the days of Tycho down, but without success until Bessel, in 1838, succeeded in demonstrating and measuring the parallax of the star 61 Cygni; and the next year Henderson, of the

Cape of Good Hope, brought out that of α Centauri. It will be remembered that it was mainly on account of his failure to detect stellar parallax that Tycho rejected the Copernican theory and substituted his own (Art. 504).

Roemer, of Copenhagen, in 1690, thought that he had detected the effect of stellar parallax in his observations of the difference of right ascension between Sirius and Vega at different times of the year. A few years later, Horrebow, his successor, from his own discussion of Roemer's observations, made out the amount to be nearly four seconds of time or 1′, and published his premature exultation in a book entitled "Copernicus Triumphans." The discovery of *aberration* by Bradley explained many abnormal results of the early astronomers which had been thought to arise from stellar parallax, and proved that the parallax must be extremely small. About the beginning of the present century, Brinkley, of Dublin, and Pond, the Astronomer Royal, had a lively controversy over their observations of α Lyræ (Vega). Brinkley considered that his observations indicated a parallax of nearly 3″. Pond, on the other hand, deduced a minute *negative* parallax, which is of course impossible, and, as some one expresses it, would put the star "somewhere on the other side of nowhere." In fact, as it turns out, Pond was nearer right than Brinkley, the actual parallax as deduced from the latest observations being only about 0″.2. The apparent negative parallax simply indicated that the actual parallax is too small to show itself decidedly, and was overborne by errors of observation. The periodical changes of temperature and air pressure continually lead to fallacious results except under the most extreme precautions.

810. Methods of Determining Parallax. — The operation of measuring a stellar parallax is, on the whole, the most delicate in the whole range of practical astronomy. Two[1] methods have been successfully employed so far — the *absolute* and the *differential.*

(*a*) The first method consists in making meridian observations of the right ascension and declination of the star in question at different seasons of the year, applying all known corrections for precession, nutation, aberration, and proper motion, and then studying the resulting star-places. If the star is without parallax, the places should be identical after the corrections have been duly applied. If it has parallax, the star will be found to change its right ascension and declination systematically, though slightly, through the year. But the changes of the seasons so disturb the constants of the instrument that the method is treacherous and uncertain. There is no possibility of getting rid of these temperature effects (in producing changes of refraction and varying expansions of the instrument itself) by merely multiplying observations and *taking averages,* since the

[1] For spectroscopic method, see note on page 506.

changes of temperature are themselves annually periodic, just as is the parallax itself.

Still, in a few cases the method has proved successful. Different observers at different places with different instruments have found for a few stars fairly accordant results; as, for instance, in the case of α Centauri, already mentioned as our nearest neighbor.

811. (*b*) **The Differential Method.** — This consists in measuring the change of position of the star whose parallax we are seeking (which is supposed to be comparatively near to us), with reference to other small stars, which are in the same telescopic field of view, but are supposed to be so far beyond the principal star as to have no sensible parallax of their own. If the comparison stars are near the large one (say within two or three minutes of arc), the ordinary wire micrometer answers very well for the necessary measures; but if they are farther away, the heliometer (Art. 677) represents special and very great advantages. It was with this instrument that Bessel, in the case of 61 Cygni, obtained the first success in this line of research.

The great advantage of the differential method is that it avoids entirely the difficulties which arise from the uncertainties as to the exact amount of precession, etc.; and in great measure, though not entirely, those arising from the effect of the seasons upon refraction and the condition of the instruments. On the other hand, however, it gives as the final result, not the absolute parallax of the star, but only the *difference between its parallax and that of the comparison star.* If the work is accurate the parallax deduced *cannot be too great; but it may be sensibly too small,* and so may make the star apparently too remote. This is because the parallax of the comparison star can never be quite zero: if the comparison star happens to have a parallax of its own as large as that of the principal star, there will be no relative parallax at all; *if larger, the parallax sought will come out apparently negative,* which is by no means unusual.

812. Determination of Parallax by Photography. — Recently photography has been pressed into the service with great advantage, first by the late Professor Pritchard in 1886. Measurements with the micrometer and heliometer are so tedious that in practice it is impossible to use more than one or two "comparison stars" in determining a parallax by their use, but there is no such restriction in the case of photography. The negative will probably show the images of a great number of stars, and all of them can be utilized.

Of course the number of negatives must be considerable, and taken at times well distributed through the year, with extreme precautions also in the development of the plates to prevent *any slipping* or distortion of the film upon the glass.

It is probable that this method will now give us before very long a large number of well-determined parallaxes.

813. Selection of Stars. — It is important to select for investigations of this kind those stars which may reasonably be supposed to be near, and, therefore, to have a sensible parallax. The most important indication of proximity is a *large proper motion*, and *brightness* is, of course, confirmatory. At the same time, while it is probable that a bright star with large proper motion is comparatively near, it is not certain. The small stars are so much more numerous than the large ones that it will be nothing surprising if we should find among them one or more neighbors nearer than α Centauri itself.

814. Unit of Stellar Distance. — *The Light-Year.* The ordinary "*astronomical unit*," or distance of the sun from the earth, is not sufficiently large to be convenient in expressing the distances of the stars. It is found more satisfactory to take as a unit the distance that light travels in a year, which is about 63000 times the distance of the earth from the sun. A star with a parallax of 1″ is at a distance of 3.26 "*light-years*" so that the distance of any star in "light-years" is expressed by the formula

$$D_y = \frac{3.26}{p''}.$$

815. Table IV. in the Appendix gives the parallaxes, the distances in light-years, and the proper motions, and "cross," or "thwartwise" motions (Art. 801, note) of certain stars whose parallaxes may be considered as now fairly determined. There are other stars for which parallaxes have been found larger than some of those included in the table; but the results are not yet sufficiently confirmed.

The student will, of course, see that the tabulated distance in the case of a remote star is liable to an enormous percentage of error. Considering the amount of discordance between the results of different observers, it is extremely charitable to assume that any of the parallaxes are certain within $\frac{1}{50}$ of a second of arc: but in the case of a star like the pole star, which appears to have a parallax of less than 0″.08, this $\frac{1}{50}$ of a second is $\frac{1}{4}$ of the whole amount; so that the distance of that star is uncertain by at least twenty-five per cent. ($\frac{1}{50}$ of a second is the angle subtended by $\frac{1}{10}$ of an inch at the distance of ten miles.)

As regards the distance of stars, the parallax of which has not yet been measured, very little can be said with certainty. It is *probable* that the remoter ones are so far away that light in making its journey occupies a thousand and perhaps many thousand years.

Exercises on Chapter XX.

1. Assuming the parallax of 61 Cygni as 0″.40, and that it is approaching the sun at the rate of 34.5 miles a second (Art. 801), how many years would it take to increase its brightness by ten per cent, supposing its radial velocity to remain unchanged? *Ans.* 2050 years.

2. Assuming the distance of 61 Cygni as 8.15 light-years, and that the radial and transverse velocities are 34.5 and 38 miles a second respectively, find how near the star will come to the sun if it keeps on uniformly, and in a straight line; also how long it will take to reach that point of nearest approach.

Ans. Nearest approach 6.03 light-years; reached after 19900 years.

Note on Spectroscopic Determination of Parallax of Binary Stars.

SPECTROSCOPIC DETERMINATION OF STELLAR PARALLAX. In the case of certain binary stars (Arts. 872–877) of which the period and angular dimensions of the orbit are accurately known, *and which have the plane of the orbit nearly directed toward the sun*, the spectroscope will enable us to determine the velocity with which they move, and therefore the *actual dimensions of the orbit in miles*. Combining this with the apparent dimensions of the orbit *in seconds of arc*, we get at once the length in miles of a second of arc at the star's distance; from which the parallax and distance immediately follow.

Wright has recently (1905) tested this method upon Alpha Centauri, and gets a parallax of 0.76″, in almost absolute agreement with that determined by the usual methods. It will, however, probably be a long time before we shall have sufficient spectroscopic observations to enable us to make many applications of the method, especially as the available binaries are not numerous.

CHAPTER XXI.

THE LIGHT OF THE STARS. — STAR MAGNITUDES AND PHOTOMETRY. — VARIABLE STARS. — STELLAR SPECTRA. — SCINTILLATION OF STARS.

816. Star Magnitudes. — The term "magnitude," as applied to a star, refers simply to its brightness. It has nothing to do with its apparent angular diameter. Hipparchus and Ptolemy arbitrarily graded the visible stars, according to their brightness, into six classes, the stars of the sixth magnitude being the smallest visible to the eye, while the first class comprises about twenty of the brightest. There is no assignable reason why *six* classes should have been constituted, rather than eight or ten.

After the invention of the telescope the same system was extended to the smaller stars, but without any general agreement or concert, so that the magnitudes assigned by different observers to telescopic stars vary enormously. Sir William Herschel, especially, used very high numbers: his twentieth magnitude being about the same as the fourteenth on the scale now generally used, which more nearly corresponds with that of Argelander.

817. Fractional Magnitudes. — Of course, the stars classed together under one magnitude are not exactly alike in brightness, but shade from the brighter to the fainter, so that exactness requires the use of *fractional* magnitudes. It is now usual to employ decimals giving the brightness of a star to the nearest tenth of a magnitude. Thus, a star of 4.3 magnitude is a shade brighter than one of 4.4, and so on.

A peculiar notation was employed by Ptolemy, and used by Argelander in his "Uranometria[1] Nova." It recognizes *thirds* of a magnitude as the smallest subdivision. Thus, 2, 2,3, 3,2, and 3 express the gradations between second and third magnitude, 2,3 being applied to a star whose brightness is a little inferior to the second, and 3,2 to one a little brighter than the third magnitude.

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[1] The term "Uranometria" has come to mean a catalogue of *naked-eye stars:* like the catalogues of Hipparchus, Ptolemy, and Ulugh Beigh.

818. Stars Visible to the Naked Eye. — Heis enumerates the stars clearly visible to the naked eye in the part of the sky north of 35° south declination, as follows: —

1st magnitude, ...	14	4th magnitude, ...	313
2d "	48	5th "	854
3d "	152	6th "	2010
Total			3391

According to Newcomb, the number of stars of each magnitude is such that united they would give, roughly speaking, somewhere nearly the same amount of light as that received from the aggregate of those of the next brighter magnitude. But the relation is very far from exact, and seems to fail entirely for the fainter magnitudes below the tenth or eleventh, the smaller stars being less numerous than they should be. In fact, if the law held out perfectly, and if light was transmitted through space without loss, the whole sky would be a blaze of light like the surface of the sun.

819. Light-Ratio and Absolute Scale of Star Magnitudes. — It was found by Sir John Herschel, about fifty years ago, that the light given by the average star of the first magnitude is just about one hundred times as great as that received from one of the sixth, and that a corresponding ratio has been pretty nearly maintained throughout the scale of magnitudes, the stars of each magnitude being about $2\frac{1}{2}$ times ($\sqrt[5]{100}$) brighter than those of the next inferior magnitude. The number which expresses the ratio of the light of a star to that of another one magnitude fainter is called the *light-ratio.*

In the star magnitudes of the maps by Argelander, Heis, and others, which are most used at present, the divergence from a strict uniformity of light-ratio is, however, sometimes very serious. About 1850 it was proposed by Pogson to reform the system, by adopting a scale with the uniform light-ratio of $\sqrt[5]{100}$, adjusting the first six magnitudes to correspond as nearly as possible with the magnitudes hitherto assigned by leading authorities, and then carrying forward the scale indefinitely among the telescopic stars. Until recently this "*absolute scale of magnitude,*" as it has been called, has not been much used; but in the New Uranometrias made at Cambridge (U. S.) and Oxford it has been adopted, and it is now rapidly supplanting the older systems.

820. Relative Brightness of Different Star Magnitudes. — In this scale the light-ratio between successive magnitudes is made exactly

$\sqrt[5]{100}$, *or the number whose logarithm is* 0.4000,[1] *viz.*, 2.512. Its reciprocal is the number whose logarithm is 9.6000, *viz.*, 0.3981. If b_1 is the brightness of a standard first-magnitude star, expressed either in candle-power or other convenient unit, and b_n be the brightness of a star of the nth magnitude on this scale, we shall therefore have

$$\log b_n = \log b_m - \tfrac{4}{10}(n-m); \quad (1)$$

$$\text{conversely, } n = 1 + \tfrac{10}{4}(\log b_m - \log b_n), \text{ or } 1 + \tfrac{10}{4}\log\left(\frac{b_m}{b_n}\right),$$

$(n-m)$ in these equations being the number of magnitudes *between* the star of the mth magnitude and the star of the nth magnitude; so that for a star of the sixth magnitude compared with the first equation (1) reads,

$$\log b_6 = \log b_1 - \tfrac{4}{10} \times 5 = \log b_1 - 2.$$

With this light-ratio, every difference of five magnitudes corresponds to a multiplication or division of the star's light by 100; *i.e.*, to make one star as bright as the standard star of the first magnitude it would require 100 of the sixth, 10000 of the eleventh, 1 000000 of the sixteenth, and 100 000000 of the twenty-first magnitude.

As nearly standard stars of the first magnitude on this scale we have α Aquilæ and Aldebaran (α Tauri). The pole star and the two "pointers" are very nearly standard stars of the *second* magnitude.

821. Negative Magnitudes.—According to this scale, stars that are one magnitude *brighter* than those of the standard first would be of the *zero* magnitude (as is the case with Arcturus), and those that are brighter yet would be of a *negative* magnitude; *e.g.*, the magnitude of Sirius is -1.43; and Jupiter at opposition, in conformity to this system, is described as a star of nearly -2d magnitude, which means that it is nearly 2.51^3, or about 16 times *brighter than* a star of the $+1$st magnitude like Aldebaran.

822. Relation of Size of Telescope to the Magnitude of the Smallest Star Visible with it.—If a telescope just shows a star of a given magnitude, then to show stars one magnitude smaller we require an instrument having its aperture larger in the ratio of $\sqrt{2.512}$ (or $\sqrt[10]{100}$) to 1; *i.e.*, as 1.59 : 1. Every *tenfold* increase in the diameter of the object-glass will therefore carry the power of vision just *five magnitudes lower*.

[1] $\frac{\text{Log. } 100}{5} = 0.4000.$

Assuming what seems to be very nearly true for normal eyes and good telescopes, that the "*minimum visibile*" for a one-inch aperture is a star of the ninth magnitude, we obtain the following little table of *apertures required to show stars of a given magnitude*, the formula being $m = 9 + 5 \times \log.$ of aperture in inches.

Star Magnitude. .	7	8	9	10	11	12
Aperture.	$0^{in}.40$	$0^{in}.63$	$1^{in}.00$	$1^{in}.59$	$2^{in}.51$	$3^{in}.98$
Star Magnitude. .	13	14	15	16	17	18
Aperture.	$6^{in}.31$	$10^{in}.00$	$15^{in}.90$	$25^{in}.10$	$39^{in}.80$	$63^{in}.10$

But on account of the increased thickness necessary in the lenses of large telescopes, they never quite equal their theoretical capacity as compared with smaller ones.

The Yerkes telescope, therefore (forty inches aperture), will barely show stars of the seventeenth magnitude, not quite one magnitude fainter than the smallest visible with the twenty-six-inch telescope at Washington. But the *number* made visible will probably be about doubled, since the smaller stars are vastly the more numerous.

823. Measurement of Star Magnitudes and Brightness. — Until recently all such measurements were mere eye-estimates. Even yet nearly all photometric measures depend *ultimately* on the judgment of the eye. But it is possible by the help of instruments to aid this judgment very much by limiting the point to be decided, to the question whether two lights as seen are, or are not, exactly equal, or else making the decision depend on the visibility or non-visibility of some appearance. See Art. 831.

824. Method of Sequences. — For some purposes the unassisted eye is quite as good as any photometric instrument. It judges directly with great precision of the *order of brightness* in which a number of objects stand. In the method of "sequences," as it is called, the observer merely arranges the stars he is comparing, say to the number of fifty or so, in the order of their brightness, taking care that the stars in each sequence list are nearly at the same altitude, and seen under equally favorable circumstances. Then he makes a second sequence, taking care to include in it some of the stars that were in the first; and so on. Finally, from the whole set of sequences, a list can be formed, including all the stars contained in any of them, arranged in the order of brightness. This process gives, however, no determination of the light-ratio, nor of the

number of times by which the light of the brightest exceeds that of the faintest.

Variable stars are still often observed in this way, the stars with which they are compared being such as have their magnitudes already well determined.

825. 2. **Instrumental Methods.**—These are nearly all based on two different principles:—

a. The measurement is made by causing the star to *disappear* by diminishing its light in some measurable way. This is usually referred to as the "method of *extinctions*."

b. The measurement is effected by causing the light of the star to appear just *equal* to some other standard light, by decreasing the brightness of the star or of the standard in some known ratio until they are perfectly equalized.

Under the first head come the photometers which act upon the principle of "*limiting apertures*." The telescope is fitted with some arrangement, often a so-called "cat's-eye," by which the available aperture of the object-glass can be diminished at will, and the observation consists in determining with what area of object-glass the star is just visible. The method is embarrassed by constant errors from the fact that the greater thickness of the glass in the middle of the lens comes into account, and, still worse, from the fact that the image of the star becomes large and diffuse (on account of diffraction) when the aperture is very much reduced.

826. The Wedge Photometer.—The method of producing the "extinction" by a "*wedge*" of dark, neutral-tinted glass is much better. The wedge is usually five or six inches long, by perhaps a quarter of an inch wide, and at the thick end cuts off light enough to extinguish the brightest stars that are to be observed. In the Pritchard form of the instrument this wedge is placed close to the eye at the eye-hole of the eye-piece; in some other forms it is placed at the principal focus of the object-glass, where micrometer wires would be put.

In observation the wedge is pushed along promptly until the star just disappears, and a graduation on the edge of the slider is read.

The great simplicity of the instrument commends it, and if the wedge is a good one of really neutral glass (which is not easy to get), the results are remarkably accurate. But the observations are very trying to the eyes on account of the straining to keep in sight an object just as it is becoming invisible. The constant of the wedge must be carefully determined in the

laboratory, *i.e.*, what length of the wedge corresponds to a diminution of the light of a star by just one magnitude (cutting off 0.602 of its light). It is convenient to have the slider graduated into inches or millimeters on the one edge and magnitudes on the other. The "Uranometria Nova Oxoniensis" is a catalogue of the magnitudes of the naked-eye stars to the number of 2784, between the pole and 10° south declination, observed with an instrument of this kind by Professor Pritchard, and published in 1885.

827. Polarization Photometers.—The instruments, however, with which most of the accurate photometric work upon the stars has been done, are such as compare the light of the star with some standard by means of an "*equalizing apparatus*" based on the application of the principles of double refraction and polarization.

The light of either the observed star or the comparison star (real or artificial) is polarized by transmission through a Nicol prism, or else both pencils are sent through a double refracting prism. The images are viewed with a Nicol prism in the eye-piece; and by turning this the polarized image or images can be varied in brightness at pleasure, and the amount of variation determined by reading a small circle attached to it. In the photometers of Seidel and Zöllner, who observed comparatively few objects, but very accurately, the artificial star with which the real stars were compared was formed by light from a petroleum lamp, shining through a small aperture, and reflected to the eye by a plate of glass in the telescope tube. Müller and Kempf at Potsdam used the same instrument in their very accurate photometric catalogue of 3500 stars above the $7\frac{1}{2}$ magnitude between the equator and dec. +20°. Pickering, in making his much more extensive but less precise photometric catalogues, published and in progress, has also used the polarization principle, but has made the *pole star* the standard, bringing it by an ingenious arrangement into the same field with the star whose brightness is to be measured.

Photometric observations in many cases require large and somewhat uncertain corrections, especially for the absorption of light by the atmosphere at different altitudes, and the final results of different observers naturally fail of absolute accordance. Still the agreement between the different recent catalogues is remarkably close, generally within one or two tenths of a magnitude.

828. The Meridian Photometer. — This instrument, contrived and used by Professor Pickering in the observations of the Harvard Photometry, consists of a telescope with two object-glasses side by side. The telescope is pointed nearly east and west, and in front of each object-glass is placed a silvered glass mirror at an angle of 45°. One of the mirrors is so set as

to bring the rays of the pole star to one object-glass; the other mirror is capable of being turned around the optical axis of the telescope, in such a way as to command a star at any part of the meridian, and bring its light into the other object-glass. At the eye end is placed a double refraction polarization apparatus, which gives an image of the pole star polarized in one plane, and an image of the star to be observed polarized in a plane at right angles, both visible at the same time in the same eye-piece. In front of the eye-piece is a Nicol prism. On looking into the instrument the observer sees two stars, the pole star at rest, the other moving along as in a transit instrument. He simply turns the Nicol until the images are equalized, setting the Nicol at all the four different positions which will produce the effect, and reading the graduated circle *C*. The whole operation consumes not more than a minute, with the help of an assistant to record the numbers as read off. The "Harvard Photometry" (usually referred to simply as "H. P.,") was made by means of an instrument with object-glasses only two inches and a half in diameter. An instrument with four-inch lenses was used at Cambridge in measuring the magnitudes of all the nearly 80000 stars of Argelander's *Durchmusterung*, which are of the eighth magnitude or brighter, and has been sent to Peru, where it is to complete the photometry of the southern heavens.

829. Photometry by means of Photography.—It has been found that, excepting a few strongly colored stars, the intensity, or more simply the *size*, of the image of a star formed upon a photographic plate may be used as a measure of its brightness as compared with other stars taken on the same plate, or on similar plates similarly exposed. The comparison becomes easier and more accurate if the photographic telescope is not made to follow the stars exactly, but is allowed to lag a little, so that the star forms a "trail." It will, therefore, be possible to use the plates of the great photographic star campaign to determine star magnitudes as well as positions. But, as has been intimated, there are some anomalies; certain stars, for instance, that are hardly visible to the naked eye, photograph as bright stars, and there are others — red stars — that are abnormally faint on the plate. The exceptions are numerous enough to make it necessary to use photographic magnitudes with caution. In the study of *variable* stars, however, the method may be used with some great advantages.

830. Star Colors and their Effects on Photometry.—The stars differ considerably in color. The majority are of a very pure white, like Sirius and the sun, but there are not a few of a yellowish hue, like Capella, or reddish, like Arcturus and Antares; and there are some, mostly small stars, which are as red as garnets and rubies. We also have, associated with larger ones in double-star systems,

numerous small stars which are strongly green or blue; and there are a few large isolated stars, which, like Vega, are of a decidedly bluish tinge.

These differences of color embarrass photometric measurements made by either of the methods described, because it is impossible to make a red star look identical with a blue one by any mere increase or diminution of brightness, and because different observers will differ in setting the wedge of an extinction photometer according to the color of the star. Some eyes are abnormally sensitive to blue light, some to red. To the writer, for instance, Vega is decidedly superior to Arcturus, while the majority of observers see the difference as decidedly the other way.

831. Spectrum Photometry. — The only completely satisfactory and scientific method would be to compare the spectra of the stars with some standard spectrum, *say that of the pole star*, dividing the spectrum into a considerable number of portions, and determining and recording the amount of light in each portion of the spectrum as compared with homologous parts of the standard spectrum. This, of course, would immensely increase the work of comparing the brightness of the stars; but it is quite feasible to do it for a few hundred of the brighter ones, and it would be well worth accomplishment. If we ever succeed in getting photographic plates equally sensitive to rays of all wave length, photography would answer the purpose well.

The day may come when we shall have "bolometers" or "radiometers" sufficiently delicate to enable us to extend our measures through the whole extent of the spectrum — the invisible portions as well as the visible — and so *determine the total "energy" sent to us from a star:* but we are far enough from it at present.

It is worth noting, also, that Minchin has recently met with encouraging success in attempting to measure the luminosity of stars by its effect in changing the electrical resistance of *selenium*, and the method may ultimately develop into something valuable.

832. Starlight compared with Sunlight. — The light *received* from Vega is about $\frac{1}{40000000000}$ (one forty thousand millionth) of that from the sun, according to the determinations of Zöllner and others. The measurement is not easy, and must be taken as having a very considerable margin of error.

Sirius is nearly equivalent to six of Vega, its light being about $\frac{1}{7000000000}$ of the sun's.

The light of the *standard first magnitude star* may be taken as about half that of Vega, or $\frac{1}{80000000000}$ of sunlight; so that on the absolute scale the sun is reckoned as of the -26.3 "magnitude."

Since the light of a sixth-magnitude star is only $\frac{1}{100}$ of that of a standard first-magnitude, it follows that it would require 8 000000-000000 stars of the 6th magnitude to give us sunlight.

833. Total Light of the Stars. — Assuming what is roughly, though not exactly, true, that Argelander's magnitudes follow the standard scale, it appears that the 324000 stars north of the equator enumerated in his *Durchmusterung* give a light about equal to that of 240 first-magnitude stars; but it is noticeable that the aggregate amount of light given by the stars in each of the fainter magnitudes increases rapidly.

The following is the estimate, substantially according to Newcomb:

10	stars	(above the 2d magnitude)			=	6.0	first-magnitude stars.
37	"	from 2d to 3d		"	=	7.3	" "
122	"	"	3d to 4th	"	=	9.6	" "
310	"	"	4th to 5th	"	=	9.8	" "
1016	"	"	5th to 6th	"	=	12.7	" "
4322	"	"	6th to 7th	"	=	21.6	" "
13593	"	"	7th to 8th	"	=	27.1	" "
57960	"	"	8th to 9th	"	=	46.	" "
247544	"	"	9th to 9½	"	=	100. ±	" "
Total					=	240.	

How much to add for the still smaller magnitudes is very uncertain. Beyond the tenth magnitude the number of small stars does not increase proportionately fast, so that if we could carry on the account of stars to the twentieth magnitude, it is practically certain that we should not find the total light of the aggregate stars of each succeeding magnitude increasing at any such rate as from the seventh to the tenth. Perhaps it would be a not unreasonable estimate to put the total starlight of the northern hemisphere as equivalent to about 1500 first-magnitude stars, or that of the whole sphere at 3000. This would make the total starlight on a clear night about $\frac{1}{80}$ of the light of the full moon, and about $\frac{1}{87\,000000}$ that of the sun. The light from the stars which are visible to the naked eye would not be as much as $\frac{1}{25}$ of the whole. Newcomb (1902) estimates the total light of all the stars as about equal to 600 stars of magnitude zero — about one-fourth of the estimate above made.

834. Heat from the Stars. — Probably the heat of a star bears to solar heat about the same ratio as that of their lights; it is at the very limit of perception. Stone (1869) thought his thermopile

showed sensible effects of heat from Arcturus and Vega; but later work discredited his results. About 1890 Boys with his much more sensitive "radio-micrometer" failed in obtaining any distinct indications. The first certain success was reached in 1898–1900 by Prof. E. F. Nichols at the Yerkes Observatory with an apparatus twenty times as sensitive as that of Boys. He obtained measurable "deflections" from Vega, Arcturus, Jupiter, and Saturn.

Arcturus appeared to give 2.2 times as much heat as Vega; Jupiter 4.7 as much; and Saturn $\frac{1}{4}$; Vega about as much as a standard candle *9 miles* distant.

835. Amount of Light emitted by Certain Stars. — This, of course, is something vastly different from that *received* by us. A star may emit hundreds of times as much light as the sun, and yet, if the star is remote enough, the amount that reaches the earth will be only an excessively minute fraction of sunlight. If l be the amount of light that we *receive* from a star, expressed in terms of sunlight at the earth, then the total amount of light *emitted*, L, is given by the simple equation,

$$L = l \times D^2, \text{ or (nearly) } 4000\,000000 \times l \times D_y^2.$$

D being the distance of the star in astronomical units, and D_y, in "light-years," while L is expressed in terms of the sun's light emission.

Turning to the table of stellar parallax (Appendix, Table IV.), we find that, according to Gill & Elkin, D for Sirius equals 529000;

$$\text{whence, for Sirius,} \quad L = \frac{529000^2}{7000\,000000} = 40.0;$$

that is, the light emitted by Sirius is forty times as much as that emitted by the sun.

Similarly for the pole-star ($p = 0''.07$), $L = 68$; for Vega ($p = 0''.16$), $L = 44$; α Centauri ($p = 0''.75$), $L = 1.9$; 70 Ophiuchi ($p = 0''.25$), $L = 0.41$; 61 Cygni ($p = 0''.40$), $L = \frac{1}{10}$; 21258 Ll. ($p = 0''.26$), $L = \frac{1}{113}$.[1]

The companion of Sirius is a little star of the ninth magnitude, which forms a double-star system with Sirius itself. The light emitted by this companion does not exceed $\frac{1}{10000}$ that of Sirius.

836. Causes of Differences of Brightness in Stars. — It used to be thought that the stars were all very much alike in magnitude and

[1] In making this calculation the magnitudes of the Harvard Photometry were used.

constitution; not, indeed, without considerable differences, but as much resembling each other as do individuals of the same race. It is now quite certain that this is not the case, as is obvious from the short list of actual light emissions just given.

If the stars *were* all alike, all the differences of apparent brightness would be traceable simply to differences of *distance;* but as the facts are, we have to admit other causes to be equally effective. The differences of brightness are due, *first,* to difference of *distance; second,* to difference of *dimensions,* or of light-giving area; *third,* to difference in the *brilliance of the light-giving surface,* depending upon difference of temperature and constitution. There are stars near and remote, large and small, intensely incandescent and barely glowing with incipient or failing light.

As Bessel puts it, there is no reason why there may not be "as many *dark* stars as bright ones," and, as we shall soon see, there is now positive evidence that they are really numerous. The companion of Sirius, though only giving about $\frac{1}{12000}$ part as much light as Sirius itself, is nearly $\frac{1}{2}$ as heavy; so that, *mass for mass,* it cannot be $\frac{1}{4000}$ part as luminous.

When we compare stars by the thousand, we can say of the tenth-magnitude stars, for instance, as compared with the fifth, that *as a class* they are *more remote;* and also, just as truly, that *their average diameters are smaller,* and also that *their surfaces are less brilliant;* but we must be careful not to make any assertions of this sort regarding any one star of the tenth magnitude compared with a particular individual of the fifth, unless we have some absolute knowledge of their relative distances. The faint star may be the larger of the two, or the hotter, or the nearer. We must know something beyond their relative "magnitudes" before it is possible to settle such questions.

837. Real Diameter of Stars. — As to the apparent *angular* diameter, we can only say negatively that it is insensible, in no case being known to reach 0''.01. If there be a star of the same diameter as our sun, at such a distance that its parallax equals one second, its apparent diameter must be $\frac{1924''}{206265}$. [The sun's mean angular diameter is 1924'' (Art. 276).] This equals 0''.0093 — a quantity far too small to be reached by any direct measurement, especially since, even in the Lick telescope, the "spurious" disc of

a star has a diameter of nearly $\frac{1}{7}''$, and in smaller telescopes is much larger (about $0''.4$ in a ten-inch telescope).

There is a theoretical connection between the diameter of the diffraction rings seen around the image of a star in the telescope, and the real (as opposed to the spurious) diameter of the image; by comparing, therefore, the actual size of the rings with the size they should have if the star were an absolute optical point, we might hope to get a determination of the star's angular diameter. Thus far, however, no satisfactory results have been obtained. Michelson has proposed a somewhat similar method based on the diffraction fringes produced when a pair of parallel slits are placed in front of the object-glass of a telescope. But the calculation of the diameter of the stellar disc depends upon the assumption *that it is uniformly luminous all over*, or, if not, that we know the law of distribution,— assumptions by no means safe.

In a single case, that of the variable star Algol (Art. 848), the diameter has actually been determined by the peculiarities of its variation combined with spectroscopic observations (Art. 851); and quite possibly other similar cases may be found before very long. Algol has a diameter of about 1 060000 miles.

VARIABLE STARS.

838. A close examination shows that many stars change their brightness, and are called "*variable.*" The variable stars may be classified[1] as follows:—

I. Cases of slow continuous change.
II. Irregular fluctuations of light: alternately brightening and darkening without any apparent law.
III. Temporary stars, which blaze out suddenly and then disappear.
IV. Periodic stars of long period (two months to two years).
V. Periodic stars of short period (a few hours to three weeks).
VI. Periodic stars in which the variation is like that which would be the result of an eclipse by some intervening body — the Algol type.

839. I. Gradual Changes. On the whole, the changes in the brightness of the stars since the time of Hipparchus and Ptolemy have been surprisingly small. There has been no general increase

[1] This classification is substantially that of Professor Pickering, slightly modified, however, by Houzeau.

or decrease in the brightness of the stars as a whole; and there are few cases where any individual star has altered its brightness by a half or even a quarter of a magnitude. The *general appearance* of the sky is the same as it was 2000 years ago; so that notwithstanding all the effect of proper motions in the meantime and the whole amount of the variation that has taken place in the brightness of the stars, there is no doubt that if either of these old astronomers were to come to life he would immediately recognize the familiar constellations.

There are a few instances, however, in which it is almost certain that change has taken place and is going on. In the time of Eratosthenes the star in the "claw of the Scorpion" (now β Libræ) was reckoned the brightest in the constellation. At present, it is a whole magnitude below Antares, which is now much superior to any star in the vicinity. So when the two stars Castor and Pollux in the constellation Gemini were lettered by Bayer, the former, α, was brighter than Pollux, which was lettered β; but β is now notably the brighter of the two. Taking the whole heavens, we find a considerable number of such cases; perhaps a dozen or more.

840. Missing Stars. — It is a common belief that since accurate star-catalogues began to be made, many stars have disappeared and not a few new ones have come into existence. While it would not do to deny absolutely that anything of the kind has ever happened, it is certainly unsafe to assert that it has.

There are a considerable number of cases where stars are now missing from the older catalogues *as published*, — nearly, if not quite, a hundred, — but in almost every instance examination of the original observations shows that the place as printed was a mistake of some sort which can now be traced, — sometimes a mistake of the recorder, sometimes in the reduction of the observation, and sometimes of the press. In a few cases the star observed was a planet (Uranus, Neptune, or an asteroid); and in some cases the missing star may have been a "temporary star," as, for instance, 55 Herculis, which was observed by the elder Herschel. So many of the missing stars are now satisfactorily explained that it is natural to suppose that the few cases remaining are of the same sort.

There is no known instance of a *new* star appearing and remaining permanently visible.

841. II. STARS THAT EXHIBIT IRREGULAR FLUCTUATIONS IN BRIGHTNESS. The most conspicuous example of this class is the

strange star η Argus (not visible in the United States). This star ranges all the way from the *zero* magnitude (in 1843, when, according to Sir John Herschel, it was brighter than every star but Sirius) down to the seventh magnitude, which is its present brightness and has been ever since 1865. It is often called η *Carinæ*, the constellation Argo-Navis being subdivided into Puppis, Vela, and Carina.

Between 1677 (when it was observed by Halley as of the fourth magnitude) and 1800, it oscillated in brightness, so far as we can judge from the few observations extant, between the fourth and second magnitudes. About 1810, it rose rapidly in brightness, and between 1826 and 1850 it was never below the standard first magnitude. When brightest, in 1843, it was giving more than 25000 times as much light as in 1865. A singular fact is that the star is in the midst of a nebula which apparently sympathizes with it to some extent in its fluctuations. (There are other instances of connection of nebulæ with variable or temporary stars, as will appear later on.) Fig. 220 represents the "light-curve" of this object from 1800 to 1870, according to Loomis. It is barely possible, though hardly probable, that the star may be periodic with a period of about 70 years; but if so, it is unique.

FIG. 220. — Light-Curve of η Argus according to Loomis.

α Orionis, α Herculis, and α Cassiopeiæ behave in a somewhat similar manner, only the whole range of variation in their brightness is less than a single magnitude, and the oscillations never extend over more than two or three years. The catalogue of variable stars shows a considerable number of other similar cases.

842. III. Temporary Stars. There are fifteen well authenticated cases in which new stars have appeared *temporarily*, — that

is, for a few weeks or months, — blazing up suddenly and then gradually fading away. The list[1] is as follows, up to 1898:—

1. 134 B.C. The star of Hipparchus.
2. 389 A.D. A star in Aquila.
3. 1572 A.D. Tycho's star in Cassiopeia.
4. 1600 A.D. *P* Cygni, 3d magnitude, observed by Jansen.
5. 1604 A.D. Kepler's star in Ophiuchus.
6. 1670 A.D. 11 Vulpeculæ, 3d magnitude, observed by Anthelm.
7. 1848 A.D. A star of the 5th magnitude, observed by Hind — also in Ophiuchus.
8. 1860 A.D. *T* Scorpii, 7th magnitude, in the star cluster M 80, observed by Auwers.
9. 1866 A.D. *T* Coronæ-Borealis, 2d magnitude.
10. 1876 A.D. Nova Cygni, 3d magnitude.
11. 1885 A.D. A star in the nebula of Andromeda, 6th magnitude.
12. 1892–93 A.D. Nova Aurigæ, 4th magnitude.
13. 1893 A.D. Nova Normæ, 7th magnitude.
14. 1895 A.D. Nova Carinæ (Argus), 8th magnitude.
15. 1898 A.D. Nova Sagittarii, 5th magnitude.

As regards the first of these stars, we know almost nothing. Hipparchus has left no record of its position or brightness; but the Chinese annals mention a star as appearing in Scorpio at just that date, and probably the same object; though the Chinese observations *may* refer to a comet. The appearance of this new star led Hipparchus to form his catalogue of stars.

As to the second, possibly a comet, we know hardly more.

843. The third is justly famous. When it was first seen by Tycho in November, 1572, it was already brighter than Jupiter, having probably appeared a few days previously. It very soon became as bright as Venus herself, being even visible by day. Within a week or two it began to fail, but continued visible to the naked eye for fully sixteen months before it finally disappeared. It is not certain whether it still exists or not as a telescopic star: Tycho determined its position with as much accuracy as was possible to his instruments; and there are a number of small stars, any one of which is so near to Tycho's place that it might be the real object.

There has been an entirely unfounded notion that this star may have been identical with the "Star of Bethlehem," it being imagined that the star is *periodically* variable, with a period of 314 years. If so, it might have been expected to reappear in 1886, and it was so expected by certain persons

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[1] See Addendum B, following page 580.

"as a sign of the second coming of the Lord." It is difficult to see how the idea came to be so generally prevalent as it certainly has been. Probably every astronomer of any note has received hundreds of letters on the subject. At Greenwich a printed circular was prepared and sent out as a reply to such inquiries.

The fifth star, observed by Kepler, was nearly, though not quite, as bright as that of Tycho, and lasted longer — fully two years. It also has disappeared so that it cannot now be identified.

844. The ninth star excited much interest. It blazed out between the 10th and 12th of May as a star of the second magnitude, remained at its maximum for three or four days, and then, in five or six weeks, faded away to its original faintness, for it now is, and was before the outburst, a nine and one-half magnitude star on Argelander's catalogue, with nothing noticeable to distinguish it from its neighbors. While at the maximum its spectrum was carefully studied by Huggins, and exhibited brightly and strongly the bright lines of hydrogen, just as if it were a sun like our own, but entirely covered with outbursting "prominences" of incandescent hydrogen.

The tenth star had a very similar history. It also rose to its full brightness (second magnitude) on November 24, within *four* hours according to Schmidt, remained at a maximum for only a day or two, and faded away to invisibility within a month. But it still exists as a very minute telescopic star of the fifteenth magnitude. It was also spectroscopically studied by several observers (by Vogel especially) with the strange result that the spectrum, which at the maximum was nearly continuous, though marked by the bright lines of hydrogen and by bands of other unknown substances, lost more and more of this continuous character, until at last it became a simple spectrum of three bright lines *like that of a nebula*.

845. The eleventh of these temporary stars was very peculiar in one respect; not in its brightness, for it never exceeded the six and one-half magnitude, but because it appeared right in the midst of the great nebula of Andromeda, only 12″ or 13″ distant from the nucleus. It came out suddenly like all the others, and faded away gradually in about six months, to absolute extinction so far as any existing telescope can show. It showed under photometric measurements many fluctuations in brightness, not losing its light smoothly

and regularly, but in a rather paroxysmal manner. Its spectrum, even when brightest, was simply continuous without anything more than the merest trace of bright lines in it. The eighth star on the list resembled it in the fact that it appeared in the midst of a star cluster.

845*. In Jan. 1892 a twelfth "Nova" appeared in the constellation of Auriga, which at its brightest, about Feb. 5th, was a star of the $4\frac{1}{2}$ magnitude. Its spectrum (Fig. 220*) was very peculiar, showing a great number of *bright* lines, especially those of hydrogen and with them also the *dark lines* of the same substances. The bright and dark lines were displaced relatively to each other as if they were respectively due to at least two different masses of gas, in swift relative motion[1] at the rate of something like 500 miles a second, — the "bright-line" mass receding from us, and the other approaching.

In the autumn the star, which had sunk to the 11th magnitude, again brightened up to about the 9th, and then the spectrum was found to be almost, if not absolutely, identical with that of a planetary nebula, but finally, according to Campbell, in 1903 became *simply continuous.*

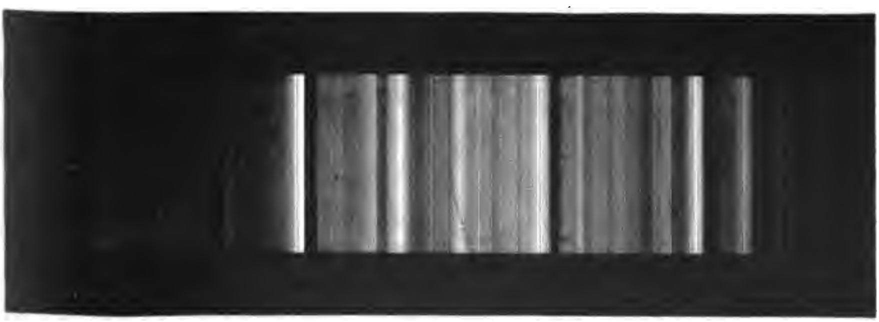

FIG. 220* is from a photograph by Frost at Potsdam.

The "Novæ" of 1883, '95 and '98 are peculiar in the fact that they were detected by *photography,* having been recognized by Mrs. Fleming of the Harvard College Observatory upon both the chart plates and spectrum photographs taken at the Harvard Station in

[1] But see Art. 802*. It is not certain that the displacements of the lines may not have been due, partly at least, to pressure-effects accompanying explosions or eruptions or possibly to "*anomalous refraction.*" See Addendum B.

South America. Their spectra very precisely resembled that of Nova Aurigæ, showing the same duplex combination of bright lines with dark. It now seems very probable that the "new stars" would be not very uncommon if the small stars could all be closely watched; and it is clear that there are important physical resemblances between them; but the phenomena are not yet clearly explained.

846. IV. LONG-PERIOD VARIABLES. o CETI TYPE. These objects resemble the temporary stars in rapidly brightening up for a short time and then fading back to the original condition; but they do it *periodically*. The periods are generally of considerable length, from six months to two years; but they are very apt to be considerably irregular, not unfrequently to the extent of several weeks.

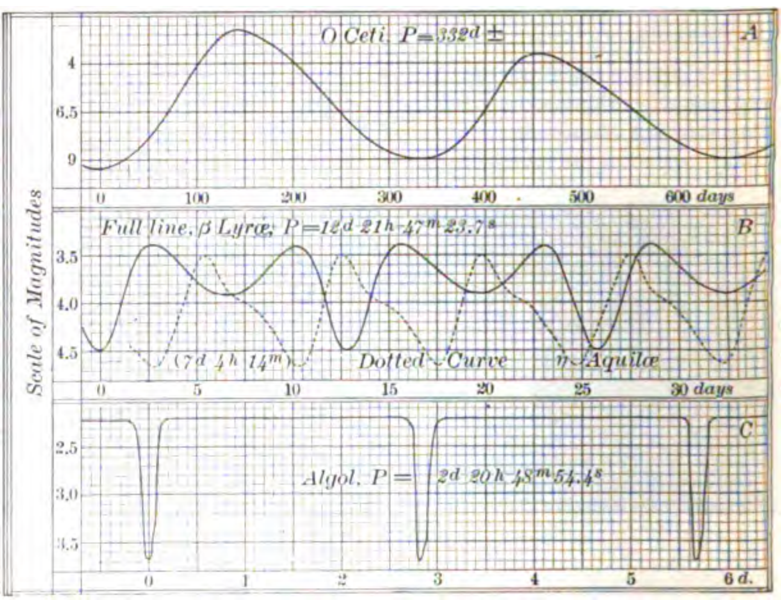

FIG. 221. — Light-Curves of Variable Stars.

The star o Ceti (often called *Mira*, that is, "the Wonderful") may be taken as the type of this class. Its variability was discovered by Fabricius in 1596. During most of the time it remains invisible to the naked eye, falling below the 9th magnitude at its minimum, but once in about eleven months it runs up to the fourth, third, or even

the second magnitude, and then back again. Its brightness increases more rapidly than it fails, and it remains at its maximum for a week or ten days, and sometimes much longer. The maxima vary very much in brightness, and are frequently several weeks ahead of, or behind, the computed time. At the maximum its spectrum is very beautiful, containing a large number of intensely bright lines, most of which are due to *hydrogen*, though some of them are still unidentified. Its light-curve is *A*, in Fig. 221. A large majority of the known variables belong to this class. Nearly all of them are notably reddish in color, and most of them show a colonnaded spectrum marked with bright lines.

847. V. SHORT-PERIOD VARIABLES.[1] TYPE OF η AQUILÆ AND β LYRÆ. In these the periods range from seven and four-tenths hours (that of S Antliæ, the shortest known at present) to three weeks, and the light of the star fluctuates all the time. In many cases there are two or more minima in a complete period, accompanied by complicated spectroscopic phenomena of bright and dark lines, which shift their places, and double and undouble themselves in a very interesting and significant way. (See Art. 872*.) The light-curves of η Aquilæ and β Lyræ are given at *B*, Fig. 221.

848. VI. VARIABLES OF THE ALGOL TYPE.[1] The sixth and last class consists of stars which seem to suffer a partial eclipse at short intervals. Of this type of stars, Algol, or β Persei, may be taken as the most conspicuous representative. Its period is $2^d\,20^h\,48^m\,55^s.4$, which is subject to almost no variation, except certain slow changes that appear to be the result of some unknown disturbance. During most of the time the star remains of the second magnitude. At the time of obscuration it loses about five-sixths of its light, falling to the fourth magnitude in about four and one-half hours, remaining at the minimum for about twenty minutes, and then in three and one-half hours recovering its original condition. During the minimum the spectrum undergoes no considerable change in its *appearance*, but during the whole period, as will be presently explained, the lines in it regularly shift their positions. (See Art. 851.)

The periods are all short, ranging from ten hours to four and one-half days. About forty stars are known at present to belong to this class.

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[1] See Addendum C.

849. Explanation of Variability.—Evidently no single explanation will hold for all the different classes. For the gradually progressive changes no explanation need be looked for; on the contrary, it is surprising that such changes are no greater than they are, for the stars are all growing older.

As for the irregular changes, no sure account can yet be given of them. Where the range of variation is small, as it is in most cases, one thinks of spots on the surface like those of our own sun (but much more extensive and numerous), and running through a period just as our sun spots do. Let a star with such spots upon it revolve on its own axis, as of course it must do, and in the combination we have at least a possible explanation of a great proportion of all the known cases, both the irregular variables and the regularly periodic.

Many of the spectroscopic phenomena of the temporary stars and of some of the long-period variables closely resemble, on a slightly magnified scale, those that are observed in the solar chromosphere and prominences. The same bright lines of hydrogen and helium appear, and the same behavior of gaseous masses, distorting and displacing the lines. The facts strongly suggest in these cases a theory of "explosions" or eruptions.

850. Collision Theory. — For the temporary stars, and those of the *o* Ceti type, Sir Norman Lockyer (in connection with a much more extended subject) suggests a "collision theory," illustrated by Fig. 221*. The fundamental idea that the phenomena of the temporary stars may be due to collisions is not new. Newton long ago brought it out, and to some extent discussed it; but considering the probable diameters of the stars as compared with the distances between them, it seems impossible that collisions could have been frequent enough to account for the number of temporary stars actually observed.

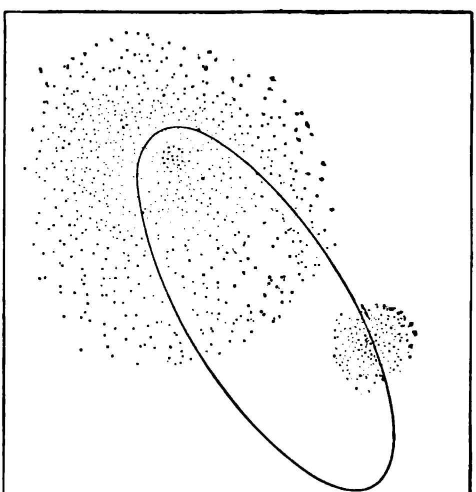

FIG. 221*.—The Collision Theory of Variable Stars.

Lockyer, however, imagines that the temporary stars, and also variable stars of the *o* Ceti class, are, in their present stage of development, not compact bodies, but only pretty dense swarms of meteorites of considerable extent, each accompanied by another smaller one revolving around it in an eccentric orbit, just as comets revolve around the sun, or as the components of double stars revolve around each other. He supposes that the perihelion distance is so small that the swarms interpenetrate and pass through each other at the perihelion, which could happen without disturbing the *general* motion of either of the two meteoric flocks; but while they are thus passing, the collisions are immensely increased in number and violence, with a corresponding increase in the evolution of light. There are many good points about this ingenious theory, but also serious objections to it — as, for instance, the great irregularity of the periods of stars of this class, an irregularity which seems hardly consistent with such an orbital revolution.

851. Stellar Eclipses. — As to the Algol type, the natural explanation is by means of an eclipse of some sort. The interposition of a more or less opaque object between the observer and the star, — a dark companion revolving around it, — would produce just the effect observed, as was suggested by Goodricke a century ago. That this is really the case has now been practically demonstrated by the spectroscopic work of Vogel in 1889, who found by the method indicated in Art. 802 * that from twelve to eighteen hours before the obscuration, Algol is receding from us at the rate of nearly twenty-seven miles a second, while after the minimum it approaches us at the same rate. This is just what it ought to do, if it had a large, dark companion, and the two were revolving around their common centre of gravity in an orbit nearly edgewise to the earth. When the dark star is rushing forward to interpose itself between us and Algol, Algol itself must be moving backwards, and *vice versa* when the dark star is receding after the eclipse. Vogel's conclusions are, that the distance of the dark star from Algol is about 3250000 miles; that their diameters are respectively about 840000 and 1060000 miles; that their united mass is about two-thirds that of the sun; and their density about one-fifth that of the sun, — not much greater than that of cork. See Fig. 222.

Furthermore, from the variations in the observed period of the star, alluded to in Art. 848, combined with certain minute irregu-

larities in its 'proper motion,' Chandler has shown it to be likely that this swiftly moving pair is itself revolving around another distant and invisible star in an orbit about as large as that of Uranus, with a period of about 130 years. Tisserand, however, suggests a different explanation, depending on a slow revolution of the apsides of the orbit.

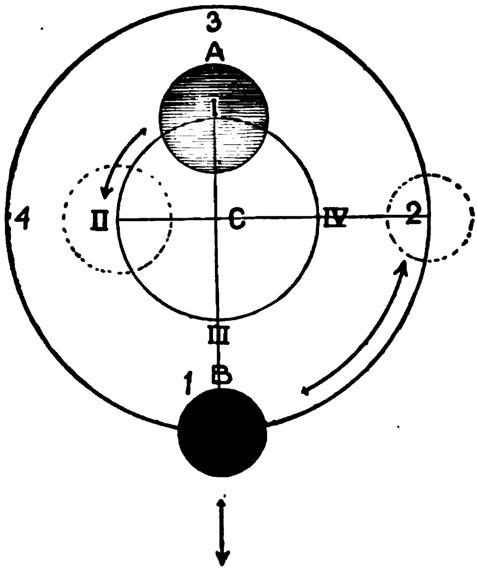

FIG. 222. System of Algol.

In the case also of δ Cephei and β Lyræ (of Class IV.) the observations of Belopolsky, Lockyer, and others have made it nearly certain that two or more bodies in orbital revolution are concerned in their phenomena, the variations in the light being in part at least due to a more or less complete eclipse, but also in part to other (tidal?) interactions which are not yet clear. It is not unlikely that all the *punctual* variables (those that keep accurate time in their light-changes) may turn out to be *spectroscopic binaries* (Art. 879).

In certain cases (*Y* Cygni and *Z* Herculis), the odd and even minima occur at unequal intervals, indicating a very eccentric orbit.

852. Number and Designation of Variables.[1] — The "Third Catalogue of Variable Stars," by Dr. S. C. Chandler, published in 1896, contains 393 stars of which the variation is regarded as certain.

[1] See Addendum C.

About 300 are unquestionably periodic, of which about 250 belong to the *o* Ceti type, 29 to the η Aquilæ class, and 14 to the Algol group. There are half a dozen or so with periods apparently between twenty-five and sixty days, leaving it doubtful how they should be classed. About thirty are distinctly *irregular* in their variation, and there are about fifty in respect to which the periodicity is not yet either ascertained or disproved. The catalogue is followed by a second list of 154 stars which are "suspected" of variation. The number of known variables had nearly doubled since Mr. Chandler published his first catalogue in 1888, and it is still growing rapidly. The new variables are mostly "telescopic."

Table VI. in the Appendix presents the principal data for the naked-eye variables which are visible in the United States.

When a star is discovered to be variable which previously had no special appellation of its own, it is customary to designate it by one of the last letters in the alphabet, beginning with R. Thus R Sagittarii is the first discovered variable in Sagittarius; S Sagittarii is the second; T Sagittarii, the third, and so on. But we have P and Q Cygni, both "temporaries."

853. Range of Variation. — In many cases the whole range is only a fraction of a magnitude (especially among the more newly discovered variables), but in a great number it extends from four to eight magnitudes, the maximum brightness exceeding the minimum by from fifty to a thousand times; and in a few cases the range is greater yet. Not unfrequently considerable changes of color accompany the changes of brightness; the star as a rule being whiter at its maximum, and frequently showing bright lines in its spectrum.

854. Method of Observation. — There is no better way than that of comparing the star by the eye, or with the help of an opera-glass or small telescope, with surrounding stars of about the same brightness at the time when its light is near the maximum or minimum; noting to which of them it is just equal at the moment, and also those which are a shade brighter or fainter.

It is possible for an amateur to do really valuable work in this way, by putting himself in relation with some observatory which is interested in the subject. The observations themselves require so much time that it is difficult for the working force in a regular observatory to attend to the matter properly, and outside assistance is heartily welcomed in gathering the needed

facts. The observations themselves are not specially difficult, require no very great labor or mathematical skill in their reduction, and, as has been said, can be made without instruments; but they require patience, assiduity, and a keen eye.

Photography also has lately come to the front as a most effective method. A very large proportion of the variables discovered within the last few years have been found by the comparison of the photographic star-charts made at Cambridge and at the Harvard South American Stations. In several cases the photographed spectrum of a star has, by its peculiar "colonnaded" character and bright lines, attracted the attention and marked it as "suspicious"; and in nearly every case the suspicion has been verified.

854*. Variable-Star Clusters.[1]—One of the most interesting and even startling results of stellar photography is the discovery of *variable-star clusters*, announced by Professor Pickering in 1895. Attention had been called to certain clusters by the visual discovery of one or two variables in them, and Mr. Bailey, who for several years has been in charge of the Southern photographic operations, soon obtained a large number of negatives of several of them, and immediately found that, while many clusters show no variables. others contain a great number. In the cluster known as "Messier 3 (in Canes Venatici)," 132 have been detected; in ω Centauri, 122; in Messier 5 (in Libra), 85; and in a cluster known as "N. G. C. 7078," 51. Forty-seven more have been announced in other clusters; in all 437 up to January, 1898. The periods are not yet fully determined, but the changes are very rapid, so that two photographs of Messier 5 taken only *two hours* apart show a dozen cases in which the variation of brightness amounts to a full magnitude.

STAR SPECTRA.

(If this book were to be written *de novo*, the sections upon stellar spectra would be placed almost at the beginning of Chapter XX., since in dealing with the star-motions, and the peculiarities of variable stars, it is now continually necessary to refer to spectroscopic phenomena, most of which have been discovered, or become practically important, since the first preparation of the work in 1888.)

855. In 1824 Fraunhofer, in connection with his study of the lines of the solar spectrum, investigated also the spectra of certain

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[1] See Addendum C, following page 580.

stars, using an apparatus essentially similar to that which is now employed at Cambridge. He placed a prism in front of the object-glass of a small telescope and looked at the stars through this, using a cylindrical lens in the eye-piece to widen the spectrum, which otherwise would be a mere line.

He found that Sirius, Castor, and many other stars show very few dark lines in their spectrum, but strong ones; while, on the other hand, the spectra of Pollux (β Geminorum) and Capella resemble closely the spectrum of the sun. In all the spectra he recognized the *D* line, although it was not then known that it had anything to do with sodium.

856. Observations of Huggins and Secchi.—Almost as soon as the spectroscope had taken its place as a recognized instrument of science it was applied to the study of the stars by Rutherfurd and Huggins, and Secchi followed hard in their footsteps. Huggins (now Sir William) studied the spectra of comparatively few stars, but with all the dispersive power he could obtain, and in detail; while Secchi, using a much less powerful instrument, examined several thousand star spectra, in a more general way, for purposes of classification.

Huggins identified with considerable certainty in the spectra of α Orionis (Betelgeuze) and α Tauri (Aldebaran) a number of elements that are familiar on the earth, and are most of them prominent in the solar spectrum. In the former he reported sodium, magnesium, calcium, iron, bismuth, and hydrogen; and in α Tauri, in addition, tellurium, antimony, and mercury; but these latter metals have not yet been verified.

857. Classification of Stellar Spectra.—Secchi, in his spectroscopic survey, found that the 4000 stars which he observed could all be reduced to four classes, and although his classification can now be regarded as provisional only, and by no means complete or satisfactory, yet it is more generally used than any other, either in its simple form, or with subdivisions and modifications such as Vogel, Pickering, and others have introduced.

According to Secchi:—

The first class comprises the white or blue stars. To it belong Sirius and Vega, and, in fact, considerably more than half of all the stars examined. The spectrum is characterized by the great strength of the hydrogen lines, which are wide, hazy bands, much like the *H*

and *K* lines in the solar spectrum. Other lines are extremely faint or entirely absent; the *K*[1] line especially, which in the solar spectrum is especially prominent, in the spectra of most of these stars is hardly visible.

The second class is also numerous, and *is composed of stars with a spectrum substantially like that of our sun.* The *H* and *K* lines are both strong. Capella and Pollux (β Geminorum) are prominent examples of this class. There are certain stars which form a connecting link between these two first classes, stars like Procyon and α Aquilæ, which, while they show the hydrogen lines very strongly, also exhibit a great number of other lines between them. The first and second classes together embrace fully seven-eighths of all the stars he observed.

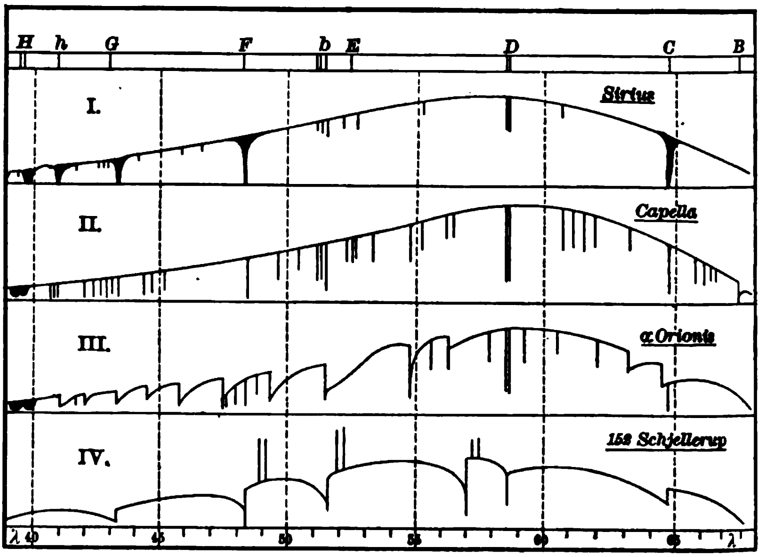

FIG. 223. — Secchi's Types of Stellar Spectra.

The third class includes most of *the red and variable stars,* some 500 in number, and the spectrum is characterized by dark *bands* instead of lines (though lines are generally present also). These

[1] In stars of this class the "*H* line" in their spectra is not the *H* line of *calcium*, but the *Epsilon* line of *hydrogen* (*Hε*), which lies close to the calcium line, as shown in Fig. 116*, Art. 326. In stars of the second or "solar" class, on the contrary, both *H* and *K* are practically due to calcium alone, the hydrogen line being very inconspicuous.

bands, not yet identified as to origin, shade from the blue towards the red; that is, they are sharply defined and darkest at the more refrangible edge. Occasionally in spectra of this type some of the hydrogen lines are bright. α Herculis, α Orionis, and *Mira* (o Ceti) are fine examples of this third class.

The fourth class is composed of a very small number of stars, less than sixty so far as known, mostly small red stars. This spectrum is also a banded one; but compared with the third class the bands (probably due to carbon) are *reversed*, that is, are shaded towards the blue. These generally show also a number of bright lines. None of the conspicuous stars belong to this class — none above the fifth magnitude. The sixth magnitude star, 152 Schjellerup, may be taken as its finest example (α, $12^h\ 40^m$; δ, $+45°\ 59'$, in the constellation of Canes Venatici). Fig. 223 exhibits the light-curves of these four types of spectrum.[1]

There are many stars which seem in their spectroscopic characteristics to lie between classes 1 and 2; and there are others which cannot be said to belong to either of the four. Pickering has proposed a *fifth* class, mainly to include a small group, known as the "Wolf-Rayet stars" with a very peculiar spectrum of bands and bright lines. Nearly seventy are known at present, all faint, and all in, or near, the milky way and Magellanic clouds. They seem to hold an important place in the still obscure theory of stellar development.

858. Vogel has revised Secchi's classification of spectra as follows, making only three main classes, but with subdivisions:

I. (*a*) Same as Secchi's I. The white stars.
(*b*) Nearly continuous; all lines wanting or very faint. β Orionis is the type.
(*c*) Showing the lines of hydrogen bright, and also the helium line D_3 (Art. 323).

II. (*a*) Same as Secchi's II.

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[1] It is difficult to represent spectra accurately by any process of engraving that can be readily reproduced in a book like the present. The curve, on the other hand, is easily managed, and, though it does not please the eye like the spectrum itself, it is capable of conveying all the information that could be obtained from the most finished engraving. *Dark lines* are represented by lines running *downward* from the upper boundary line of the curve, and *bright lines* by lines running *upward*, while the bands and their shading are represented by variations in the contour of the curve.

(*b*) Like II. (*a*), but showing bright lines which are *not* the lines of hydrogen or helium. (The Wolf-Rayet stars.)

III. (*a*) Same as Secchi's III.

(*b*) Same as Secchi's IV.

Vogel's classification is based in part on the very doubtful assumption that stars of Class I. are hottest and also youngest, while the other classes belong to stars which are either beginning to fail or are already far gone in decrepitude. But it is very far from certain that a red star is not just as likely to be younger than a white one, as to be older. It probably is now at a *lower temperature*, and possesses a more extensive envelope of gases; but it may be increasing in temperature as well as decreasing. At any rate we have no certain knowledge about its age.

Since the identification of *helium* and its numerous lines in the solar spectrum, its lines have also been recognized in the spectra of many stars (especially numerous in Orion), and seem likely to throw much light on their classification. Vogel has taken them into account in a modified form of his system, which we have not space to present.

Sir Norman Lockyer has also proposed a very elaborate classification, based on his "Meteoritic Theory" of stellar-development (see Art. 926).

859. Photography of Stellar Spectra. — As early as 1863 Huggins attempted to photograph the spectrum of Vega, and succeeded in getting an impression of the spectrum, but without any of the lines. In 1872 Dr. Henry Draper of New York, working with the reflector which he had himself constructed, succeeded in getting an impression of the spectrum of the same star, showing for the first time four of its hydrogen lines. The introduction of the more sensitive dry plates in 1876 induced Mr. Huggins to resume the subject (as did Dr. Draper soon after), and they soon succeeded in getting pictures showing many lines. The spectra were about half an inch long by $\frac{1}{12}$ or $\frac{1}{16}$ of an inch wide. After the lamented death of Dr. Draper in 1882, Professor Pickering took up the work at Cambridge (U. S.); and with such success that Mrs. Draper, who had intended to establish and to endow her husband's observatory as an establishment for astro-physical research, and a most fitting monument to his memory, concluded to transfer the instruments to Cambridge, and there establish the "Draper Memorial," which has already accomplished so much for spectroscopic astronomy. Other observers have followed on, both in Europe and in this country; and it is hardly too much to say that four-fifths of all stellar spectroscopic work is now done by photography. Spectra too faint to be even seen by the eye can be photographed, and so studied in their minutest peculiarities; and with the continual improvement of our plates the range of possible observation increases daily.

860. The Slitless Spectroscope. — Professor Pickering has attained his remarkable success by reverting to the "slitless spectro-

scope," arranged in the manner first used by Fraunhofer, and later revived by Secchi. The instrument consists of a telescope with the *objective corrected not for the visual, but for the photographic rays*, equatorially mounted and carrying *in front of the object-glass* one or more "objective-prisms" with a refracting angle of from 10° to 30°, and large enough to cover the whole lens.

The refracting edge of the prism is placed east and west, so that the linear spectrum of a star formed on a plate at the focus of the object-glass runs north and south. If, now, the clock-work of the instrument is adjusted to follow the star exactly, the image (*i.e.*, the spectrum) will be a mere line, broken here and there where the dark lines of the spectrum should appear. By merely retarding or accelerating the clock a trifle, the linear spectrum will drift a little sidewise upon the plate, and so will form a spectrum having a width depending on the amount of this drift during the time of exposure. If the air is calm the lines of the spectrum thus formed are as clean and sharp as if a slit were used; otherwise not.

861. The instrument hitherto most used in this work at Cambridge is Dr. Draper's eleven-inch photographic refractor, with four huge glass prisms in a box in front of the object-glass, arranged as indicated in Fig. 224. With this apparatus, photographic spectra of the brighter stars are now obtained having, before enlargement, a length of fully three inches from F in the blue of the spectrum to the extremity of the ultra violet. It is a pity, of course, that the lower portions of the spectrum below F cannot be reached in the same way; but no plates sufficiently sensitive to green, yellow, and red rays have yet been found. The exposure necessary to obtain the impression of even the most powerful photographic rays is from half an hour to an hour. Fig. 225 is enlarged about one-third from one of these photographs of the spectrum of Vega, which extends far into the ultra violet.

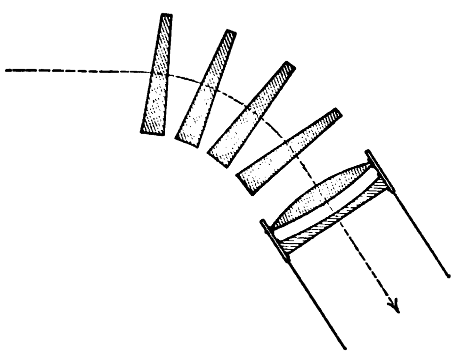

FIG. 224.
Arrangement of the "objective-prisms.

At the extreme left, the figure fails to show properly the perfect regularity with which the hydrogen lines crowd closer and closer together, in exact

accordance with a remarkable formula discovered by Balmer in 1885, *viz.*, $\lambda = \lambda_0 \frac{m^2}{m^2 - 4}$, in which m takes successively the values 3, 4, 5, etc., λ_0 being 3646.1. (See note at end of the chapter, p. 538.)

These spectra bear tenfold enlargement perfectly, making them more than two feet long by two inches in width, and then in the spectrum of such a star as Capella they show hundreds of lines. It is simply amazing that the feeble, twinkling light of a star can be made to produce such an autographic record of the substance and condition of the inconceivably distant luminary.

Several other photographic telescopes of much larger size, both in Europe and in America, are now fitted with objective-prisms. The Bruce telescope of 24 inches aperture, which has already been mentioned (Art. 798*), is at

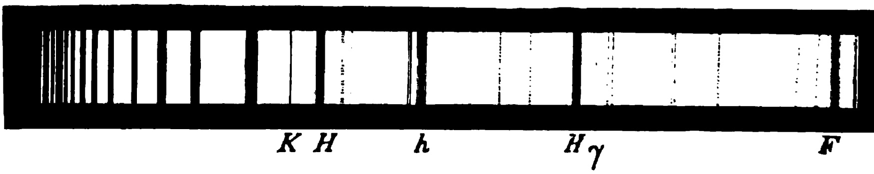

FIG. 225. — Photographic Spectrum of Vega. Cambridge, 1887.

present the largest. Theoretically, a *diffraction grating* would answer the same purpose as a prism, and some experiments have been made as to the practicability of constructing such a grating for use with the great Yerkes telescope, by following Fraunhofer's original plan of winding fine wires upon a pair of finely threaded screws. Promising experiments are also being made with concave gratings.

In still another form of instrument, the spectroscope is attached to the eye end of the telescope just as usual, the slit of the collimator being simply omitted.

862. Peculiar Advantages of the Slitless Spectroscope. — The slitless spectroscope has three great advantages. First, that it utilizes all the light that comes from the star to the object-glass, much of which in the usual form of the instrument is lost in the jaws of the slit. Secondly, that by taking advantage of the length of a large telescope, it produces a very high dispersion with even a single prism. Thirdly, and most important, it gives on the same plate and with a single exposure the spectra of all the many stars whose images fall upon it. With the smaller eight-inch instrument made at Cambridge, and one prism, as many as 100 or 150 spectra are sometimes taken together; as, for instance, in a spectrum photo-

graph of the Pleiades. For purposes of "reconnoissance," therefore, where the object is to obtain, compare, and classify the spectra of thousands of stars, this form of instrument is unrivalled.

863. Disadvantages of the Slitless Spectroscope. — *Per contra*, the giving up of the slit precludes all the usual methods of identifying the lines by actually confronting them with comparison spectra; the comparison prism (Art. 315) cannot be used. This makes it extremely difficult to utilize these magnificent pictures for purposes of scientific measurement.

Several methods have been proposed by which, theoretically, the difficulty might be overcome; none of them, however, offer any practical approach to the accuracy obtainable with slit-spectroscopes, which up to the present time have been exclusively used by Vogel, Keeler, and Campbell in all absolute measurements of motion in the line of sight, all determinations of wave-length, and all trustworthy identifications of stellar elements. In certain cases of *relative* motion, however (Art. 879), objective-prism spectroscopes have already done good work, and Professor Pickering has very lately (1896) devised a most ingenious method of extending their range to a determination of the motions of all the stars of a group relative to some one selected as a reference point. Two photographs of the spectra are made, one with the telescope on one side of the pier, and the red ends of all the spectra towards the *north*, say. Then the telescope is reversed to the other side of the pier, and a second negative is made in which the spectra will have their red ends to the *south*. Moreover, this second negative is made with the sensitive film *turned away from the object-glass*. On putting the two plates together, with films next each other, and making the two spectra of the "guide-star" coincide, all the other spectra will also coincide; — exactly, in cases where the "radial-motion" of the stars is the same as that of the guide-star: otherwise there will be a slight want of coincidence in the lines that ought to agree, and *half* this difference of position will be the "displacement" of the spectrum lines due to the difference between the radial velocity of the guide-star and that of the star in question. The method has been tried upon the Pleiades, but, rather disappointingly, did not bring out any evidence of a general rotation of the cluster around its centre.

864. Twinkling or Scintillation of the Stars. — This is a purely atmospheric effect, usually violent near the horizon and almost null at the zenith. It differs greatly on different nights according to the steadiness of the air.

If the spectrum of a star near the eastern horizon be examined with a spectroscope so held as to make the spectrum vertical, it will

appear to be continually traversed by dark bands running through the spectrum from the blue end towards the red. At the western horizon the bands move in the opposite direction, from red to blue; on the meridian they merely oscillate back and forth.

Cause of Scintillation. — Authorities differ as to the exact explanation of scintillation, but probably it is mainly due to *two* causes (optically speaking), both depending on the fact that the air is full of streaks of unequal density that are carried by the wind.

(1) In the first place, light transmitted through such a medium is concentrated in some places and turned away from others *by simple refraction:* so that, if the light of a star were strong enough, a white surface illuminated by it would look like the sandy bottom of a shallow, rippling pool of water illuminated by sunlight, with light and dark mottlings which move with the ripples on the surface. So, as we look towards the star, and the mottlings due to the irregularities of the air move by us, we see the star alternately bright and faint; in other words, it *twinkles;* and if we look at it in a telescope we shall see that it not only twinkles, but *dances*, *i.e.*, it is slightly displaced back and forth by the refraction.

(2) The other cause of twinkling is "*interference*." Pencils of light coming from the star (which optically is a mere point), and feebly refracted by the air in the way above explained, reach the observer by slightly different routes, and are just in a condition to interfere. The result of the interference is the temporary destruction of rays of certain wave-lengths, and the reinforcement of others. At a given moment the *green* rays, for instance, will be destroyed, while the red and blue will be abnormally intense; hence the quivering dark bands in the spectrum. If the star is very near the horizon, the effects are often sufficient to produce marked changes of color.

865. Why Planets Twinkle Less than Stars. — This is mainly because they have *discs of sensible diameter*, so that there is a general unchanging *average* of brightness for the sum total of all the luminous points of which the disc is composed. When, for instance, point *A* of the disc becomes dark for a moment, point *B*, very near it, is just as likely to become bright; the interference conditions being different for the two points. The different points of the disc *do not keep step*, so to speak, in their twinkling.

865*. Note on "Series" in Spectra. — In 1896 Professor Pickering found on the Draper Memorial photographs a remarkable series of lines in the spectra of certain stars, of which ζ Puppis is the most conspicuous. These lines fall regularly intermediate between the lines of hydrogen, and

he soon after discovered that a single formula, which includes that of Balmer, gives the positions both of these lines and those of hydrogen formerly known: it is, $\lambda = 3646.1 \frac{n^2}{n^2 - 16}$. If in this we give to n the *even* values, 6, 8, 10, etc., we get the wave-lengths of the lines of hydrogen as formerly known: the *odd* values 7, 9, 11, etc., give the new lines. It seems, therefore, extremely likely that these new lines also belong to hydrogen, in some condition, however, which differs from that which obtains in our laboratories, and in stars like Vega.

The investigations of Kayser and Runge (1888–1896) have shown that the lines in the spectra of many, if not most, of the elements are spaced in a somewhat similar manner, expressible by a simple formula; usually, however, two or more distinct series are found. In the spectrum of helium, according to Runge, there are two "sets," each set consisting of a principal series, and two subordinate series, — six in all. In most cases, the regular "series" exclude some of the lines that appear in the spectrum of a given element, and not unfrequently these independent lines are among the most conspicuous and important of all. Thus, the H and K lines do not belong to either of the two regular series in the spectrum of calcium. The explanation of these series is not yet known, but it probably depends somehow upon the manner in which the atoms are arranged in the molecule.

Exercises on Chapter XXI.

1. What is the brightness of a star of the 10.5 magnitude (on the absolute scale) compared with that of a star of the standard first magnitude?

From Art. 820 we have $\log b_{10\cdot5} = \log b_1 - \frac{4}{10} \times 9.5$. If we take the brightness of the first magnitude star as the unit of brightness $\log b_1 = 0$, and we have $\log b_{10\cdot5} = 0 - 0.4 \times 9.5 = -3.8000$. To bring this entirely negative logarithm into the usual tabular form, in which the characteristic only is negative while the mantissa is positive, we numerically increase the characteristic by unity, making it -4, and at the same time take for the new mantissa $1 - 0.8000$, or $.2000$; we have, therefore, $\log b_{10\cdot5} = \bar{4}.2000$; whence, from the logarithmic table, we find $b_{10\cdot5} = 0.000158$.

$$\text{Also } \log \frac{b_1}{b_{10\cdot5}} = 0 - (-3.8000) = +3.8000\text{ ; whence, } b_1 = 6309.6 \times b_{10\cdot5}.$$

(In all computations respecting stellar magnitudes four-place tables are sufficient.)

2. What is the brightness of an eleventh magnitude star in terms of the first?

Ans. 0.0001, or $\frac{1}{10000}$.

3. What is the brightness of a 4.8 magnitude star in terms of the first?

Ans. 0.0302, or $\frac{1}{33.1}$.

4. What is the magnitude of a star whose brightness is one one-hundred-thousandth that of a first magnitude star? (Art. 820, Eq. 2.)

Ans. 13.5 magnitude.

5. What is the magnitude of a star a millionth as bright as a first magnitude?

Ans. 16th magnitude.

6. What is the magnitude, on the absolute scale, of a luminary 80000000000 times as bright as a first magnitude star? (Log 80000 000000 = 10.9031.)

Ans. -26.26 magnitude.

(This is about the estimated brightness of the sun.)

7. What is the apparent magnitude of a double star whose components are of the first and second magnitudes respectively?

Ans. 0.64 magnitude.

8. What, if the components are of the second and fourth magnitudes?

Ans. 1.85 magnitude.

9. If the distance of a fourth magnitude star were diminished one-half, of what magnitude would it appear?

Ans. 2.50 magnitude.

10. If the distance of a star were increased by forty per cent, how much would its magnitude be changed?

Ans. 0.73 of a magnitude, *numerical increase.*

11. If the distance of a star were diminished by forty per cent, how would its magnitude be affected?

Ans. 1.11 of a magnitude, *numerical decrease.*

12. If a star of the 9th magnitude has a parallax of 0.25″, how does the light emitted by it compare with that of the sun?

Ans. $\frac{1}{18}$.

13. With the data given in Table IV. compute the light-emission of other stars compared with that of the sun.

CHAPTER XXII.

DOUBLE AND MULTIPLE STARS. — ORBITS AND MASSES OF DOUBLE STARS. — CLUSTERS. — NEBULÆ. — THE MILKY WAY. — DISTRIBUTION OF STARS. — CONSTITUTION OF THE STELLAR UNIVERSE. — COSMOGONY AND THE NEBULAR HYPOTHESIS.

866. Double and Multiple Stars. — The telescope shows numerous instances in which two stars lie very near each other, in many cases so near that they can be seen separate only under a high

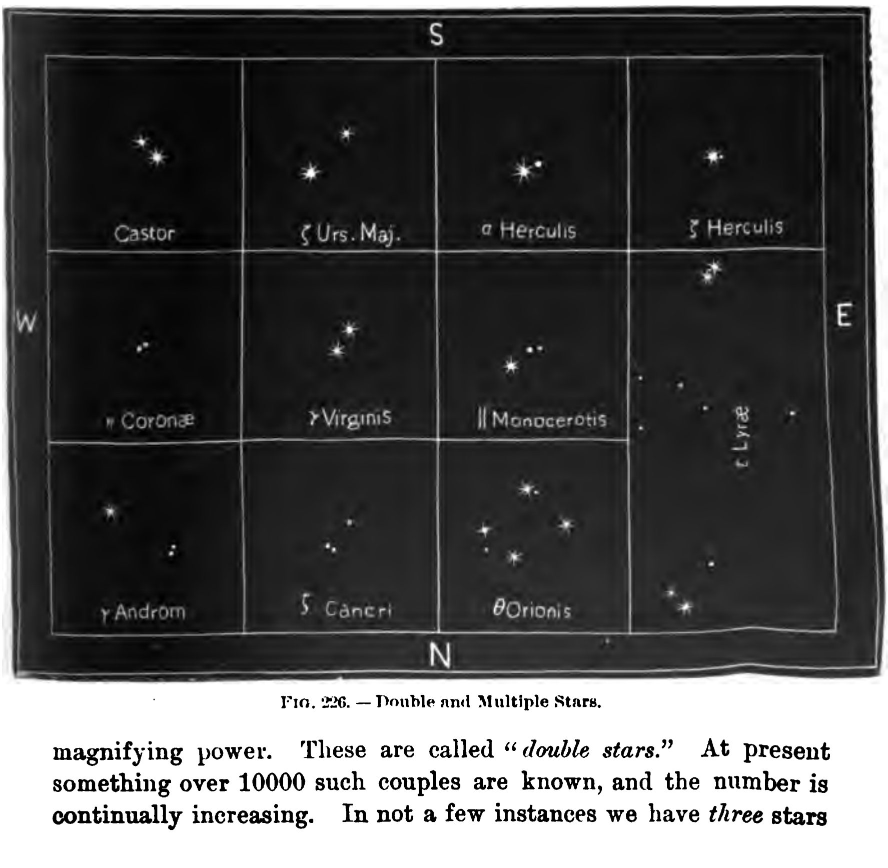

Fig. 226. — Double and Multiple Stars.

magnifying power. These are called *"double stars."* At present something over 10000 such couples are known, and the number is continually increasing. In not a few instances we have *three* stars

together, two of which are usually very close and the third farther away; and there are several cases of *quadruple stars*, where there are two pairs of stars lying close together (as in ε Lyræ), or a pair of stars with two single stars close by; and there are some cases where more than four form a "*multiple star.*" Fig. 226 represents a number of such double and multiple stars.

867. Distance, Magnitudes, and Colors.—The apparent distances usually range from 30″ to ½″, few telescopes being able to separate double stars closer than ½″.

In a very large proportion of cases (perhaps about one-third of all) the two stars are nearly equal; in many others they are extremely unequal, a minute star near a large one being usually known as its "companion."

Not infrequently the components of a double star present a fine contrast of color; *never, however, in cases where they are nearly equal in magnitude.* It is a remarkable fact, as yet wholly unexplained, that when we have such a contrast of color the tint of the smaller star always *lies higher in the spectrum* than that of the larger one. The larger one is *reddish or yellowish*, and the smaller one *green or blue*, without a single exception among the many hundreds of such tinted couples now known. γ Andromedæ and β Cygni are fine examples for a small telescope.

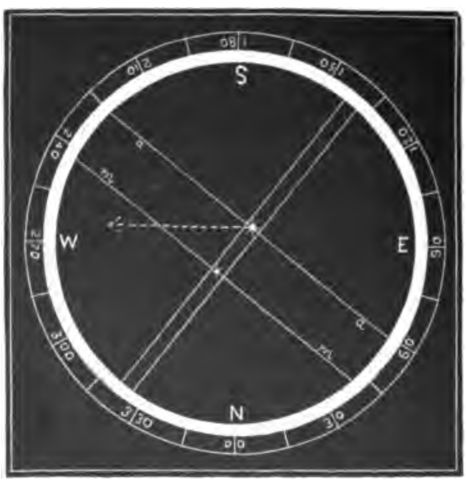

FIG. 227.
Measurement of Distance and Position-Angle of a Double Star.

868. Measurement of Double Stars.—Such measures are generally made with a filar position-micrometer, essentially such as shown in Figs. 28 and 29 (Art. 73). The quantities to be determined are the distance and position-angle of the couple. By "distance" we mean simply the apparent distance in seconds of arc between the centres of the two star dics. The *position-angle* of a double star is the *angle*

made with the hour-circle by the line drawn from the larger star to the smaller, reckoning around from the north through the east, as shown in Fig. 227.

Photography may also be used, and promises to become very useful in cases where the distance is not less than 4″ or 5″.

869. Stars Optically and Physically Double.— Stars may be double in two different ways. They may be merely *optically* double, — that is, simply in line with each other, but one far beyond the other; or they may be really very near together, in which case they are said to be "*physically connected*," because they are then under the influence of their mutual attraction, and move accordingly.

870. Criterion for distinguishing between Physically and Optically Double Stars.— This cannot be done off-hand. It requires a series of measurements long enough continued to determine whether the relative movement of the stars is in a curve or a straight line. If the stars are really close together their attraction will force them to describe curves around each other. If they are really at a great distance and only accidentally in line, then their proper motions, being sensibly uniform and rectilinear, will produce a *relative* motion of the same kind. Taking either star as fixed, the other star will appear to pass it in a straight line, and with a steady, uniform drift.

871. Relative Number of Stars Optically Double and Physically Connected.— Double-star observations practically began with Sir William Herschel only a little more than a hundred years ago. When he took up the subject less than 100 such pairs had been recognized, such as had been accidentally encountered in making observations of various kinds. The great majority of double stars have been discovered so recently that sufficient time has not yet elapsed to make the criterion above given effective with more than a small proportion of them. But it is already perfectly clear that the optically double stars are, as the theory of probability shows they ought to be, very few in number, while several hundred pairs have shown themselves to be physically connected, *i.e.*, to be what are known as "*binary*" stars, or couples which revolve around their common centre of gravity.

872. Binary Stars. — Sir W. Herschel began his observations of double stars in the hope of ascertaining stellar parallax. He had supposed in the case of couples where one was large and the other small that the smaller one was usually a long way beyond the other (as *sometimes* is really the fact). In this case there should be perceptible variations in the distance and position of the two stars during the course of the year; precisely such variations as those by which, fifty years later, Bessel succeeded in getting the parallax of 61 Cygni (Art. 811). But Herschel, instead of finding the yearly oscillation of distance and position which he expected, found quite a different and, at the time, a surprising thing, — a regular, progressive change, which showed that one of the stars was slowly describing a regular orbit around the other. To use his own expression, he "went out like Saul to seek his father's asses, and found a kingdom," — the dominion of gravitation[1] extended to the stars, unlimited by the bounds of the solar system. γ Virginis, ξ Ursæ Majoris, and ζ Herculis were among the most prominent of the systems which he pointed out.

At present the number of pairs *known* to be binary is at least 250, and as many more begin to show signs of movement. (Up to the present time of course only the quicker moving ones are obvious). About sixty have progressed so far, — having made at least one entire revolution or a great part of one, — that their orbits have been computed more or less satisfactorily.

873. Orbits of Binary Stars. — The real orbit described by *each* of such a pair of stars is always found to be an ellipse, and assuming the applicability of the law of gravitation, the common centre of gravity must be at the focus. The two ellipses are precisely similar, the one described by the smaller star being larger than the other in inverse proportion to the star's mass.

So far as the *relative* motion of the two bodies goes, we may regard either of them (usually the larger is preferred) as being at rest, and the other as moving around it in a *relative orbit* of precisely the same *shape* as either of the two actual orbits which are described around the centre of gravity. But the relative orbit is larger, having for

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[1] It is not yet fully *demonstrated* that the motions of binary stars are due to gravitation, though it is extremely *probable*, and the burden of proof seems to be shifted upon those who are disposed to doubt it. See, however, the foot-note to Art. 901.

its semi-major axis the *sum* of the two semi-axes of the real orbits (Art. 427).

Usually the *relative* orbit is all that we can ascertain at present, as this alone can be deduced from the micrometer measures when they consist only of position-angles and distances measured between the two stars.

In a few cases where such measures have been made from small stars in the same field of view with the couple, but not belonging to the system, or when the couple has long been observed with the meridian circle, it becomes possible to work out separately the apparent orbit of each star of the pair with reference to their common centre of gravity. It will also be possible ultimately to compute the actual orbits of many double stars, *independent of any hypothesis*, by help of their radial motions spectroscopically determined in different parts of the orbit. But this can be done only in cases where the components are not too close to permit their spectra to be separately observed. The spectroscopic and micrometer measures combined (if strictly correct) would absolutely determine the form, size, position (and *distance also*) of the binary system, the law of the central force, and the masses of the component stars.

874. Calculation of the Orbit of a Binary Star. — If the observer is so placed as to view the orbit perpendicularly, he will see it in its true form and having the larger star in its focus, while the smaller moves around it, describing "equal areas in equal times." But if the observer is anywhere else, the orbit will be apparently more or less distorted. It will still be an ellipse (since every projection of a conic is also a conic), but the large star will no longer occupy its focus, nor will the major and minor axes be apparently at right angles to each other; nor will they even coincide with the longest and shortest diameters of the ellipse. In this distorted ellipse the smaller star will, however, still describe equal areas in equal times around the larger one.

Theoretically, five absolutely accurate observations of the position and distance are sufficient to determine the elements of the relative orbit, *if we assume that the orbital motion is described under the law of gravitation.* Practically a greater number are needed in most cases, because the motions are so slow and the stars so near each other that observation-errors of $0''.1$ (which in most calculations are of small account) here become important. The work requires not only labor, but judgment and skill, and unless the pair has completed or nearly completed an entire revolution the result is apt to be seri-

ously uncertain. So far, as has been said, about sixty such orbits are fairly well determined. Catalogues, more or less complete, will be found in Flammarion's book on "Double Stars," also in Gledhill's "Hand-book of Double Stars," and Houzeau's "Vade Mecum." Table V. in the Appendix gives the elements of twenty-two of the best-known orbits, mostly from the recent calculations of Dr. See.

875. Sirius and Procyon. — The cases of these two stars are remarkable. In both instances the large stars have been found from meridian-circle observations to be slowly moving in little ellipses, although when this discovery was first made neither of them was known to be double. In 1862 the minute companion of Sirius was discovered by Clark with the object-glass of the Chicago telescope, then just finished, and at that time the largest object-glass in the world. And this little companion was found to be precisely the object needed to account for the peculiar motion of Sirius itself.

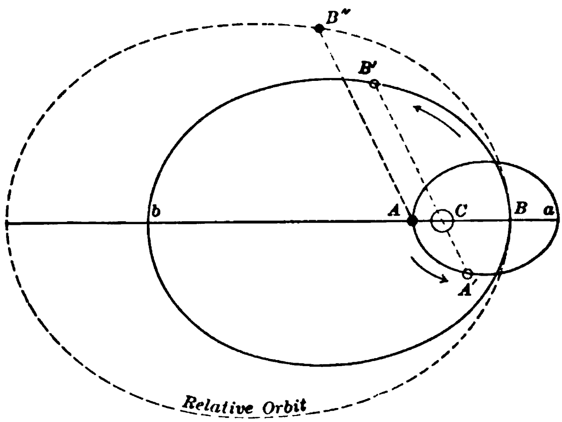

FIG. 228. — Orbits of Sirius and his Companion.

Fig. 228 represents the absolute orbits of the two stars and also the relative orbit of the smaller star around the larger when the latter is regarded as being at rest. The small circle at *C* is the earth's orbit drawn on the same scale.

In the case of Procyon, the companion was discovered only in Nov. 1896, by Schaeberle at the Lick Observatory, and its orbit is not yet worked out.

876. Periods. — The periods of binary stars, so far as at present determined by micrometric measures, vary from $5\frac{1}{2}$ years (the period of Ll. 9091 according to See: Burnham denies it) to several hundred years; though none of the very long periods are yet accurately ascertained.

It is possible that one or two others may be found with periods even shorter than eleven years, and it is practically certain that as time goes on, pairs of longer period than 1500 years will present themselves.

Fig. 228* shows the apparent orbits of several of the most interesting binaries.

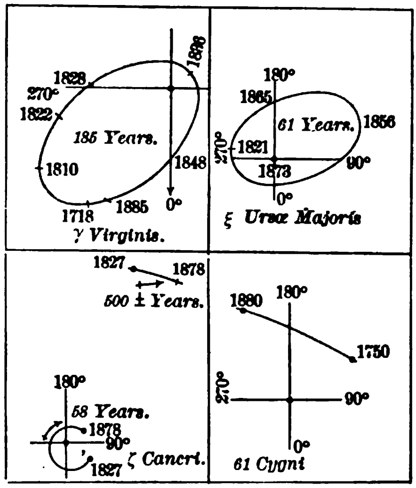

FIG. 228*. — Orbits of Binary Stars.

877. Size of the Orbits. — The angular semi-major axes of the orbits thus far computed range from about 0″.3 for δ Equulei, to 18″ for α Centauri. The real dimensions are, of course, only to be obtained when we know the star's parallax and distance.[1] Fortunately several of the stars whose parallaxes have been determined are also binary stars. Assuming the data as to parallax and orbits given in the tables in the Appendix we find the following results: —

NAME.	Assumed Parallax.	Angular Semi-axis.	Real Semi-axis.	Period.	Mass. ⊙ = 1.
η Cassiopeiæ	0″.35	8″.21	23.5	195y.8	0.33
Sirius	0.39	8.03	20.6	52.2	3.13
α Centauri	0.75	17.70	23.6	81.1	2.00
70 Ophiuchi	0.25	4.54	18.2	88.4	0.77

In the case of Sirius, the companion appears to have a mass about four-tenths that of the principal star, while the two components of α Centauri are very nearly equal in mass, though not in brightness.

It is obvious from the table that the double-star orbits are comparable with the larger orbits of the solar system, all four of them being approximately equal to that of Uranus. Of course the many binary stars whose distance is so great as to make their parallax insensible while their apparent orbits are as large as those given in the list must have real orbits of still vaster dimensions.

[1] The *real* semi-axis of the orbit in astronomical units is simply the *angular* semi-axis divided by the parallax.

The most characteristic peculiarity of the double-star orbits is their *large eccentricity*, which for the sixty, now fairly determined, averages very nearly 0.50. Dr. See of Chicago explains the fact on the hypothesis that the binary stars are the result of a process of "tidal-evolution" (Arts. 484 and 916). It is supposed that what was originally a nebulous "lump" assumed a dumb-bell form in whirling, and then the two parts separated, still revolving around their common centre of gravity, and rotating on their axes. In this case the tidal reaction between the two would be extremely powerful and effective, and See, following very nearly the methods of Darwin's original investigation, has shown that the result would be to cause them to move apart, and assume orbits more and more eccentric up to limits depending upon the initial conditions. The theory is now very generally accépted as the substantial truth.

878. Masses of Binary Stars.— When we know both the size of the orbit of a binary and its period, the mass, according to the law of gravitation, follows at once from the equation of Art. 536,

$$M + m = 4\pi^2 \frac{a^3}{t^2}.$$

If t and a are given respectively in *years* and astronomical units of *distance*, then, by omitting the factor $4\pi^2$, $M + m$ comes out in *terms of the sun's mass.* The final column of the little table above gives the *masses* of the four pairs of stars as compared with the mass of the sun. But the student must bear in mind that the parallaxes of stars are so uncertain that these results are to be accepted with a very large margin of error.

879. Spectroscopic Binaries.[1]— One of the most interesting of recent astronomical results is the detection by the spectroscope of several pairs of double stars so close that no telescope can separate them. In 1889 the brighter component of the well-known double star Mizar (Zeta Ursæ Majoris, Fig. 226) was found by Pickering to show the dark lines *double* in the photographs of its spectrum, at regular intervals of about fifty-two days. The obvious explanation is that this star is composed of two, which revolve around their common centre of gravity in an orbit which is turned nearly edgewise to us. When, twice in their revolution, the line that joins the two stars is perpendicular to the line along which we view them, one of the two will be moving towards us, while the other is moving in an opposite direction; and as a consequence, the lines in their spectra will be shifted opposite ways, according to Doppler's principle. Now since the two stars are so close that their spectra

[1] See footnote on page 580.

overlie each other, the result will be simply to make the lines in the compound spectrum *apparently double.* From the distance apart of the lines, the relative velocity of the stars can be found, and from this the size of the orbit and the mass of the stars. Thus it appears that in the case of Mizar the relative velocity of the two components is about 100 miles per second, and the period about 104 days. If we assume that the two stars are of about the same size (which is likely since their spectra are equally bright) and that the orbit is circular, we find that the distance between them is about 140 000000 miles, and their united mass about forty times that of the sun.

Vogel, from later observations (1900-1901) shows that the period is only 20.6 days ($\frac{1}{5}$ of 104), which makes the distance 28 330 000 miles, and the mass about 9 times that of the sun.

The lines in the spectrum of Beta Aurigæ exhibit the same peculiarity, but the doubling occurs once in *two* days; the periodic time being *four* days, the velocity about 150 miles a second, and the diameter of the orbit about 8 000000 miles, the united mass of the two stars comes out about two and a half times that of the sun. These observations of Professor Pickering's were made by photographing the spectrum with the *slitless spectroscope* (Art. 861), and are possible only where the stars which compose the binary are both of them reasonably bright.

In 1896 two other similar cases were announced by Professor Pickering, discovered on the South American spectrum photographs. The first of the two stars is μ' Scorpii, a star of the 3d magnitude in the reptile's tail. The period is $34^h\ 42^m.5$, the relative velocity 300 miles a second, and the radius of the relative orbit 6 055000 miles. The other star is 3105 Lacaille, of the $4\frac{1}{2}$ magnitude in the constellation Puppis; the period is $74^h\ 46^m$, with a velocity of 385 miles a second, the radius of the orbit being 16 500000 miles. In both cases the two components of the binary differ considerably in brightness. Still more recently, Jan. 1898, the star β Lupi has been added to the list.

879*. In 1889 Vogel also, as already stated in Art. 851, detected with his spectrograph the orbital motion of the Algol system, and a few months later he discovered similar behavior in the case of Spica (α Virginis). At first the spectroscopic measures upon this star appeared very discordant, but he soon found that everything was reconciled by assuming the existence of a companion, too faint to be *seen*, but massive enough to make Spica itself swing around

their common centre of gravity once in $4^d\ 0^h\ 19^m$, with a velocity of about 57 miles a second, and in an orbit which, if circular, has a diameter of about 6 million miles. This orbit cannot be quite edgewise to the earth, since if it were there would be "eclipses" and variations in the brightness of Spica, such as do not occur.

Very recently (1895–96) Belopolsky has found by the same method that the brighter component of the double star Castor has, like Spica, an invisible companion, which causes it to swing backwards and forwards once in a little less than three days, with an orbital velocity of about 15.5 miles a second. He has also ascertained that the short-period variable δ Cephei gives spectroscopic evidence of orbital motion distinctly elliptical, with a mean velocity of about 13 miles, and corresponding with the period of variation ($9^d\ 8^h.8$). Sir Norman Lockyer announces the same thing with respect to η Aquilæ, ζ Geminorum, T Vulpeculæ and S Sagittæ, though the details are not yet at hand. In all these cases the evidence lies in the *shift* of lines and not their *doubling;* the star which causes the observed motion is *dark*. And it is to be noted that in several of the cases the minima and maxima of the star do not occur where they ought to if it were a case of simple eclipse.

β Lyræ is also abundantly proved by the observations of Pickering, Lockyer, and several others to be a spectroscopic binary, with two, or perhaps more than two, bright components. The lines of its spectrum shift and double, and behave in a very complicated and interesting way. The main variations in its light are due to partial eclipses; but other causes are certainly involved. The dissimilarity of the component spectra which make up the observed spectrum of this star leads Pickering to suggest that we may be able to infer that certain other stars with anomalous spectra, combining the characteristics of two or more recognized classes, are really double, even if the spectroscope does not show motion.

In this connection the two variable stars Y Cygni and Z (not ζ) Herculis should be mentioned. From the peculiarities of their light-curves Dunér has shown that they are close binaries with distinctly elliptical orbits, having periods of about $1^d\ 12^h$, and 4^d respectively. They obviously belong to the same class as the "spectroscopic binaries," though as yet their spectra have not been satisfactorily investigated, owing to their faintness.

If See's "tidal-evolution" theory is correct, these close binaries are to be regarded as infant systems, which in time will grow and widen out. But the connection of bodies nearly equal in mass, though differing greatly in their brightness, is still a puzzling mystery.

880. Have the Stars Planets attending them?—It is a very natural supposition that the minute companions which attend some of the larger stars may be really planetary in their nature, shining

more or less by reflected light. As to this we can only say, that while it is quite possible that other stars besides our sun may have their retinues of planets, it is quite certain that such planets could not be seen by us with any existing telescope. If our sun were viewed from α Centauri, Jupiter would be a star of less than the twenty-first magnitude, at a distance of only 5″ from the sun, which itself would be a smallish first-magnitude star.

881. The statement can be verified as follows: Jupiter at opposition is certainly not equivalent in brightness to twenty stars like Vega (most photometric measurements make it from eight to fourteen). Assuming, however, that it is equal to twenty Vegas, its light received by the earth would be about $\frac{1}{2000\,000000}$ of the sun's. At opposition our distance from Jupiter is about four astronomical units, so that seen from the same distance as the sun, its light would be sixteen times that quantity, or (nearly) $\frac{1}{125\,000000}$ of the sun's.

Now a ratio of 125 000000 between the light of two stars corresponds to a difference of 20 + magnitudes

$$\left(\log 125\,000000 = 8.0969\,;\ \text{but}\ \frac{8.0969}{0.4000} = 20.24\ \text{magnitudes.}\quad (\text{Art. } 820.)\right)$$

Accordingly, if the observer were removed to such a distance that the sun would appear like a first-magnitude star (as would be the case from α Centauri), Jupiter would be a star of the twenty-first magnitude. According to Art. 822, it would require a 25-inch telescope to show a star of the sixteenth magnitude; it would therefore require an instrument with an aperture of 250 inches, or nearly 21 feet, to show a star five magnitudes fainter, even if there were no large star near to add to the difficulty.

882. Triple and Multiple Stars. — There are a considerable number of objects of this kind, and some of them constitute physical systems. In the case of ζ Cancri the two larger stars revolve around their common centre in a nearly circular orbit less than 2″ in diameter, and with a period of about sixty years; while the third star, smaller and more distant, moves around the closed pair in an orbit not yet well determined, but with a period that must be several hundred years; and in its motion there is evidence of a peculiar perturbation, which Seeliger has satisfactorily explained as the result of motion around an invisible star, in an orbit of about the same size as that of the principal pair. Dr. See considers that he has discovered a similar perturbation in the system of 70 Ophiuchi due to an invisible body; but his conclusion is not everywhere accepted as yet. In ϵ Lyræ we have two pairs, each making a very slow revolution, of periods not yet determined, but probably ranging from 300 to 500 years. And since the pairs have also a common proper motion it is practically certain that they also are physically connected, and revolve around their common centre of gravity in a

period to be reckoned by millenniums — the motion during the last hundred years being barely perceptible. In other cases, as, for instance, in the multiple star θ Orionis, we have a number of stars not organized in pairs, but at more or less equal distances from each other: we are confronted by the problem of n bodies in its most general and unmanageable form.

STAR-CLUSTERS.

883. Clusters. — There are in the sky numerous groups of stars containing from one hundred to many thousand members. Some of them are made up of stars visible separately to the naked eye, as the Pleiades; some of them require a small telescope to resolve them, as, for instance, the Præsepe in Cancer, and the group of stars in the sword-handle of Perseus; while others yet, even in telescopes of some size, look simply like wisps or balls of shining cloud, and break up into stars only in the most powerful instruments.

In a large instrument some of the telescopic clusters are magnificent objects, composed of thousands of stellar sparks compressed into a ball which is dazzlingly bright at the centre and thinning out towards the edge. In some of them vividly colored stars add to the beauty of the group and some are full of *variable* stars (Art. 854*). In the northern hemisphere the finest cluster is that known as Messier 13 Herculis (α, $16^h\ 37^m$ and δ, $36°\ 40'$) not very far from the "apex of the sun's way."

884. The Pleiades. — Of the naked-eye clusters the Pleiades is the most interesting and important. To an ordinary eye six stars are easily visible in it, the six largest ones indicated in the figure (Fig. 229). Eyes a little better see easily five more — those next in size in the figure (the two stars of Asterope being seen as one). A very small telescope (a mere opera-glass) increases the number to nearly a hundred; and with large instruments more than 400 are catalogued in the group. A few of the stars, apparently in the cluster, are really only accidentally on the same line of vision, and are distinguished by proper motions different from those of the rest of the group; but the great majority have proper motions nearly the same in amount and direction. Their spectra are all of the first class, but Alcyone and Pleione show some of the hydrogen lines *bright*.

The distances and positions of the principal stars with respect to the central star Alcyone have been carefully measured three or four times during the last fifty years. The relative motions during the period have

not proved large enough to admit of satisfactory determination, but it is clear that such motions exist. A curious and interesting fact is the presence of *nebulous matter* in considerable quantity. A portion of this nebulosity hanging around Merope (the northeast star of the dipper-bowl in the figure) was discovered many years ago; but it was reserved to photography to

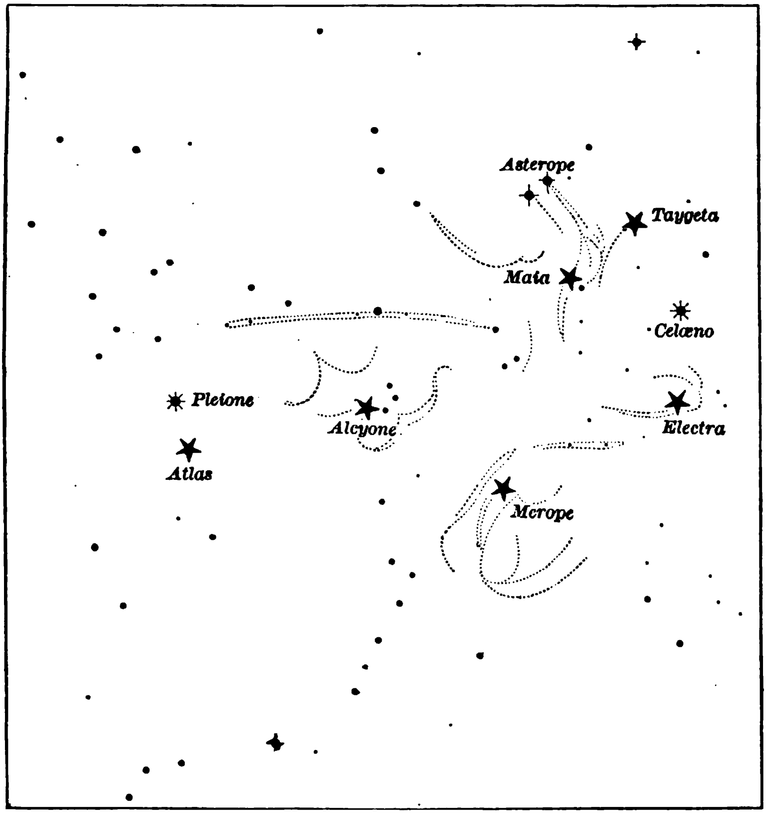

FIG. 229. — The Pleiades.

detect very recently other clouds of nebulosity attached to other stars, especially to Maia, and to show that the whole space is covered with streaks and streamers of it, emitting light of such a character as to impress the photographic plate much more strongly than the eye. The figure shows roughly the outlines of some of the principal nebulous filaments, but one must see the photographs of Roberts and others in order to get any adequate idea of their extent and beauty.

885. Distance of Star-Clusters and Size of the Component Stars. — The question at once arises whether clusters, such as the one

mentioned in Hercules, are composed of stars each comparable with our sun, and separated by distances corresponding to the distance between the sun and its neighboring stars, or whether the bodies which compose the swarm are really very small, — mere sparks of stellar matter: whether the distance of the mass from us is about the same as that of the stars among which it seems to be set, or whether it is far beyond them. Forty years ago the accepted view was that the stars composing the clusters are no smaller than ordinary stars, and that the distance of the star-clusters is immensely greater than that of the isolated stars. There are many eloquent passages in the writings of that period based upon the belief that these star-clusters are *stellar universes*, — "galaxies," like the group of stars to which the writers supposed the sun to belong, but so inconceivably remote that in appearance they shrank to these mere balls of shining dust.

It is now, however, quite certain that the other view is correct, — that star-clusters are among *our* stars and form part of *our* universe. Large and small stars are so associated in the same group in many cases as to leave us no choice of belief in the matter. It is true that as yet no parallax has been detected in any star-cluster; but that is not strange, since a cluster is not a convenient object for observations of the kind necessary to the detection of parallax.

THE NEBULÆ.

886. The Nebulæ. — There are also in the sky a multitude of faintly shining bodies, — shreds and balls of cloudy stuff that are known as "*nebulæ*" (the word meaning strictly a "little cloud"). About 9000 of these objects are already catalogued.

Two or three of them are visible to the naked eye. The nebula in the girdle of Andromeda is the brightest of them, in which, it will be remembered, the temporary star of 1885 appeared.

The next brightest is the wonderful nebula of Orion, which, in the beauty and variety of its details, in the interesting relations of the included stars, the delicate tint of the filmy light, and in its spectroscopic interest, far exceeds the other, — indeed, all others.

It is so difficult to represent these delicate objects by any process available in a text-book that we limit ourselves to giving two cuts, one copied from Mr. Roberts' exquisite photograph of the great nebula of Andromeda, and the other from a drawing of the curious Ring-nebula in Lyra.

The first successful photograph of a nebula (the Orion Nebula) was made by Dr. H. Draper in 1880, and he was soon followed by Common. At present Mr. Roberts, working with a 20-inch reflector, takes the lead. Visual observation and draughtsmanship cannot here at all compete with the photographic process, which continually brings out features before unrecognized

FIG. 230. — Mr. Roberts' Photograph of the Nebula of Andromeda.

in the most powerful telescopes, — sometimes new and startling revelations, like the concentric rings in the Andromeda Nebula, an apparent parallel of Saturn and his rings. The photograph has one drawback, however: *stars* in the Nebula are not properly shown; nor is the relative brightness of different portions fairly given on any single negative. The exposure necessary to bring out faint details is far too great for the brighter parts.

With a small telescope a nebula cannot be distinguished from a close star-cluster, and it is quite likely that the clusters and nebulæ shade into each other by insensible gradations. Forty years ago it was supposed that there was no distinction between them except that of mere remoteness, — that all nebulæ could be resolved into stars by sufficient increase of telescopic power. When Lord Rosse's

great telescope was first erected, it was for a time reported (and the statement is still often met with) that it had "resolved" the Orion nebula. This was a mistake, however. No telescope ever *has* resolved that nebula into stars or ever will, for we now *know* that it is not composed of stars.

887. Forms and Magnitudes of Nebulæ.—The larger and brighter nebulæ are, many of them, very irregular in form, stretching out sprays and streamers in all directions, and containing dark openings or "lanes." The so-called "fish-mouth" in the nebula of Orion, and the dark streaks in the nebula of Andromeda, are striking examples. Some of these bodies are of enormous volume. The nebula of Orion, with its outlying streamers, extends over several square degrees, and the nebula of Andromeda covers more than one. Now, as seen from even the nearest star, the apparent distance of Neptune from the sun is only 30″, and the diameter of its orbit 1′. It is perfectly certain that neither of these nebulæ is as near as α Centauri, and therefore the cross-section of the Orion nebula, as seen from the earth, must be *at least* many thousand times the area of Neptune's orbit, and the "hole" in the Annular Nebula of Fig. 231 below must be somewhat larger than that orbit, which at the distance of α Centauri would subtend an angle of only 45″ on the scale given in the figure.

And the nebulæ as *seen* with the telescope are only the brightest portions of vaster clouds.

Recent photographs of Orion, made with instruments of short focus and with a long exposure, show that the whole constellation is enveloped in a nebulosity, which for the most part attaches itself to the principal stars, like the nebulosity in the Pleiades (Art. 884). The well-known nebula of Orion is only the brightest portion of this inconceivably enormous mass.

We do not know what is the real shape of either of the nebulæ, whether it is a thin, flat sheet, or a voluminous bulk; but some things about these two nebulæ and several others favor strongly the idea that their thickness does not correspond to their apparent area.

888. The Smaller Nebulæ.—The smaller nebulæ are for the most part elliptical in outline, some nearly circular, others more elongated, and some narrow, slender streaks of light. Generally they are

brighter at the centre, and in many cases the centre is occupied by a star. Indeed, there is a considerable number of so-called "*nebulous stars*," that is, stars with a hazy envelope around them.

There are some nebulæ which present nearly a uniform disc of light, and are known as "*planetary*" nebulæ, and there are some which are dark in the centre and are known as "*annular*" or ring nebulæ. The finest of these annular nebulæ is the one in the constellation of Lyra, about half-way between the stars β and γ; it is shown in Fig. 231.

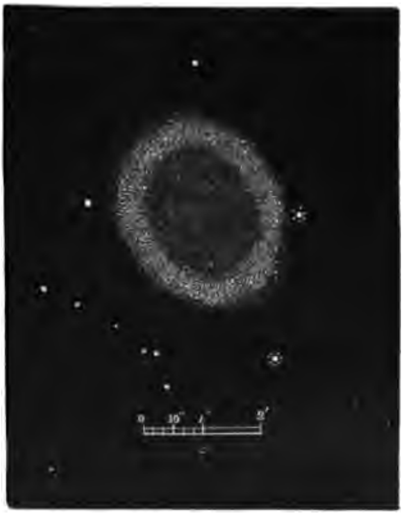

FIG. 231. — The Annular in Lyra.

There are also a number of double nebulæ, and some of these exhibit a remarkable spiral structure when examined by telescopes of the largest aperture or by means of photography. The so-called "*whirlpool*" nebula in the constellation "Canes Venatici" is the most striking specimen. This spiral structure, however, is to be seen *only* in large telescopes; in fact, very little of the real beauty of most of these objects is *visually* accessible to instruments of less than 12 inches aperture.

889. Variable Nebulæ.—There are several nebulæ which *vary in their brightness* from time to time; one especially, near ϵ Tauri, at times has been visible with a small telescope, while at other times it is entirely invisible even with large ones. So far no regular periodicity has been ascertained in such cases.

890. Their Spectra.— One of the earliest and most remarkable achievements of the spectroscope was its demonstration of the fact that the light of many of the nebulæ proceeds mainly from *luminous gas*. They give a visual spectrum of six or seven bright lines[1]

[1] The wave-lengths of these lines are the following, in the order of brightness: (1) 5007.05 ? (2) 4959.02 ? (3) 4861.50, Hydrogen (F); (4) 4340.66, Hydrogen (γ); (5) 4101.85, Hydrogen (h); (6) 5875.98, Helium (D_3); (7) 4472, Helium.

(Fig. 232), three of which are fairly conspicuous. This most important and brilliant discovery was made by Huggins in 1864.

Three of the lines, *F*, H_γ, and *h*, are due to *Hydrogen*. Two, the faintest of all, first observed by Copeland, are lines of *Helium*, so conspicuous in the solar chromosphere; but the origin of the rest remains unknown. The brightest line of the whole number is in the green, λ,5007, and was for a while referred to nitrogen; but under closer examination the identification breaks down, though the student will find it still called "the nitrogen line" now and then. The unidentified element is provisionally called "nebulium." Mr. Lockyer identified it with a "fluting" in the low-temperature spectrum of magnesium, which he has found in the spectrum of meteorites; but it is now certain that this also was an error, and this line and its neighbor 4959 still remain a mystery.

FIG. 232. — Visual Spectrum of the Gaseous Nebulæ.

All the nebulæ which give a gaseous spectrum at all present this same spectrum entire or in part. If the nebula is faint only the brightest lines appear, while the H_γ line and the other fainter lines are seen only in the brightest nebulæ and under favorable circumstances.

891. Photography since 1886 has proved itself as effective in the study of nebular spectra as of stellar: we have at present a list of over 70 lines photographed in the spectra of half a dozen nebulæ, 55 in that of the Orion nebula alone, among them all the hydrogen lines clear to the last of the ultra-violet series. Some of the lines appear also in the spectra of the trapezium stars, showing that these stars are of the same material as the surrounding nebula, only more condensed.

During the summer of 1890, Keeler at the Lick Observatory observed a number of the planetary nebulæ with a spectroscope of high dispersive power, and was able to detect and to measure the motion of several of them in the line of sight. The velocity of their motion appears to be of the same order as that of the stars, the nebulæ observed giving results ranging from zero up to nearly forty miles a second, — some approaching and some receding.

892. But not all the nebulæ by any means give a gaseous spectrum: those which do so — about half the whole number — are of a more or less distinct greenish tint, which is at once recognizable in the telescope. The *white* nebulæ, the nebula of Andromeda at their head, give only a continuous and perfectly expressionless spectrum, unmarked by any lines or bands, either bright or dark. This must not be interpreted as showing that these nebulæ cannot be gaseous; for a gas *under pressure* gives just such a spectrum; but so also do masses of solid or liquid when heated to incandescence. The spectroscope simply declines to testify in this case. The *telescopic* evidence as to the nature of the white nebulæ is the same as for the green. They withstand all attempts at resolution, none more firmly than the Andromeda nebula itself, the brightest of them all.

893. Changes in the Nebulæ. — The question has been raised whether some of the nebulæ have not sensibly changed, even within the few years since it has become possible to observe them in detail. It is quite certain that in important respects the early *drawings* differ seriously from those of recent observers; but the appearance of a nebula depends so much upon the telescope and the circumstances under which it is used, the features are so delicate and indefinite, and the difficulty of representing them on paper is such, that very little reliance can be placed on discrepancies between drawings, unless supported by the evidence of *measures* of some kind.

Thus far, the best authenticated instance of such a change, according to Professor Holden, is in the so-called "trifid" nebula, in Sagittarius. In this object there is a peculiar three-legged area of darkness which divides the nebula into three lobes. A bright triple star, which in the early part of the century was described and figured by Herschel and other observers as in the *middle* of one of these dark lanes, is now certainly in the edge of the nebula itself. The star does not seem to have moved with reference to the neighboring stars, and it seems therefore necessary to suppose that the nebula itself has drifted and changed its form.

As to the nebula of Orion, Professor Holden's conclusion is, that while the *outlines* of the different features have probably undergone but little change, their *relative brightness* and *prominence* have been continually fluctuating. This, however, can hardly be considered certain; to settle the question will probably require another fifty years or so, and the comparison, not of drawings, but of *photographs*.

894. Nature of the Nebulæ. — As to the constitution of these clouds we can only speculate. In the green nebulæ we can say with

confidence that hydrogen, helium, and some other gas are certainly present, and that the gases emit most of the light that reaches us from such objects. But how much solid or liquid matter in the form of grains and drops may be included within the gaseous cloud we have no means of knowing.

The idea of Mr. Lockyer (a part of his wide induction as to what we may call the "meteoritic constitution of the universe") is that they are clouds of "sparse meteorites, the collisions of which bring about a rise of temperature sufficient to render luminous one of their chief constituents," which, when he wrote the sentence, he imagined to be magnesium.

How far this theory will stand the test of time and future investigations remains to be seen. At first view it seems very doubtful whether the *collisions* in such a body could be frequent or violent enough to account for its luminosity, and one is tempted to look to other causes for the source of light. *"Luminescence" does not require a high temperature.*

895. Number and Distribution of Nebulæ. — Sir William Herschel was the first extensive investigator of these interesting objects, and left his unfinished work as a legacy to his son, Sir John Herschel, who completed the survey of the heavens by a residence of several years at the Cape of Good Hope. His "General Catalogue" was long the standard of reference for objects of this kind, but has been superseded by Dreyer's "New General Catalogue," which, with an addition published a few years later, contains between 9000 and 10 000 of them. Probably few important ones remain to be found.

As to their distribution, it is a curious and important fact that it is *in contrast* to the distribution of the stars. The stars, as we shall soon see, gather especially in and about the Milky Way, as do also the star-clusters; but the nebulæ specially crowd together in regions as far from the Milky Way as it is possible to get. As has been pointed out by more than one, this shows, however, not a want of relation between the stars and the nebulæ, but some "relation of contrariety." Precisely what this is, and why the nebulæ avoid the regions thickly starred, is not yet clear. Possibly the stars *devour them*, that is, gather in and appropriate surrounding nebulosity so that it disappears from their neighborhood.

896. Distance of the Nebulæ. — On this point we have very little absolute knowledge. Attempts have been made to measure the parallax of one or two, but so far unsuccessfully. Still it is probable, indeed almost certain, that they are at the same *order of distance* as

the stars. The wisps of nebulosity which photography shows attached to the stars in the Pleiades (and a number of similar cases appear elsewhere), the nebulous stars of Herschel, and numerous nebulæ which have a star exactly in the centre,—these compel us to believe that in such cases the nebulosity is really *at the star*. Then in the southern hemisphere there are two remarkable luminous clouds which look like detached portions of the Milky Way (though they are not near it), and are known as the Nubeculæ or "Magellanic clouds." These are made up of stars and star-clusters, and of nebulæ also, all swarming together, and so associated that it is not possible to question their real proximity to each other.

897. Fifty years ago a very different view prevailed. As has been said already, astronomers at that time very generally believed that there was no distinction between nebulæ and star-clusters except in regard to distance, the nebulæ being only clusters too remote to show the separate stars. They considered a nebula, therefore, as a "universe of stars," like our own "galactic cluster" to which the sun belongs, but as far beyond the "star-clusters" as these were believed to be beyond the isolated stars. In some respects this old belief strikes one as grander than the truth even. It made our vision penetrate more deeply into space than we now dare think it can.

THE SIDEREAL SYSTEM.

898. The Galaxy, or Milky Way.—This is a luminous belt which surrounds the heavens nearly in a great circle. It varies much in width and brightness, and for about a third of its extent, from Cygnus to Scorpio, is divided into two nearly parallel streams. In several constellations, as in Cygnus, Sagittarius, and Argo Navis, it is crossed by dark straight-edged bars that look as if some light cloud lay athwart it, and in the constellation of Centaurus there is a dark pear-shaped orifice, — the "coal sack," as it is called.

The galaxy intersects the ecliptic at two opposite points near the solstices, making with it an angle of about 60°. The northern "galactic pole," as it is called, lies, according to Sir John Herschel, in declination +27°, and right ascension $12^h\ 47^m$; the southern "galactic pole" is of course at the opposite point in the southern hemisphere. As Herschel remarks, the "galactic plane" "is to sidereal what the ecliptic is to planetary astronomy, a plane of ultimate reference, the ground plan of the sidereal system."

The Milky Way is made up almost wholly of small stars from the eighth magnitude down. It contains also a large number of star-

clusters, but (as has been already mentioned) very few true nebulæ. In some places the stars are too thickly packed for counting, especially in the bright knots which abound here and there.

(An excellent detailed description of its appearance and course may be found in Herschel's "Outlines of Astronomy."

899. Distribution of Stars in the Sky: Star-Gauges. — It is obvious that the stars are not uniformly scattered over the heavens. They show a decided tendency to collect in groups here and there, and to form connected streams; but besides this, an enumeration of the stars in the great star-catalogues shows that the number increases with considerable regularity from the galactic poles, where they are most sparse, towards the galactic circle, where they are most crowded The "star-gauges" of the Herschels make this fact still more obvious.

These gauges consisted merely in the counting of the number of stars visible in the field of view (15′ in diameter) of the twenty-foot reflector. Sir William Herschel made 3400 of these gauges, directing the telescope to different parts of the sky; and his son followed up the work at the Cape of Good Hope. Struve's discussion of these gauges in their relation to the galactic circle gives the following result: —

Distance from Galaxy.	Number of Stars in Field.
90°	4.15
75°	4.68
60°	6.52
45°	10.36
30°	17.68
15°	30.30
0°	122.00

900. Structure of the Heavens. — Our space does not permit a discussion of the untenable conclusions reached by Herschel and others by combining the unquestionable data derived from observation, with the unfounded and untrue assumptions that the stars are substantially of a size and spaced at approximately equal distances. Many of those conclusions relating to the form and dimensions of the Milky Way, and of the stellar universe to which our sun belongs, have become almost classical; but they are none the less incorrect.

It is certain, however, that the faint stars *as a class* are smaller and darker and more remote than are the bright ones *as a class:* and accepting this, we can safely draw from the star-gauges a few general conclusions, as follows: —

We present them substantially as given by Newcomb in his "Popular Astronomy," p. 491.

1. "The great mass of the stars which compose this (stellar) system are spread out on all sides in or near a widely extended plane, passing through the Milky Way. In other words, the large majority of the stars which we can see with the telescope are contained in a space having the form of a round, flat disc, the diameter of which is eight or ten times its thickness.

2. "Within this space the stars are not scattered uniformly, but are for the most part collected into irregular clusters or masses, with comparatively vacant spaces between them." They are "gregarious," to use Miss Clerke's expression.

3. Our sun is near the centre of this disc-like space.

4. The *naked-eye stars* "are scattered in this space with a near approach to uniformity," the exceptions being a few star-clusters and star-groups like the Pleiades and Coma Berenices.

5. "The disc described above does not represent the form of the stellar system, but only the limits within which it is mostly contained." The circumstances are such as to "prevent our assigning any more definite form to the system than we could assign to a cloud of dust."

6. "On each side of the galactic region the stars are more evenly and thinly scattered, but probably do not extend out to a distance at all approaching the extent of the galactic region," or if they do they are very few in number; but it is impossible to set any definite boundaries.

7. On each side of the galactic and stellar region we have a nebular region, comparatively starless, but occupied by great numbers of nebulæ.

As to the Milky Way itself, it is not yet certain whether the stars which compose it are distributed pretty equally near the galactic circle, or whether they form something like a ring with a comparatively vacant space in the middle.

As to the distance of the remotest stars in the stellar system, it is impossible to say anything very definite, but it seems quite certain that it must be at least as great as 10000 to 20000 light-years. If one asks what is beyond, whether the star-filled space extends indefinitely or not, no certain answer can be given.

Nor is there now any reason to suppose that *our own stellar* system is separated from other stellar systems by any vast abyss of practically empty

space, relatively proportioned to that which separates our *planetary* system from the possible planetary systems of other suns.

901. Do the Stars Form a System? — That is, do they form an *organized unit*, in which, as in the solar system, each of the different members has its own function and permanently maintains its relation to the rest? Gravitation probably operates, as indicated[1] by the binary stars, and the stars are moving swiftly in various directions with enormous velocities, as shown by their proper motions, and by the spectroscope. The question is whether these motions are controlled by gravitation, and whether they carry the stars in *orbits* that can be known and predicted.

That the stars are organized into a system or systems *of some sort* can hardly be doubted. But that the system is one at all after the pattern of the solar system, in which the different members move in *closed orbits*, — orbits that are permanent except for the slow changes produced by perturbation, — this is almost certainly impossible, as was said a few pages back.

902. Is there a Revolution of the Whole Mass of Stars? — A favorite idea has been that the mass of stars which constitutes our system has a slow rotation like that of a body on its axis, the plane of this general revolution coinciding with the plane of the galaxy. Such a general motion is not in any way inconsistent with the independent motions of the individual stars, and there is perhaps a slight inherent probability in favor of such a movement; but thus far we have no evidence that it really exists — indeed, there hardly *could* be any such evidence at present, because exact Astronomy is not yet old enough to have gathered the necessary data.

903. Central Suns. — A number of speculative astronomers, Mädler perhaps most prominently, have held the belief that there is a "*central sun*," standing in some such relation to the stellar system as our sun does to the

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[1] They fall short of "*demonstrating*" it, because, although their apparent motions are perfectly consistent with the universality of gravitation, they are equally so with several other imaginable laws of force. (See Article by A. Hall, Astron. Journal, Vol. VIII.) But all other laws involve the improbable condition that the force must vary with the *direction* as well as the *distance*. Spectroscopic observations of the *velocity* of binaries are, however, theoretically competent to decide the question by enabling us to compute the *actual* orbit of a binary from its apparent orbit (as given by the micrometer) without any assumptions as to the law of the central force. (See Art. 873.) Dr. See has recently worked out the necessary formulæ very completely.

solar system. It is hardly necessary to say that the notion has not the slightest foundation, or even probability.

Lambert supposed *many* such suns as the centres of subordinate stellar systems, and because we cannot see them, he imagined them to be dark.

If we conceive of boundaries drawn around our stellar system, and count all the stars within the limits as members of it, leaving out of the account all that fall outside, then, of course, our system so limited has at any moment a perfectly definite *centre of gravity*. There is no reason why some particular star may not be very near that centre, and in that sense a "central sun" is possible; but its central position would not give it any preëminence or rule over its neighbors, or put it in any such relation to the rest of the stars as the sun bears to the planets.

904. Orbits of Sun and Stars. — It is practically certain that the motions of the stars are not *orbital* in any strict sense. Excepting stars which are in clusters, all other stars are simultaneously acted upon by many forces drawing in various and opposite directions; and these forces must in most cases be so nearly balanced that the resultant cannot be very large. The motions of the stars must consequently, as a rule, be *nearly* rectilinear.

Still the balancing of the forces will seldom be exact, and accordingly the path of a star will almost always be *slightly* curved; and since the amount and direction of the resultant force which acts on the star is continually changing, the curvature of its motion will alter correspondingly, and the result will be a path which does not lie in any one plane, but is bent about in all ways like a piece of crooked wire. It is hardly likely, however, that the curvature of a star's path would, in any ordinary case, be such as could be detected by the observations of a single century, or even of a thousand years.

As has been said before, in connection with the proper motions of the stars, the probability is that the separate stars move nearly independently, "like bees in a swarm." In the solar system the central power is supreme, and perturbations or deviations from the path which the central power prescribes are small and transient. In the stellar system, on the other hand, the central force, if it exists at all (as an attraction towards the centre of gravity of the whole mass of stars) is trifling. Perturbation prevails over regularity, and "*individualism*" is the method of the greater system of the stars, as solar despotism is that of the smaller system of the planets.

905. Cosmogony. — Unquestionably one of the most interesting, and also most baffling, topics of speculation is the problem of the way in which the present condition of the universe came about. By what processes have moons and earths and Jupiters and Saturns, come to their present state and into their relation to the sun? What has been their past history, and what has the future in store for them? How has the sun come to his present glory and dominion? and in the stellar universe, what is the meaning and mutual relation of the various orders of bodies we see, — of the nebulæ, the star-clusters, and the stars themselves?

In a forest, to use a comparison long ago employed by the elder Herschel, we see around us trees in all stages of their life-history. There are the seedlings just sprouting from the acorn, the slender saplings, the sturdy oaks in their full vigor, those also that are old and near decay, and the prostrate trunks of the dead. Can we apply the analogy to the heavens, and if we can, which of the objects before us are to be regarded as in their infancy, and which of them as old and near dissolution?

906. Fundamental Principles of a Rational Cosmogony. — In the present state of science many of the questions thus suggested seem to be hopelessly beyond the reach of investigation, while others appear like problems which time and patient work will solve, and others yet have already received clear and decided answers. In a general way it may be said that the *condensation and aggregation of rarefied masses of matter under the force of gravitation; the conversion into heat of the (potential) "energy of position" destroyed by the process of condensation; the effect of this heat upon the contracting mass itself, and the radiation of energy into space and to surrounding bodies as waves of light and heat,* — these principles contain nearly all the explanations that can thus far be given of the present state of the heavenly bodies.

907. The Planetary System. — We see that our planetary system is not a mere accidental aggregation of bodies. Masses of matter coming hap-hazard towards the sun would move, as comets do, in orbits, always conic sections to be sure, but of every degree of eccentricity and inclination. There are a multitude of relations actually observed in the planetary system which are wholly independent of gravitation and demand an explanation.

1. The orbits are all *nearly circular.*
2. They are all nearly *in one plane* (excepting the cases of some of the little asteroids).
3. The revolution of all is *in the same direction.*
4. There is a curiously *regular progression of distance* (expressed by Bode's law, which, however, breaks down at Neptune).
5. There is a roughly regular *progression of density,* increasing both ways from Saturn, the least dense of all the planets in the system.

As regards the planets themselves, we have

6. The *plane* of the planets' rotation *nearly coinciding with that of the orbit* (probably excepting Uranus).
7. The *direction* of the rotation *the same as that of the orbital revolution* (excepting probably Uranus and Neptune).
8. The *plane* of *orbital revolution of the satellites* coinciding nearly with that of the planet's rotation.
9. The *direction* of the satellites' revolution also coinciding *with that of the planet's rotation.*
10. The largest planets rotate most swiftly.

908. Origin of the Nebular Hypothesis. — Now this is evidently a good arrangement for a planetary system, and therefore some have inferred that the Deity *made* it so, perfect from the first. But to one who considers the way in which other perfect works of nature usually come to their perfection — their processes of growth and development — this explanation seems improbable. It appears far more likely that the planetary system *grew* than that it was *built* outright.

Three different philosophers in the last century, Swedenborg, Kant, and La Place (only one of them an astronomer), independently proposed essentially the same hypothesis to account for the system as we now know it. La Place's theory, as might have been expected from his mathematical and scientific attainments, was the most carefully and reasonably worked out in detail. It was formulated before the discovery of the great principle of the "conservation of energy," and before the mechanical equivalence of heat with other forms of energy was known, so that in some respects it is defective, and even certainly wrong. In its main idea, however, that the solar system once existed as a nebulous mass and has reached its present state as the result of a series of purely physical processes, it seems certain to prove correct, and it forms the foundation of all the current speculations upon the subject.

909. La Place's Theory. — (*a*) He supposed that at some past time, which may be taken as the starting-point of our system's

history (though it is not to be considered as *the beginning of the existence of the substance* of which our system is composed), the matter now collected in the sun and planets was in the form of a *nebula.*

(*b*) This nebula was a *cloud of intensely heated gas*, perhaps hotter, as he supposed, than the sun is now.

(*c*) This nebula, under the action of its own gravitation, assumed an approximately globular form with a rotation around an axis. As to this movement of rotation, it appears to be necessary to account for it by supposing that the different portions of the nebula, before the time which has been taken as the starting-point, had motions of their own. Then, unless these motions happened to be balanced in the most perfect and improbable manner, a motion of rotation would set in of itself as the nebula contracted, just as water whirls in a basin when drawn off by an orifice in the bottom. The velocity of this rotation would become continually swifter as the volume of the nebula diminished, the so-called "moment of momentum" remaining necessarily unchanged.

910. (*d*) In consequence of this rotation, the mass, instead of remaining spherical, would become much flattened at the poles, and as the rotation went on and the motion became accelerated, the time would come when the centrifugal force at the equator of the nebula would become equal to gravity, and "rings of nebulous matter" would be *abandoned* (not thrown off), resembling the rings of Saturn, which, indeed, suggested this feature of the theory.

(*e*) A ring would revolve for a while as a whole, but in time would *break*, and the material would *collect into a single globe.* La Place supposed that the ring would revolve as if it were solid, the outer edge, therefore, moving more swiftly than the inner. If this were so, the mass formed from the collection of the matter of the ruptured ring *would necessarily rotate in the same direction* as the ring had revolved.

(*f*) The planet thus formed would continue to revolve around the central mass, and might itself in turn abandon rings which might break, and so furnish it with a retinue of satellites.

911. It is obvious that this theory meets completely most of the conditions of the problem. It explains every one of the facts just mentioned as demanding explanation in the solar system. Indeed,

it explains them almost *too* well; for as the theory stands it meets a most serious difficulty in the *exceptional* cases of the planetary system, such as the anomalous and retrograde revolutions of the satellites of Uranus and Neptune. Another difficulty lies in the swift revolution of Phobos (Art. 589), the inner satellite of Mars. According to the unmodified nebular hypothesis, no planet or satellite could have a time of revolution less than the time of rotation which the central body would have, if expanded until its radius becomes equal to the radius of the satellite's orbit; still less could it have a period shorter than the central body *now* has.

912. Necessary Modifications. — The principal modifications which seem essential to the theory in the light of our present knowledge, are the following. (The small letters indicate the articles of the original theory to which reference is made.)

(*b*) It is not probable that the original nebula could have been *at a temperature* even nearly as high as the present temperature of the sun. The process of condensation of a gaseous cloud from loss of heat by radiation would cause the temperature to *rise*, according to the remarkable and almost paradoxical law of Lane (Art. 357), until the mass had begun to liquefy or solidify. And it appears probable that the original nebula, instead of being *purely gaseous*, was rather a *cloud of dust* than a "*fire-mist*"; *i.e.*, that it was made up of finely divided particles of solid or liquid matter, each particle enveloped in a mantle of permanent gas. Such a nebula in condensing would *rise* in temperature at first as if purely gaseous, so that its central mass after a time would reach the solar stage of temperature, the solid and liquid particles melting and vaporizing as the mass grew hotter. At a subsequent stage, when yet more of the original energy of the mass had been dissipated by radiation, the temperature of the bodies which were formed from and within the nebula would fall again.

And yet La Place *may* have been right in ascribing a high temperature to the original nebula. If that were really the case, the only difference would be that the nebula would be longer in reaching the condition of a solar system; but it is not *necessary*, as he supposed, to assume that the original temperature was high, and that the matter was originally in a purely gaseous condition, in order to account for the present existence of such a group of bodies as the incandescent sun and its cool attendant planets.

913. (*d*) As regards the manner in which the planetary bodies were probably liberated from the parent mass, it seems to be very doubtful whether the matter accumulated at the equator of the rotating mass would usually separate itself as a ring. If a plastic mass in swift rotation is not absolutely homogeneous and symmetrical, it is more likely to become distorted by a lump formed somewhere on its equator, which lump may be finally detached and circulate around its primary. The formation of a *ring*, though possible, would seem likely to be only a rare occurrence.

La Place seems to have believed also that the outer rings must necessarily have been abandoned first, and the others in regular succession, so that the *outer* planets are much the older. It seems, however, quite possible, and even probable, that several of the planets may be of about the same age, more than one ring having been liberated at the same time; or several planets having been formed from different zones of the same ring.

(*e*) In the case where a ring was formed, it is practically certain that it could not have revolved as a solid sheet; *i.e.*, with the same angular velocity for all the particles, and with the outer portions, therefore, moving more swiftly than the inner. If, for instance, the matter which now constitutes the earth were ever distributed to form a ring occupying anything like half the distance from Venus to Mars, it must have been of a tenuity comparable only to that of a comet. The separate particles of such a ring could have had very little control over each other, and must have moved independently; the outer ones, like remoter planets, making their circuits in longer periods and moving *more slowly* than those near the inner edge, as is now known to be the case with Saturn's rings (Art. 641*).

914. Trowbridge's Explanation of the Anomalous Rotation of Uranus and Neptune.—When such a ring concentrates into a single mass, the direction of the rotation of the resultant planet depends upon the manner in which the matter was originally distributed. If the ring be nearly of the same density throughout, the resulting planet (which would be formed at about the middle of the ring's width) must have a *retrograde* rotation like Uranus and Neptune. But if the particles of the ring are more closely packed near its inner edge, so that the resultant planet would be formed much within the middle of its width, its axial rotation must be *direct*. In the first case, illustrated in Fig. 233 (*a*), the particles near the *inner edge*

of the ring would control the rotation, having a greater moment of rotation with respect to *M*, where the planet is supposed to be formed, than those at the outer edge. The rotation, therefore, will be *retrograde*, on account of their greater velocity.

In the other case, Fig. 233 (*b*), where the inner edge of the ring is densest, and the planet is formed as at *N*, much nearer the inner than the outer edge of the ring, the aggregate moment of rotation

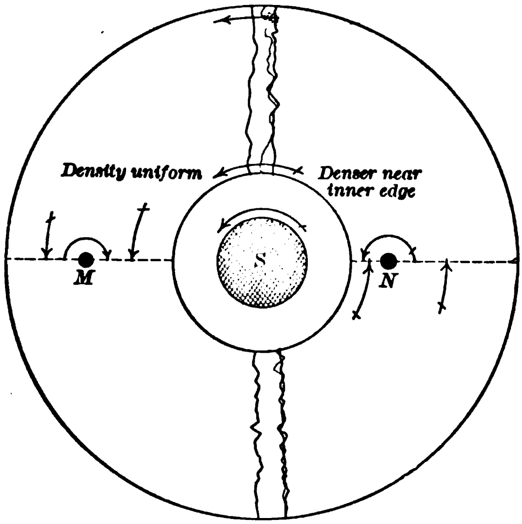

FIG. 233. — Rotation of Planets formed from Rings according to Trowbridge.

with respect to *N* is greater for the *particles beyond N* (because of their greater distance from it) than that of the swifter moving particles within, and this determines a *direct* rotation.

The fact that the satellites of Uranus and Neptune revolve backwards is not, therefore, at all a bar to the acceptance of the nebular hypothesis, as sometimes represented. If a new planet should ever be discovered outside of Neptune it is altogether probable that its satellites would be found to retrograde.

This is not the only way in which the retrograde rotation of the outer planets may be accounted for. The theory of "tidal evolution" (Art. 916) indicates ways in which an original rotation might be *reversed*, and its period greatly changed. See also the next Article.

915. Faye in 1884 propounded a modification of the nebular hypothesis which makes the planets of the "terrestrial group" (Mercury, Venus,

the Earth, and Mars) older than the outer ones.[1] He supposes that the planets were formed by local condensations (not by the formation of rings) *within* the revolving nebula. At first, before the nebula was much condensed at the centre, the inward attraction would be at any point *directly proportional to the distance of that point from the centre of gravity* of the nebula; *i.e.*, the force could be expressed by the equation $F = ar$. After the condensation has gone so far that practically almost the whole of the matter is collected at the centre of the nebula, the force is *inversely proportional to the square of the distance*, — the ordinary law of gravitation,

i.e.,
$$F = \frac{b}{r^2}.$$

At any intermediate time, during the gradual condensation of the nebula, the intensity of the central force will, therefore, be given by an expression having the form

$$F = ar + \frac{b}{r^2},$$

r being the distance of the body acted upon from the centre of gravity of the nebula, while a and b are coefficients which depend upon its age; a continually decreasing as the nebula grows older, while b increases. The planets formed within the nebula when it was young, *i.e.*, when a was large and b was small, would have *direct* rotation upon their axes, while those formed after a had sensibly vanished would have a retrograde rotation; and this he supposes to be the case with Uranus and Neptune, which he considers *younger* than the inner planets. Faye's work "L'Origine du Monde," 1885, contains an excellent summary of the views and theories of the different astronomers who have speculated upon the cosmogony.

916. Tidal Evolution. — About 1885 Prof. George H. Darwin (son of the great naturalist) made some important investigations upon the effect of *tidal reaction* between a central mass and a body revolving about it, both of them being supposed to be of such a nature (*i.e.*, not absolutely *rigid*) that tides can be raised upon them by their mutual attraction. We have already alluded to the subject in connection with the tides (Art. 484). He finds in this reaction an explanation of many puzzling facts. It appears, for instance, that if a planet and its satellite have ever had their times of rotation of the same length as the time of their orbital revolution around their common centre of gravity, then, starting from that time, either of two things might happen, — the satellite might begin to recede from the planet, or it might fall back to the central mass. The condition is one of unstable equilibrium, and the slightest cause

[1] If Mars ultimately proves to be warmer than the earth (see Art. 589) it will be a strong argument in favor of Faye's hypothesis.

might determine the subsequent course of things in either of the two opposite directions. Whenever the time of rotation of the planet is *shorter* than the orbital period of the satellite (as it would naturally become by condensation continuing after the separation of the satellite), the tendency would be, as explained in Art. 484, slightly to accelerate the satellite, and so to cause it continually to recede by an action the reverse of that produced by the hypothetical resisting medium which is supposed to disturb Encke's comet. This, it will be remembered, is thought to be the case with our moon.

917. But if by any means the rotation of the planet were *retarded*, so that its day should become *longer* than the period of the satellite, the tides produced by the satellite upon the planet will then retard the motion of the satellite like a resisting medium, and so will cause a continual shortening of its period, precisely as in the case of Encke's comet. If nothing intervenes, this action will in time bring down the satellite upon the planet's surface. Now in the case of Mars there is a known cause operating to retard its rotation (namely, the tides which are raised by the *sun* upon the planet), and those who accept the theory of tidal evolution suggest that this was the cause which first made the length of the planet's day to exceed the period of the satellite, and so enabled the planet to establish upon the satellite that retardation which has shortened its little month, and must ultimately bring it down upon the planet.

Processes such as these of tidal evolution must necessarily be extremely slow. How long are the periods involved, no one can yet estimate with any precision, but it is certain that the years are to be counted by the million.

(We have already referred the reader (Art. 484*) to the last chapter of Ball's "Story of the Heavens" as containing an excellent and easily understood explanation of this subject.)

918. Conclusions derived from the Theory of Heat. — As Professor Newcomb has said, "Kant and La Place seem to have arrived at the nebular hypothesis by reasoning *forwards.* Modern science obtains a similar result by reasoning *backwards* from actions which we now see going on before our eyes."

We have abundant evidence that *the earth* was once at a much higher temperature than now. As we penetrate below the surface we find the temperature continually rising at a rate of about 1° F. for every fifty or sixty feet, thus indicating that at the depth of a

few miles the temperature must be far above incandescence. Now, since the surface temperature is so much lower, this implies one (or *both*) of two things, — either that heat-making processes are going on within the earth (which may be true to some extent), or else that the earth has been much hotter than it now is, and is cooling off, — and this seems to be a most probable supposition. It is just as reasonable, as Lord Kelvin expresses it, to suppose that the earth has lately been intensely heated as to suppose that a warm stone that one picks up in the field has been lately somewhere in the fire.

919. Evidence derived from the Condition of the Moon and Planets. — In the case of the *moon* we find a body bearing upon its surface all the marks of past igneous action, but now in appearance intensely cold. The *planets*, so far as we can judge from what we can see through the telescope, corroborate the same conclusion. Their testimony is not very strong, but it is at least true that nothing in the aspect of any of them militates against the view that they also are bodies cooling like the earth; and in the cases of Jupiter and Saturn many phenomena go to show that they are still (or at least *now*) at a high temperature, — as might be expected of bodies of such an enormous mass, which, necessarily, other things being equal, would cool much more slowly than smaller globes like the earth.

The ratio of surface to mass is smaller as the diameter of a globe grows larger, and upon this ratio the rate of cooling of a body largely depends. In short, everything we can ascertain from the observation of the planets agrees completely with the idea that they have come to their present condition by *cooling down from a molten or even gaseous state.*

920. The Sun's Testimony. — In the sun we have a body steadily pouring forth an absolutely inconceivable amount of heat, without any visible source of supply. Thus far the only reasonable hypothesis to account for this, and for a multitude of other phenomena which it shows us, is the one which makes it a great cloud-mantled ball of incandescent gases, slowly shrinking under its own central gravity, converting continually a portion of its "potential energy of position"[1] into the *kinetic-energy* of heat, which at present is mainly radiated off into space.

[1] By "*potential* energy of position" is meant the energy due to the separated condition of its particles from each other. As they fall together and towards the centre in the shrinkage of the sun, they "do work" in precisely the same way as any falling weight.

We say *mainly*, because it is not impossible that the sun's temperature is even yet slowly rising, and that the maximum has not yet been reached. We are not sure whether *all* the heat produced by the sun's annual shrinkage is radiated into space, or whether a portion is retained within its mass, thus raising its temperature; or whether, again, it radiates *more* than the amount thus generated, so that its temperature is slowly diminishing.

921. That the sun is really shrinking is admittedly only an inference, for the shrinkage must be far too slow for direct observation. Our case is like that of a man who, to use one of Professor Newcomb's illustrations, when he comes into a room and finds a clock in motion, concludes that the clock-weight is descending, even though its motion is too slow to be observed. Knowing the construction of the clock and the arrangement of its gearing, and the number of teeth in each of its different wheels, he states confidently just how many thousandths of an inch the weight sinks at each vibration of the pendulum; and looking into the clock-case and measuring the length of the space in which the weight can move, and noting its present place, he proceeds to state how long ago the clock was wound up, and how long it has yet to run. We must not push the analogy too far, but it is in some such way that we conclude from our measurements of the sun's annual output of energy in the form of heat, how fast it is shrinking, and we find that its diameter must diminish not far from 300 feet in a year; at least, the loss of potential energy corresponding to that amount of shrinkage would account for one year's running of the solar mechanism.

922. Age of the Solar System. — Looking backward, then, in imagination we see the sun growing continually larger through the reversed course of time, expanding and becoming ever less and less dense, until at some epoch in the past it filled all the space now included within the largest orbit of the solar system.

How long ago that was no one can say with certainty. If we could assume that the amount of potential energy lost by contraction, converted into the actual energy of heat and radiated into space, has been the same each year through all the intervening ages, and, moreover, that *all* the heat radiated has come from this source *only*, without subsidy from any original store of heat contained in an original "fire mist," or from energy derived from outside sources, then it is not difficult to conclude that the sun's past history must cover some 15 000000 or 20 000000 years.

But the assumption that the loss of heat has been even nearly uniform is extremely improbable, considering how high the present temperature of the sun must be as compared with that of the original

nebula, and how the ratio of surface to solid content has increased with the lessening diameter.

Nor is it unlikely that the sun may have received energy from other sources[1] than its own contraction. Altogether it would seem that we must consider the 15 000000 years to be the least possible value of a duration which may have been many times more extended. If the nebular hypothesis and the theory of the solar contraction be true, the sun must be as old as that,—how much older no one can tell with certainty. Lord Kelvin, however, from considerations based mainly on the observed rise of temperature downwards from the surface of the earth, and the heat-conducting power of our rocks is disposed to set a maximum limit of from 100 000000 to 200 000000 years for the possible age of the earth.

It is precisely here that the nebular hypothesis encounters its most serious difficulty. It would seem that vastly longer periods of time must have been required for the formation of rings and nebulous planets, and for their concentration into such bodies as we now find circulating around the sun.

923. **Future Prospects.**—Looking forward towards the future, it is easy to conclude also that at its present rate of radiation and contraction the sun must, within 5 000000 or 10 000000 years, become so dense that the conditions of its constitution will be radically changed, and to such an extent that life on the earth, as we now know life, would probably be impossible. *If nothing intervenes to reverse the course of things*, the sun must at last solidify and become a dark, rigid globe, frozen and lifeless among its lifeless family of planets. At least, this is the necessary consequence of what now seems to science to be the true account of its present activity and the story of its life.

924. **Stars, Star-Clusters, and the Nebulæ.**—It is obvious that the same nebular hypothesis applies satisfactorily to the explanation of the relation of these different classes of bodies to each other. In fact, Herschel, appealing only to the law of continuity, had concluded before La Place formulated his theory, that nebulæ develop sometimes into clusters, sometimes into double or multiple stars, and sometimes into single ones. He showed the existence in the sky of all the intermediate forms between the nebula and the finished star. For a time, about the middle of our century, while it was generally

[1] From the atomic disintegration of radio-active substances, for instance.

supposed that all nebulæ were nothing but star-clusters, too remote to be resolved by existing telescopes, his views fell rather into abeyance; but when the spectroscope demonstrated the substantial differences between the gaseous nebulæ and the star-clusters, they regained acceptance in their essential features; with perhaps the reservation, that many are disposed to believe that the rarest even of nebulous matter, instead of being purely gaseous, is full of solid and liquid particles like a cloud of fog or smoke.

925. The Present System not Eternal.—One lesson seems to stand out clearly,—that the present system of stars and worlds is not an eternal one. We have before us irrefragable evidence of continuous, uncompensated progress, inexorable in one direction. The hot bodies are losing their heat, and distributing it to the cold ones, so that there is a steady, unremitting tendency towards a uniform (and therefore useless) temperature throughout the universe: for heat does *work*, and is *available* as energy *only when it can pass from hotter to cooler bodies*, so that this warming up of cooler bodies at the expense of hotter ones always involves a loss, not of energy (for that is indestructible), but of *available* energy. To use the technical language now usually employed, energy is unceasingly "*dissipated*" by the processes which maintain the present life of the universe; and this dissipation of energy can have but one ultimate result,—that of absolute stagnation when a uniform temperature has been everywhere attained. If we carry our imagination backwards we reach at last a "beginning of things," which has no intelligible antecedent: if forwards, an end of things in stagnation. That by some process or other this end of things will result in "new heavens and a new earth" we can hardly doubt, but science has as yet no word of explanation.

926. Sir Norman Lockyer's Meteoritic Hypothesis.—The idea that the heavenly bodies in their present state may have been formed by the aggregation of *meteoric* matter, rather than by the condensation of a *gaseous* mass, is not new, and not original with Mr. Lockyer, as he himself points out. But his adoption and advocacy of the theory, and the support he brings to it from spectroscopic experiments on the light emitted by fragments of meteoric stones under different conditions, has given it such currency that his name will always be justly associated with it. We have already referred to it in several places (Arts. 850 and 894 especially).

He believes that he finds in the spectra of meteorites, under various conditions, an explanation of the spectra of comets, nebulæ, and all the different types of stars, as well as the spectra of the Aurora Borealis and the Zodiacal Light.

Assuming this, he considers that *nebulæ* are meteoric swarms in the initial stages of condensation, the separate individuals being still widely separated, and collisions comparatively infrequent.

As aggregation goes on, the nebulæ become *stars*, which run through a long life-history, the temperature first increasing slowly to a maximum, and then falling to non-luminosity. During this life-history the stars pass through successive stages, each stage characterized by its own typical spectrum. Lockyer has also proposed an elaborate classification of stellar spectra arranged according to these hypothetical stages; but it has not yet secured general acceptance, probably because its theoretical basis appears to be insufficiently established.

The hypothesis receives a certain support from a most interesting mathematical investigation of Prof. George Darwin, who shows that, if we assume a meteoric swarm comparable in dimensions with our solar system, composed of individual masses such as fall on the earth, and endowed with such velocities as meteors are known to have, such a swarm, seen from the distance of the stars, *would behave like a mass composed of a continuous gas.* This is not strange, since, according to the kinetic theory of gases, a gas is simply a swarm of *molecules*, behaving in just the way the meteorites are supposed to act. But it follows that the *meteoric* theory of a nebula does not in the least invalidate, or even to any great extent modify, the reasoning of La Place in respect to the development of suns and systems from a *gaseous* nebula.

A new form of the meteoric theory, known as the "planetesimal hypothesis," has been recently proposed and developed by Chamberlin and Moulton. It assumes as the origin of the solar system, a spiral nebula, composed largely of little masses (planetesimals), moving around the centre, generally in the same direction, but in orbits that vary in inclination, eccentricity, and period, and are subject to continual perturbation. There results a very slow accretion of the planetesimals into planets, with very little development of heat, since the relative velocities of the colliding bodies are very small.

A great advantage of this theory is that it allows time enough to satisfy the most exorbitant demands of geology and biology.

Exercises on Chapter XXII.

1. Find the mass of the system of Alpha Centauri from the data given in Tables IV. and V. — namely, parallax (p) = 0″.75, semi-major axis of orbit (a″) = 17″.70, and period (t) = 81.1 years.

Ans. Mass of system = 2.00 × mass of the sun.

2. Find the mass of the system of Sirius from the tabular data.

Ans. 2.78 × mass of the sun.

3. Find the mass of the system of Eta Cassiopeiæ from the tabular data.

Ans. 0.34 × mass of the sun.

4. Find the mass of the system of 70 Ophiuchi from the tabular data.

Ans. 0.77 × mass of the sun.

5. Find the radius of the apparent orbit of the spectroscopic binary 3105 Lacaille, the relative velocity of the components being 385 miles a second, and the period 3 days, 2 hours, and 46 minutes, as indicated by the doubling of the lines in the spectrum. Assume that the orbit is circular, that its plane is directed towards the sun, and that the two components are equal.

Ans. Radius of orbit = 16 493000 miles.

6. Compute the mass of the system on the same assumptions as above, remembering that the radius of this apparent orbit is also the radius of the *relative* orbit which each component describes around the other regarded as at rest.

Ans. 76.75 × mass of the sun.

7. Carry out similar computations for the systems of Zeta Ursæ Majoris, Beta Aurigæ, and Mu Scorpii, using the data of Art. 879.

8. Determine the radius of the orbit described by Spica Virginis, as shown by the shift of the lines in its spectrum. Velocity = 56.6 miles a second; period = 4 days and 19 minutes. Orbits assumed circular and in plane of the sun.

Ans. Radius = 3 123500 miles.

9. From this determine the mass of the system, assuming that the mass of the bright star is infinitesimal as compared with that of the dark star; *i.e.*, that it is a small planet revolving around a dark central sun. (A very improbable hypothesis of course.)

Ans. 0.315 × mass of the sun.

10. What is the mass of the system if the dark star is equal to the bright one? (In this case the radius of the *relative* orbit is the diameter of the apparent orbit of Spica, or *double* its value in the last example.)

Ans. 8 × 0.315, or 2.520 × mass of the sun.

11. What is the mass if the dark star has a mass only one-fourth that of the bright one? (In this case the orbit of the dark star has a radius four times as great as that of Spica, and the radius of the *relative* orbit is five times as great as that of the apparent orbit of Spica.)

Ans. 125 × 0.315, or 39.37 × mass of the sun; the mass of the bright star being 31.50, and that of the dark star being one-fourth as great, or 7.87.

NOTE. — The assumption that the bright star is a mere planet, revolving around a dark central body vastly more massive than itself, gives us a minor limit to the possible mass of the system, but the major limit cannot be fixed without knowledge as to the relative mass of the dark body.

If the dark body is larger than the bright one, the mass of the system cannot exceed eight times that minor limit.

The general formula is easily obtained: let n be the ratio between the masses of the bright and dark stars, so that if r is the radius of the circle described by the bright star around the common centre, the radius of the circle described by the other will be nr, and the radius of the *relative* orbit will be $(n+1)r$. Also let μ be the united mass of the two stars. Then, expressing the period, t, in years, r in astronomical units, and μ in terms of the sun's mass, we have

$$\mu = (n+1)^3 \frac{r^3}{t^2}$$

The factor $(n+1)^3$ becomes unity when $n = o$, *i.e.*, when the *bright star* is a particle; and infinity when n becomes infinite, *i.e.*, when the *dark star* is a particle revolving at the infinite distance, $r(n+1)$. It becomes 8 when $n = 1$, the two stars being equal.

It may be added that the assumption that the orbit is circular, and that its plane passes through the solar system, is entirely gratuitous and not likely to be correct. But the general character of the results would not be seriously changed unless the inclination and eccentricity of the orbit were great.

NOTE TO ART. 879.

The number of spectroscopic binaries recently detected by various observers in this country and Europe is so large that it is useless to try to keep up the enumeration in this text-book. The list at present (January, 1906) includes about 150, and is growing continually. Professor Campbell of the Lick Observatory who has been specially successful in this line of work, estimates that about one star in every twelve examined turns out a "spectroscopic binary." Perhaps the two most notable additions to the catalogue are Polaris and Capella, — the former with a period a little less than four days, and a range of radial velocity of only about four miles a second; the other with a period of 105 days and a velocity range of about 36 miles. The actual orbital velocity cannot, of course, be determined until we know the inclination of the orbit to the line of sight.

ADDENDUM A.

THE SPECTROHELIOGRAPH AND ITS APPLICATION TO SOLAR RESEARCH. THE SIDEROSTAT AND CŒLOSTAT.

THE spectroheliograph received a passing mention in Art. 326*, but it has recently become so important that something more is now required. The principle of the instrument, first suggested by Janssen in 1870, was first successfully, and independently, applied by Hale in Chicago, and Deslandres in Paris, about 1890. Its essential feature is the introduction in a photographic spectroscope of a *second slit,* parallel to the collimator slit, but at the other end of the instrument, close in front of the sensitive plate, thus isolating a narrow line of homogeneous light in the spectrum. If now the telescope which carries the spectroscope is pointed at the sun and its image made to pass over the collimator slit, while at the same time the sensitive plate itself is moved in precisely corresponding manner, the result will be a photograph of whatever passed over the slit, *produced solely by the light of that single wave-length* which was isolated by the second slit.

If, for instance, the spectroscope were so adjusted as to bring upon the second slit the *K* line of calcium, then the image photographed would be due to calcium vapor only, and that vapor in such physical condition as to emit this "*K* light," if the expression may be permitted. If the adjustment were such as to bring a line of hydrogen, magnesium, or iron into the slit, we should get an image due solely to the corresponding element. In order that an image of the entire sun be obtained it is of course necessary that the two slits should be longer than the diameter of the sun's image upon the slit-plate.

Other arrangements of the instrument are possible. In that first used by Hale the telescope was kept directed at the centre of the sun and the sensitive plate was fixed; but the two slits were movable and so connected by a system of levers that while the first was made to traverse the sun's image, the second was made to move correspondingly before the plate. But this arrangement is much more difficult to construct and adjust, and is in other ways inferior.

Still, at Kenwood, between 1892 and 1896, a number of beautiful photographs of prominences was thus obtained, and many interesting plates showing the calcium floccules over the entire disc.

The Rumford spectroheliograph, now used on the great Yerkes telescope, has slits 6 inches long, nearly but not quite sufficient to take in the whole diameter of the sun's image formed by the 40-inch telescope. The slit is placed east and west, and the telescope is moved in declination by a slow-motion screw driven by an electric motor, this motion being also communicated directly to the photographic plate.

It must be remembered that the *dark* lines of the solar spectrum are dark only *relative to the background;* they are really *bright*, and somewhat brighter than the same lines in the "flash spectrum," for their actual brilliance is not reduced by the light which reaches the reversing layer from the photosphere beneath, but somewhat increased (Art. 314, note). When, therefore, the second slit is set on a "dark line" of the solar spectrum, the background of continuous spectrum being excluded, an observer looking through an eye-piece would see the line *bright* and capable of impressing itself upon a sensitive film. The instrument therefore makes it possible to study separately and in detail what may be called the calcium surface, the hydrogen surface, the magnesium surface, etc., of the sun. And in the differences and correspondences of these surfaces there is a mine of important information.

In the new solar observatory which Professor Hale, with the subvention of the Carnegie Institution, is now (1904) erecting upon Wilson's Peak in Southern California, a fixed horizontal telescope fed by a "cœlostat" will be used. The image of the sun will be at least a foot in diameter (perhaps much larger) and there will be a corresponding spectroheliograph firmly mounted on piers. Several other spectroheliographs of smaller size and different constructions are being provided for stations in Europe and India, which will coöperate in solar studies, and it is expected that important results will follow from the data obtained.

In various lines of astrophysical work there are great advantages in having the telescope fixed in a convenient position with the object under observation reflected into it steadily by a mirror suitably moved by clockwork.

The *Siderostat* of Foucault (1865) consists of a plane mirror carried by a polar axis which revolves once in 24 hours. The mirror can thus be so set as to throw the reflected light in any chosen

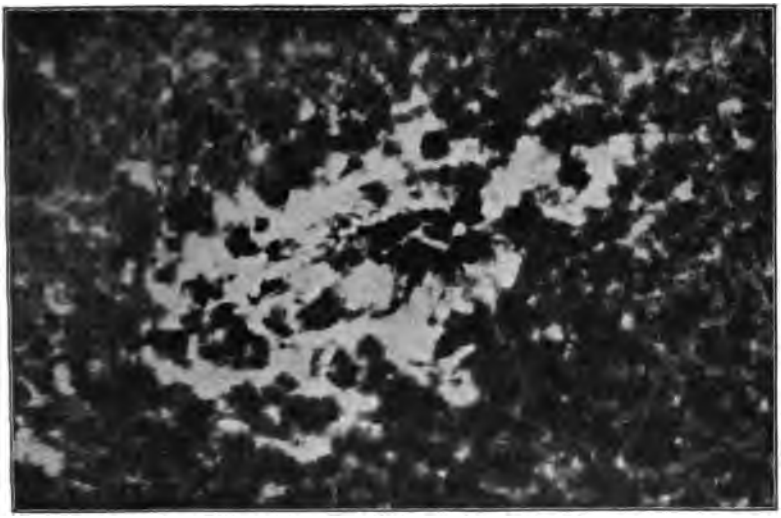

FIG. 247. — Oct. 9, 3 h. 30 m. Calcium Flocculi. Slit set on H.

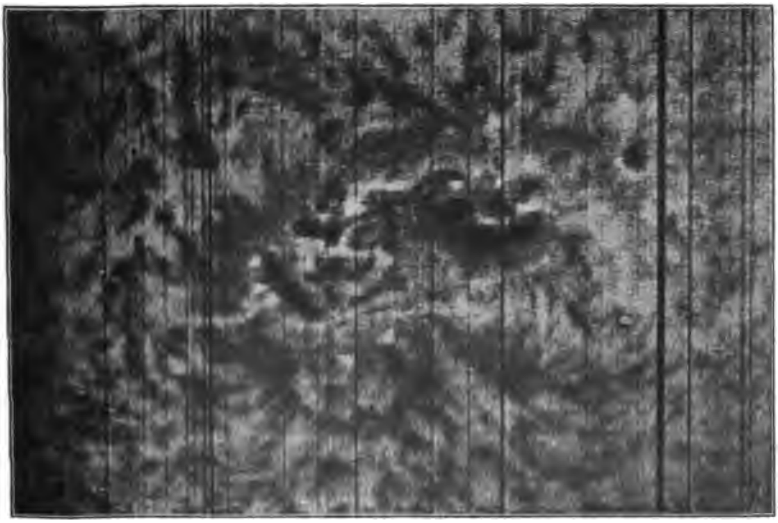

FIG. 248. — Oct. 9, 1 h. 04 m. Hydrogen Flocculi. Slit set on H_β.

THE GREAT SUN-SPOT OF OCTOBER, 1903.

Scale: Sun's Diameter = 9¾ inches.

direction (usually horizontally north or south), and means are provided by which the observer can adjust it from his distant position. This is perfectly satisfactory for all work which requires merely a fixed and convenient direction for the *central line* of the reflected beam — as for the study of star-spectra or the general spectrum of the sun. If, however, the instrument is used to form an image of the sun or of a group of stars, it has the serious disadvantage that the image revolves around its central point and with a speed that *continually varies*, thus rendering the arrangement unsuitable for most photographic work.

The *Cœlostat* differs in that the polar axis which carries its mirror turns once in 48 hours instead of 24. The reflected image in this case does not revolve, but the possible directions in which it can be thrown are not arbitrary, being determined by the declination of the object. The difficulty can be overcome by using a second mirror to send the reflected beam when it is wanted, but of course this is expensive and involves loss of light. This arrangement, however, is proposed for Mt. Wilson, since a non-rotating image is essential, as is also the possibility of mounting the spectroheliograph and its accessories upon solid piers.

As an interesting specimen of the photographs obtained by the spectroheliograph we give Figs. 247 and 248, reduced by one-thrd, from Plate XI of Professor Hale's paper describing the instrument and its work. The upper figure of the two is a "calcium" photograph of a great sun spot; the lower, a "hydrogen" photograph of the same, both taken the same afternoon. The differences are instructive, especially the peculiarly "wispy" character of the dark markings of the hydrogen plate as compared with the "lumpiness" of the calcium floccules.

ADDENDUM B.

NOVA PERSEI AND NOVA GEMINORUM.

A remarkable temporary star, the most brilliant since Kepler's star of 1604, suddenly appeared in 1901, probably on February 20, though first *seen* (by Dr. Anderson in Edinburgh) on the 21st, when it was about as bright as Polaris. Photographs covering the region of the star made at Cambridge, United States, on several dates preceding and including the 19th, show that on the 19th the star was not yet as bright as the 12th magnitude. Within three days it increased its brightness fully 25,000 fold, and on the 22d was for several hours the brightest star in the heavens, Sirius alone excepted, and more than a match for Capella and Vega. It was then distinctly ruddy, very like Arcturus. It faded at first rapidly but fitfully, and by the end of March was barely visible to the naked eye; after that its decrease was more gradual, and it is still (1904) visible in large telescopes as a little star of 12th or 13th magnitude.

The spectrum, as photographed at Cambridge on the 22d, was not that of the usual "Nova" type, but much resembled that of α Orionis (Rigel), being mainly continuous, though crossed by about 30 not very conspicuous dark lines. Clouds prevented photographs on the 23d, but on the 24th it was clear, and in the meantime a complete change had taken place. The spectrum was now essentially like that of Nova Aurigæ, with the same broad bright bands of hydrogen and their dark and more refrangible companions. It may be noted here that the recent (1903) investigations by Ebert of Munich go far towards proving that for the explanation of these doubled lines we need not resort to the hypothesis of conflicting masses of hydrogen, cool and hot, moving towards and from us with a speed of several hundred miles a second, nor even to explosive pressures. It seems probable that the phenomena may be due merely to "anomalous refraction" in portions of the star's gaseous envelope under powerful compression and intense luminous excitement.

Since then the spectrum has followed the usual course, having become nebular before the end of the year, though with some non-nebular peculiarities in the extreme width of its bright hydrogen

lines and in the presence of some conspicuous lines not yet found in nebulæ. It does not yet appear whether its spectrum will finally revert to the purely continuous, stellar type like that of Nova Aurigæ.

During the star's decline its brightness oscillated as much as a whole magnitude, the irregular interval between maxima ranging from about two days in February to six or eight in the autumn.

In September it became possible to photograph the invisible nebulosity around it with the reflectors (not the great *refractors*) of the Lick and Yerkes observatories. It was found to be very extensive, roughly circular, with an apparent diameter about half that of the moon; and since the most careful observations have been unable to detect any parallax or proper motion of the star, it is clear that its distance exceeds that of any of the nearer stars, — very likely it is as much as 100 light years, and not improbably greater yet. If so, the diameter of the nebula must have been *at least* 1400 times that of the earth's orbit, and this is probably an underestimate.

There were in it several pretty well-defined knots and streaks of condensation, and the photographs soon brought to light an almost astounding phenomenon. These knots were found to be all moving swiftly away from the star at various rates *averaging about 10″ a week,* — a motion apparently not very rapid as seen from the earth. But if the Nova were as near as our next neighbor, α Centauri, this would mean more than *two thousand miles a second;* not improbably the distance is a hundred times as great, and if so, the speed becomes greater than that of light itself. Indeed, the most plausible explanation of the phenomenon yet offered is that suggested by Kapteyn, — that the motion is only apparent, not an actual rush of masses of matter, but simply the progressive illumination of spiral streams of nebulosity, advancing along them with the speed of light, an illumination originating when the star first flashed out. If this explanation is correct, the distance of the star must be about 300 light years, and the actual outburst occurred about the time when Columbus was discovering America.

Another small "Nova" was discovered in Gemini in January, 1903, by Professor Turner of Oxford in examining star-chart photographs there made. It was of only the 8th magnitude and presented nothing of special interest.

It is a notable and perhaps significant fact that without exception all the temporary stars thus far observed have been in or near the Milky Way. It will be remembered that the same is true of all the Wolf-Rayet stars.

ADDENDUM C.

SUPPLEMENTARY TO ARTICLES 847, 848, AND 852.

(To Art. 847.) The short-period variables of the γ Aquilæ and β Lyræ type are mostly "punctual variables," to use Miss Clerke's expression; *i.e.*, like the Algol variables, their periods are uniform without any such irregularities as are usual with the stars of the o Ceti type. It is natural to ascribe such "punctuality" to an orbital revolution, and this is justified in many cases by the fact that the star is proved to be a spectroscopic binary by the periodical shifting or doubling of its spectrum-lines. But the changes of brightness cannot, as with the Algol stars, be explained "geometrically," by eclipses; while the periods are regular, the times of maxima and minima do not accord with such an explanation, and besides, the changes are continuous, and not confined to any short portion of the period. It seems necessary to suppose, therefore, that the orbital revolution is accompanied by some mutual action of the two (or more) revolving stars upon each other (possibly *tidal*, possibly some kind of physical influence) and that this action causes fluctuations in the radiating power of their surfaces.

But precise explanation is still difficult and uncertain, and very possibly no one explanation will apply to all the stars now grouped together in this class.

The great majority of these variables, like the temporary stars and the Wolf-Rayet stars, are near, or in, the Milky Way. This is not true of the stars of the o Ceti or Algol types, which are found indiscriminately in all parts of the heavens.

(To 848.) An interesting fact respecting the "eclipse stars" of the Algol type was first pointed out in 1899, simultaneously but independently, by Russell at Princeton and by Roberts of Lovedale, South Africa; namely, that an *upper limit* to the density of the eclipsing stars can be determined from the ratio of the whole period of the variable to the time between the beginning and the end of the obscuration. In the cases of the seven or eight Algol stars, for which we now have sufficiently accurate data, it was found that this *upper limit* is far below the density of the sun, and of course the

actual density lower yet: some of them must be hardly more substantial than clouds.

(To 852.) Since 1900 the catalogue of known variables has increased enormously. The sporadic variables (excluding the hundreds which have been found in star-clusters, in the Magellanic clouds, and in a region of some square-degrees including the Nebula of Orion) must now number nearly 1000, if not more; and the list is continually and rapidly growing.

The discoveries are now made mostly by photography, partly from negatives made specially for the purpose of detecting variables, and partly in the examination of the photographs of the star-charting campaign. Numerous amateur observers devote themselves to work in this line, and at several observatories the accurate study of the light-curves of variables with large telescopes and elaborate photometric apparatus forms a considerable part of the work of the institution.

Professor Pickering in his observatory report, dated Sept. 30, 1905, states that since the Harvard photographic work began in 1886, 2750 variables have been discovered, — 2197 at Cambridge, and about 555 elsewhere. Mrs. Fleming has discovered 8 "novæ" and 197 variables, mainly by bright hydrogen lines in their spectra; Professor Bailey has detected 509 in globular star-clusters; and Miss Leavitt 1442, mostly in and near the Magellanic clouds. And since September the list has been considerably lengthened.

Of course nearly all of these new variables are extremely faint, observable only by great telescopes or by photography; and for the great majority nothing is yet known as to the period and type of variation.

The total number of variables which can be reached by our present instruments must be hundreds of thousands, and not improbably millions. Among the 6000 naked-eye stars about 70 variables are already known, and there is no reason to suppose that the proportion is different for the telescopic stars.

APPENDIX.

SUPPLEMENTARY ARTICLES.

1000. Reduction of Sidereal Time to Mean Solar Time and Vice Versa. (*Supplementary to Art. 112.*) — Since the tropical year (Art. 216) contains 365.2421 mean solar days, and exactly one more sidereal day, it follows that the number of *sidereal* seconds in any time interval is equal to the number of *mean solar* seconds multiplied by $\frac{366.2421}{365.2421}$, *i.e.* by 1.00273791. Hence, if I and I' are respectively the number of mean solar and sidereal seconds in any time interval, we have $I' = I + 0.00273791\,I$. (The logarithm of the decimal coefficient is [7.4374191]). Also,

$$I = I' \times \frac{365.2421}{366.2421} = I' \times 0.9972696 = I' - 0.00273043\,I'.$$

The log. of the coefficient is [7.4362311].

The "American Ephemeris" gives at the end of the book two tables containing the values of the second terms of the two formulæ for every value of I and I' up to 24 hours, and the reduction of any sidereal interval to solar, or the reverse, is accomplished by simply adding or subtracting the tabular correction.

To reduce a given *moment* of sidereal time to solar, or the reverse, we must first have the local sidereal time of the preceding mean solar noon. This "sidereal time of mean noon" is given in the Almanac for every day, and for the meridians of Washington and Greenwich. It must be corrected for the longitude, λ, of the observer by applying the correction $235^{s}.91 \times \frac{\lambda}{24^{h}}$, which may be taken directly from the table for reducing *sidereal intervals to solar*, using λ as the argument; the correction is to be added for *West* longitude, subtracted for *East*.

To convert sidereal time into solar, we *subtract* from the given time the sidereal time at preceding noon (duly corrected for longi-

tude). The difference is the interval since noon, expressed in sidereal units, and is at once reduced to solar time units by *subtracting* the correction taken from the proper almanac table (Table II). If the given sidereal time is numerically smaller than the time at preceding mean noon, 24 hours must be added.

If the given time is solar, we *add* it to the sidereal time at preceding mean noon, and also *add* the correction from the other almanac table (Table III), which gives the reduction of solar time intervals to sidereal.

Example: Reduce to local mean solar time the sidereal time $18^h\,33^m\,27^s.71$ at Princeton, N. J. ($\lambda = 9^m\,32^s.6$ east of Washington), on Aug. 27, 1896.

	h. m. s.
Sidereal time of mean noon at Washington	10 25 43.03
Reduction for longitude (Alm. p. 528)	− 1.57
Princeton sidereal time of noon	10 25 41.46
Sidereal time given	18 33 27.71
Sidereal interval since noon	8 07 46.25
Reduction to mean time interval (p. 529) (Table II)	− 1 19.91
Corresponding mean time (local)	8 06 26.34

If "standard time" is wanted (Art. 122) we must further correct the *local* mean solar time by subtracting $1^m\,22^s.50$, the difference between five hours and the longitude of Princeton. Conversely, to reduce this mean time, $8^h\,06^m\,26^s.34$, to sidereal we proceed as follows:

	h. m. s.
Sidereal time of Princeton noon	10 25 41.46
Given mean time	8 06 26.34
Reduction to sidereal (p. 532) (Table III)	+ 1 19.91
Sidereal time	18 33 27.71

A rough reduction, usually correct within five or ten minutes, is easily made without any computation or tables by assuming that the sidereal time at noon increases uniformly two hours each calendar month, reckoning from March 21st; passing the *even* hours on the 21st of each month and the odd hours on the 6th. For days between, allow 4^m each, and 1^m additional for each hour elapsed since noon. Thus, taking the example above, we have for the Princeton sidereal time of noon on Aug. 27th $10^h\,24^m$.

	h. m.
Princeton sidereal time of noon	10 24
Given sidereal time	18 33.5
Sidereal interval	8 09.5
Correction	− 1.3
Corresponding mean time	8 08.2

The result, however, is seldom quite so nearly correct, as the method takes no account of the equation of time, or the inequality of the months.

1001. Azimuthal Motion of a Star at the Horizon. (*Supplementary to Art. 141.*)—In Fig. 240 SS' is part of the diurnal circle of a star that is rising, the arc SS' being described in a unit of time, —say, one minute. Then ab, the corresponding arc of the equator, is 15′, and SS', expressed in minutes of a great circle, is $15' \cos \delta$, δ being the star's declination aS.

In passing over SS' its azimuth changes by the arc SM, which (from the relations in the small, sensibly plane triangle $SS'M$) equals

$$SS' \times \cos S'SM$$
$$= 15' \cos \delta \times \cos S'SM. \quad (1)$$

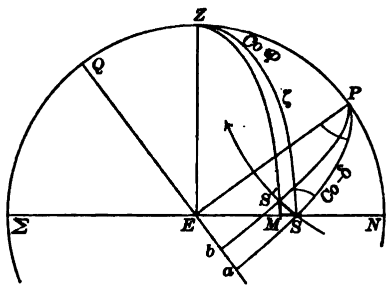

FIG. 240.

Now $S'SM = ZSP$ in the ZPS spherical triangle, and in this, by the fundamental equation of spherical trigonometry, we have

$$\cos PZ = \cos ZS \times \cos PS - \sin ZS \times \sin PS \times \cos ZSP.$$

Substituting the astronomical values, this gives

$$\sin \phi = \cos \zeta \sin \delta - \sin \zeta \cos \delta \times \cos ZSP.$$

Whence $$\cos ZSP \text{ (or } SS'M) = \frac{\sin \phi - \cos \zeta \sin \delta}{\sin \zeta \cos \delta}. \quad (2)$$

When the star is at the horizon, ζ is 90°; $\sin \zeta = 1$ and $\cos \zeta = 0$; so that the expression becomes $\cos SS'M = \dfrac{\sin \phi}{\cos \delta}$ for a star either rising or setting. Substitute this in (1) and we have, for such a star, $SM = 15' \cos \delta \times \dfrac{\sin \phi}{\cos \delta} = 15' \sin \phi$. *This is independent of the star's declination*, is therefore the same for all stars when they are rising or setting, and depends only on the latitude of the observer.

Since this motion of the star in azimuth is merely apparent, and due to the rotational "skewing" of the horizontal plane of the observer, the demonstration is equivalent to that given in Art. 141, in connection with the Foucault pendulum.

The same thing may of course be proved by differentiating the equations of the ZPS triangle so as to find the value of $\dfrac{dZ}{dP}$ in terms of ζ, δ, and ϕ, making $\zeta = 90°$ in the result; δ then disappears as above.

1002. Kepler's Problem. (*Supplementary to Arts. 188, 189.*) — The problem is to find the place which a body will occupy at any given moment when moving in accordance with Kepler's laws around the sun, having given the dimensions of its orbit, the period of revolution T, and the time t elapsed since the planet passed perihelion.

In Fig. 241 $ABA'B'$ is the orbital ellipse, having AC and BC as its semiaxes, respectively designated as a and b. S is the focus, and $CS = ae$, e being the eccentricity of the ellipse. P is the position of the planet; PS, or r, is the *radius vector*, and the angle ASP is the *true anomaly* designated by v. The shaded sector ASP is the area described by the radius vector (in accordance with Kepler's second law), and since the entire area of the ellipse is πab, the area of the sector will be $\pi ab\frac{t}{T}$, which may be written $\frac{1}{2}ab \times 2\pi\frac{t}{T}$. The second factor is the angle (in radians) found by multiplying the whole circumference by the fraction of the period $\frac{t}{T}$ elapsed since perihelion, and is known as the *Mean Anomaly*, being the anomaly which the planet would have if its angular motion was uniform. It is usually designated by M; and, if expressed in *degrees*, $M = 360°\frac{t}{T}$.

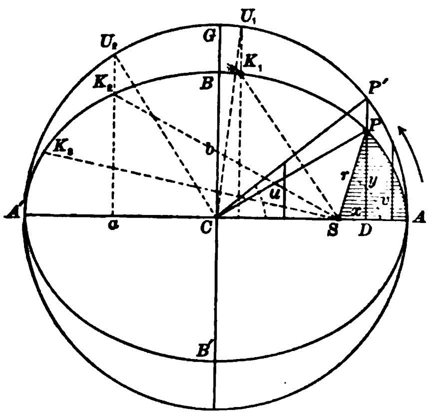

FIG. 241.—Kepler's Problem.

We have therefore

$$\text{Shaded Sector } ASP = \tfrac{1}{2}ab \times M. \qquad (1)$$

Circumscribe a circle around the ellipse, through P draw the ordinate DPP', and join CP and CP'. The angle ACP' is called the *Eccentric Anomaly*, and is usually designated by u, and when determined it gives the means of easily calculating v and r.

The shaded sector is the difference between the elliptical sector ACP and the triangle SCP.

The sector ACP is less than the circular sector ACP', which equals $\frac{1}{2}a^2u$, in the ratio of b to a, since from the properties of the ellipse and the construction of the figure every ordinate in the elliptical sector is shorter than the corresponding ordinate of the circular sector in the ratio of PD to $P'D$, or of BC to GC. Hence, we have

$$\text{Sector } ACP = \frac{b}{a} \times \tfrac{1}{2}a^2u = \tfrac{1}{2}abu. \tag{2}$$

In the triangle SCP, which equals $\frac{1}{2}PD \times SC$, we have

$$PD = \frac{b}{a} \times P'D = \frac{b}{a} \times a \sin u = b \sin u;$$

and

$$SC = ae.$$

Therefore, for the area of the triangle, we have

$$SCP = \tfrac{1}{2}abe \sin u. \tag{3}$$

Subtracting this from (2), we get

$$ASP = \tfrac{1}{2}ab(u - e \sin u). \tag{4}$$

Comparing this with (1), we have the equation sought, and known as Kepler's equation, viz.,

$$u - e \sin u = M.$$

This is a "transcendental" equation, and cannot be solved by ordinary trigonometrical formulæ, but the value of u can easily be found by approximation when M is given. We start with an approximate value for u, ascertain how nearly it satisfies the equation, and then correct the assumed value so as to diminish the error, repeating the process until a satisfactory value is obtained. There are also various formulæ which give the value of u in rapidly converging series, some of them adapted to orbits of small eccentricity like those of the planets, and others to orbits nearly parabolic.

When u is found, v and r are readily determined from the triangle PDS, in which $PD = b \sin u$, and

$$SD (= CD - CS) = a \cos u - ae = a(\cos u - e).$$

Hence, $\tan v = \frac{b}{a} \cdot \frac{\sin u}{\cos u - e}$, which can be reduced to the more convenient formula $\tan \frac{1}{2}v = \sqrt{\frac{1+e}{1-e}} \times \tan \frac{1}{2}u$. For r we have

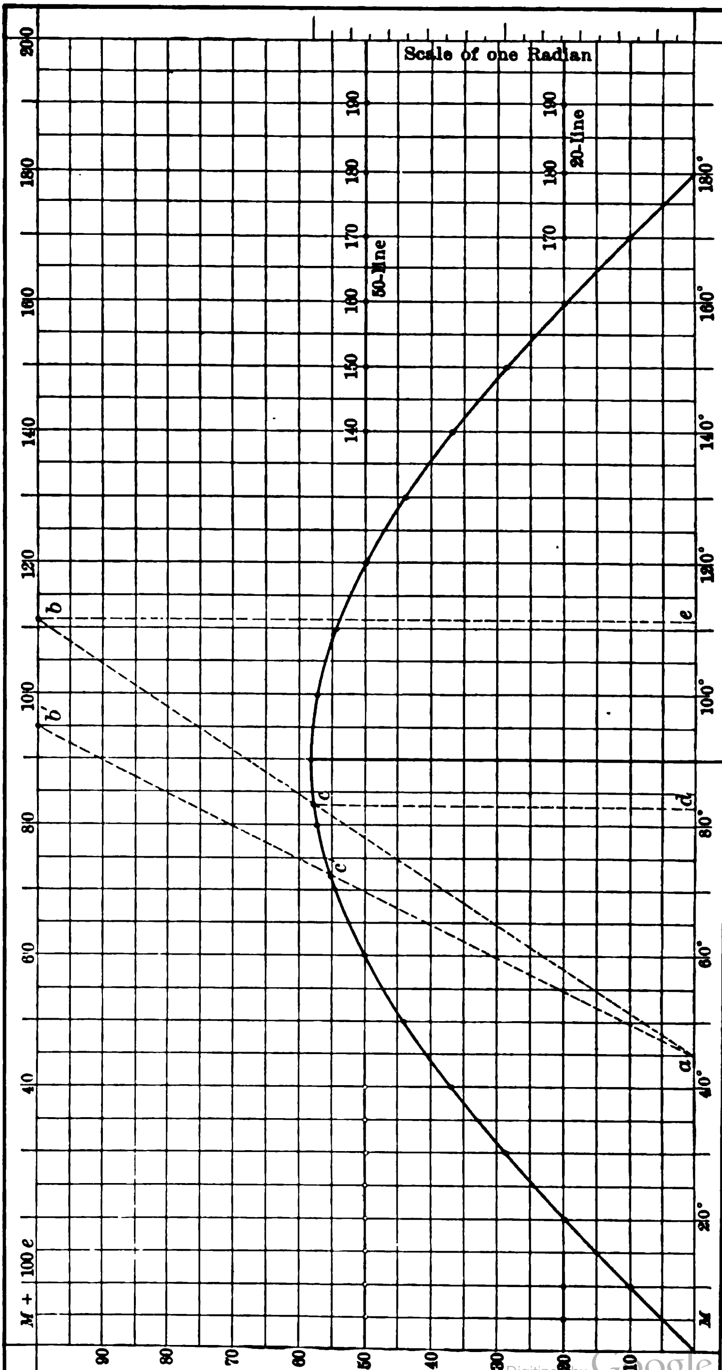

FIG. 242.

$r = \dfrac{PD}{\sin v} = b\dfrac{\sin u}{\sin v}$; or we may use the equation $r = \sqrt{PD^2 + SD^2}$. Substituting the values of PD and SD given above and reducing, we get $r = a - ae \cos u$. Still again we may use the general equation of the conic (Art. 423), $r = \dfrac{p}{1 + e \cos v}$.

1002*. Graphical Solution for u. — An approximate value of u is easily obtained by means of the *curve of sines*, as shown in Fig. 242. If from any point c on the curve a line be drawn at such a slope that $ad \div cd$ equals the eccentricity of the orbit in question (*i.e.* if $\tan acd = e$), then $ad = e \times dc = e \sin (c)$, (c) being the angle indicated by the position of c and d ($82\frac{1}{2}°$ in the case shown in the figure), and $Ma = (c) - e \times \sin (c)$, *i.e.* (c) *is the eccentric anomaly corresponding to the mean anomaly* Ma in an orbit having e for its eccentricity. To facilitate the construction of the inclined line at the proper slope, the upper horizontal line, marked $M + 100\,e$, is drawn at the distance of 100 units from the base line M, upon which the degrees are marked. To find the *eccentric anomaly* corresponding to a given *mean anomaly*, it is therefore only necessary to mark the point a, corresponding to the given mean anomaly (supposed to be 45° in the figure), and on the upper line the point b, corresponding to $M + 100\,e$, which is 111.1 in the illustration, the eccentricity being taken as 0.661 from the example given in the next article. Join a and b, and the point c gives the corresponding *eccentric anomaly*. (Generally it will be best to use a fine thread to join a and b, rather than to trust an ordinary ruler.) Of course the joining line need not be actually drawn, as we want only its point of intersection with the curve. If the mean anomaly had been 90° instead of 45°, b would of course have been 156.1, the eccentricity remaining unchanged. If the ellipse had an eccentricity of 0.50 exactly, then for a *mean anomaly* of 45° the point on the upper line would be at 95 (b' in the figure) and then u would be at c', *i.e.* $72\frac{1}{4}°$. When the mean anomaly exceeds 100°, it may be found impossible to lay off $M + 100\,e$ as directed. In that case $M + 50\,e$ may be laid off on the "fifty-line"; or, if M is near 180°, $M + 20\,e$, laid off on the "twenty-line," will answer the same purpose.

Owing to slight imperfections in the diagram, due to inaccuracies in the drawing, in the ruling of the squares, unequal shrinkage of

the paper, etc., the values of u obtained from it are only approximate, but can generally be relied on to within about $\frac{1}{4}°$. This is near enough for many purposes (double-star orbits, for instance), and is always sufficient as the starting-point for a numerical calculation.

1003. Examples.—1. Given the orbit of a comet, with *semi-major axis* equal to *four* astronomical units, and *eccentricity*, 0.66144 (or, what is the same thing, $b = \frac{3}{4} a$, as drawn in Fig. 241). Required the place of the comet in its orbit *one year* after perihelion passage. Since $a = 4$, the period $= 4^{\frac{3}{2}} =$ eight years. M is therefore one-eighth of the circumference, or 45°, and Kepler's equation becomes $45° = u - 0.66144 \times \sin u$. Since, in the tables, $\sin u$ is given in *radians*, it will be necessary to reduce the term containing it to *degrees* in solving the equation: this may be done by multiplying the term by $\frac{360°}{\pi}$, or 57°.2958, which gives 1.57861 as the logarithm of the coefficient of the $\sin u$. We then have $45° = u° - [1.57861]° \sin u$ as the equation to be solved.

We get the first approximation from the sine-curve, as shown in the preceding article, and find $u_1 = 82° \, 30'$; we then proceed to test it as follows:

	sin 82° 30′.	9.99627
	log of coeff. . . .	1.57861
	37°.573.	1.57488
	82°.500	
Difference	44°.927	(instead of 45°).

This value of u is not quite large enough and must be increased: it will be noticed that, as the angle is very near 90°, its sine will change very slowly, and the term, $[1.57861]° \sin u$, will be only very slightly altered by any small change in u. We must therefore increase u by only a very little more than the difference, 0°.073, between 44°.927 and 45°. We assume accordingly for a second approximation $u_2 = 82°.58$ or $82° \, 34'.8$. We then have:

	sin 82° 34′.8	9.99635
	coeff.	1.57861
	37°.580	1.57496
	82°.580	
Difference	45°.000,	

and this satisfies the equation exactly, so far as can be determined with five-place logarithms. When great precision is required, it is

necessary, of course, to use seven places. In this example the exact value, so computed, is $82°.58042 = 82° 34' 49''.5$.

To get v, we use the formula for $\frac{1}{2}v$.

$1 + e = 1.66144$	log	0.22049
$1 - e = 0.33856$		9.52962
		$2\overline{)0.69087}$
$\sqrt{\dfrac{1+e}{1-e}}$	log	0.34543
$\frac{1}{2}u = 41° 17'.4$	tang	9.94360
	tang $\frac{1}{2}v$	0.28903
$\frac{1}{2}v = 62° 47'.8.$	Hence, $v = 125° 35'.6.$	

For r, we use the formula $r = a - ae \cos u$.

$a = 4$	log 0.60206
$e = 0.66144$	log 9.82049
$\cos 82° 34'.8$	9.11106
$ae \cos u = 0.3417$	9.53361
$a = 4.0000$	
$r = 3.6583.$	

2. In the same orbit, let $t = 2$ years. Find u, v, and r.

Ans. $u = 122° 06'.2$
$v = 151° 57'.7$
$r = 5.4061.$

3. Let $t = 3$ years in same orbit.

Ans. $u = 152° 30'.0$
$v = 167° 33'.6$
$r = 6.3468.$

1004. Projection of a Lunar Eclipse. (*Supplementary to Art. 378.*)—We take as an example the eclipse of Sept. 3, 1895, for which we find the following data in the "American Ephemeris," p. 414:

Greenwich mean time of opposition in right ascension, Sept. 3, $17^h 47^m 46^s.6$.

Sun's declination	$+ 7° 17' 2''.5$	Moon's declination	$- 7° 25' 54''.8$
" hourly motion in decl.	$- 55''.3$	" hourly motion in decl.	$+ 13' 44''.6$
" " " " R.A.	$9^s.04$	" " " " R.A.	$105^s.82$
" semi-diameter	$15' 52''.1$	" semi-diameter	$14' 41''.8$
" horizontal parallax	$8''.5$	" horizontal parallax	$53' 58''.4$

A convenient scale is 1000" to the inch: this will bring the diagram within the limits of an 8 by 10 sheet of paper and is large enough to give all the accuracy that is required. Fractions of a second are of course neglected.

I. The first step is to *lay off the "relative orbit" of the moon with respect to the shadow.* Draw two lines accurately perpendicular to each other, their point of crossing O (Fig. 243) being the position of the moon's centre at the given moment of opposition.

(a) On the horizontal line EW lay off the difference of the hourly motions of the sun and moon in right ascension, in seconds

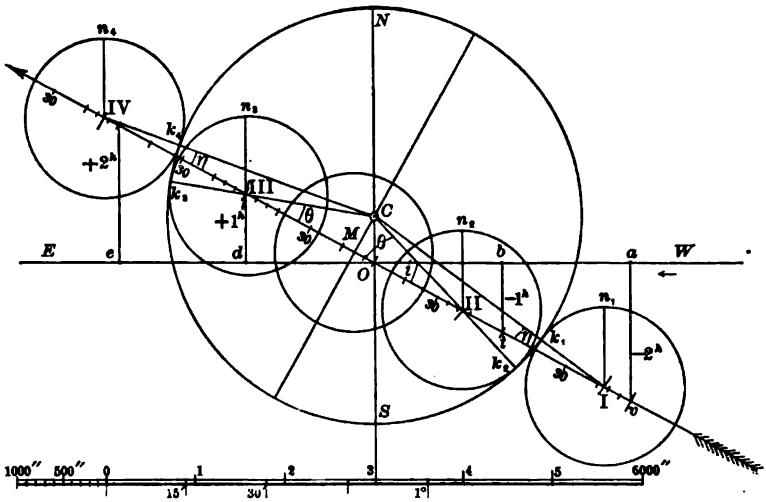

FIG. 243.

of arc, *reduced to seconds of a great circle by multiplying by the cosine of the moon's declination.* In this case we have

$$(105.82 - 9.04) \times 15 \times \cos 7° 25'.9$$
$$= 96.78 \times 15 \times \cos 7° 25'.9 = 1439''.5.$$

Ob and Od are each laid off with this value, while Oa and Oe are twice as great.

(b) At b and d lay off the difference of the hourly motions of declination, remembering that the centre of the shadow moves *north* when the sun moves *south.* We have in this case

$$13' 44''.6 - 55''.3 = 824'.6 - 55''.3 = 769''.3.$$

(This requires no reduction, since declination is measured on the hour-circle.) We lay off there the ordinates at b and d, each equal to 769.3, and those at a and e are twice as great. Since the moon is moving *northwards,* the ordinates to the west (right) of O are laid off downwards, and those on the east side upwards.

(*c*) If the work has been properly done, the four points thus determined will all lie precisely on a straight line passing through O, and will be the points occupied by the moon's centre, exactly one hour and two hours before and after the moment of opposition. The line is to be divided to mark the half and quarter hours, and when afterwards found necessary, the fifteen-minute spaces can be divided into three five-minute spaces.

II. *Mark the centre of the shadow.* Lay off a distance OC north or south of O, equal to the difference between the declinations of the sun and moon; in this case

$$(7^\circ\ 25'\ 54''.8 - 7^\circ\ 17'\ 2''.5) = 8'\ 52''.3 = 532''.3.$$

It is laid off to the *north* of O because the centre of the shadow (opposite the sun) has a smaller southern declination than the moon.

III. *Draw the shadow.* Its radius (Art. 372) is $(P + p - s)\,\frac{61}{60}$; or, in this case,

$$\tfrac{61}{60}\,(53'\ 58''.4 + 8''.5 - 15'\ 52''.1) = \tfrac{61}{60}\,(2294''.8) = 2333''.$$

With C as a centre and this radius, describe the large circle which represents the shadow. (It is not necessary actually to draw the shadow circle, but it is usual to do so: the *radius of the shadow* is needed for the next step.)

IV. *Mark the points on the relative orbit occupied by the moon's centre at the moments of contact with the shadow.* (*a*) To the radius of the shadow *add* the semi-diameter of the moon (in this case, $2333'' + 882'' = 3215''$), and with this distance as a radius, from the centre C strike two arcs cutting the relative orbit at I and IV, which will be the position of the moon's centre at the first and last (external) contacts.

(*b*) Subtract the moon's semi-diameter from the radius of the shadow, and with this difference ($2333'' - 882'' = 1451''$) as a radius from C find the points II and III, of internal contact. The figure may be completed by drawing the circles to represent the moon (radius $882''$), using I, II, M, III, and IV as centres. But it is not necessary. M, the middle of the eclipse, is half-way between II and III.

V. Finally, *read off the times of the contact on the relative orbit regarded as a scale of time.* For contacts I and II subtract the

reading from the time of opposition ($17^h\ 47^m.8$ in this case); for III and IV, add. In the present case the results come out as follows:

I.	II.	MIDDLE.	III.	IV.
h. m.	h. m.	h. m.	h. m.	h. m.
−1 47.5	−0 41.5	+0 9.5	+1 00.0	+2 7.0
17 47.8	17 47.8	17 47.8	17 47.8	17 47.8
16 00.3	17 06.3	17 57.3	18 47.8	19 54.8
(15 59.9) G.M.T.	(17 06.4)	(17 57.0)	(18 47.5)	(19 53.9)

The figures in parentheses are the calculated results given in the almanac. To get Eastern Standard time, subtract 5 hours.

1005. Calculation of a Lunar Eclipse. — I. *Preparation of data.*

(*a*) Moon's motion in R.A.	$105^s.82$	
Sun's " " "	$9^s.04$	
	96.78	
× 15 (add half and multiply by 10)	48.39	
	$1451''.7$	log 3.16188
cos 7° 25′.9		9.99633
$Ob = 1439''.5$		3.15821

(*b*) $bt = 13'\ 44''.6 - 55''.3 = 12'\ 49''.3 = 769''.3$

(*c*) $OC = 7°\ 25'\ 54''.8 - 7°\ 17'\ 2''.5 = 8'\ 52''.3 = 532''.3$

(*d*) Radius of shadow $= (P + p - S)$

P	$53'\ 58''.4$
p	$8''.5$
	$54'\ 06''.9$
S	$15'\ 52''.1$
	$38'\ 14''.8$
add $\frac{1}{60}$	$38''.2$
	$r = 38'\ 53''.0 = 2333''.0$

(*e*) Semi-diameter of moon $= s = 14'\ 41''.8 = 881''.8$

(*f*) $\begin{cases} \rho = C, \text{I} = (r + s) = 3214''.8 \\ \rho' = C, \text{II} = (r - s) = 1451''.2 \end{cases}$

(*g*) Time of opposition $= 17^h\ 47^m\ 46^s.6 = 17^h\ 47^m.78$

II. *Calculation of relative orbit (triangle Obt) to find angle bOt (i), and hourly orbital motion Ot.*

(*a*) $\text{Tang}\ i = \dfrac{bt}{Ob}$	$769''.3$	log 2.88610
	$1439''.5$	3.15821
$i = 28°\ 07'.3$		tang 9.72789
(*b*) $Ot = \dfrac{Ob}{\cos i}$		3.15821
	cos 28° 07′.3	9.94544
Hourly motion (only the logarithm is needed)		3.21277

III. *Calculation of CM and OM (triangle OCM).*

(*a*) $CM = OC \cos i$	$532''.3$	log 2.72616
	$\cos i$	9.94544
	$CM = 469''.6$	2.67160

(*b*) $OM = OC \sin i$	532''.3	2.72616
	$\sin i$	9.67335
Log OM (seconds of arc)		2.39951
To reduce to time, divide by hourly motion		3.21277
OM (time) = $0^h.1537$		9.18674
= $0^h\ 09^m.22$		
Time of opposition = $17^h\ 47^m.78$		
$17^h\ 57^m.00$ = middle of eclipse.		

IV. *Calculation of* M,I $(= M,\mathrm{IV})$, *and angle* η, *and of time of first and last contacts (triangle* $MC\mathrm{I}$).

(*a*) $\operatorname{Sin} \eta = \dfrac{CM}{C,\mathrm{I}} = \dfrac{469''.6}{3214''.8}$		2.67160
		3.50715
$\eta = 8°\ 23'.8$	$\sin \eta$	9.16445
$M,\mathrm{I} = C,\mathrm{I} \cos \eta$		3.50715
$\cos 8°\ 23'.8$		9.99533
(*b*) Log M,I (seconds of arc)		3.50248
Divide by hourly motion		3.21277
M,I (time) = $1^h.9485 = 1^h\ 56^m.91$		0.28971

Middle	$17^h\ 57^m.00$	$17^h\ 57^m.00$
	$-\ 1^h\ 56^m.91$	$+\ 1^h\ 56^m.91$
(I)	$16^h\ 00^m.09$	$19^h\ 53^m.91$ (IV)

V. *Calculation of* M,II $(= M,\mathrm{III})$ *and of angle* θ, *and of time of the two internal contacts (triangle* $MC\mathrm{II}$).

(*a*) $\operatorname{Sin} \theta = \dfrac{CM}{C,\mathrm{II}} = \dfrac{469''.6}{1451''.2}$		2.67160
		3.16173
$\theta = 18°\ 52'.5$	$\sin \theta$	9.50987
(*b*) $M,\mathrm{II} = C,\mathrm{II} \times \cos \theta$	$\cos 18°\ 52'.5$	9.97600
		3.16173
M,II (seconds of arc)		3.13773
Divide by orbital hourly motion		3.21277
M,II (time) = $0^h.841 = 50^m.46$		9.92496

Middle	$17^h\ 57^m.00$	$17^h\ 57^m.00$
	$-\ 50^m.46$	$+\ 50^m.46$
(II)	$17^h\ 06^m.54$	$18^h\ 47^m.46$ (III)

VI. The angles η and θ determine the arcs of the moon's limb intercepted between the north point of the limb and the point of contact.

For the 1st contact the arc $n_1k_1 = (90° - i) - \eta = 53°\ 29'$; for the 2d, the arc $n_2k_2 = (90° + i) + \theta = 137°$; for the 3d contact, $n_3k_3 = (90° - i) + \theta = 80°\ 45'$; for the 4th we have $n_4k_4 = (90° + i) - \eta = 109°\ 44'$. The 1st and 3d are reckoned from the north towards the east, the 2d and 4th towards the west.

Attention is called to the fact that the assumption of a uniform motion of the moon during the four hours involved in the calculation is not correct and would not be permissible if the phenomena of the eclipse could be precisely observed. Since, however, it is impossible to be sure of the tenths or even quarters of a minute in observation, the method of calculation is abundantly accurate for its purpose.

1006. Proof that the Orbit described under the Law of Gravitation is a Focal Conic. (*Supplementary to Arts. 421, 424.*) — (The demonstration that follows is substantially the same as one given in Williamson's "Treatise on Dynamics.")

1. *General differential equations of the motion.* Let the particle P (Fig. 244) be urged towards O along r by a force F, making the angle θ with OX, the axis of X. The force can be resolved into two components along the axes of X and Y, viz., $F\cos\theta$ and $F\sin\theta$, or $F\frac{x}{r}$ and $F\frac{y}{r}$. Hence, the accelerations along the axes will be given by the equations $\frac{d^2x}{dt^2} = -F\frac{x}{r}$, and $\frac{d^2y}{dt^2} = -F\frac{y}{r}$, the minus sign being used because the force F tends to diminish both x and y.

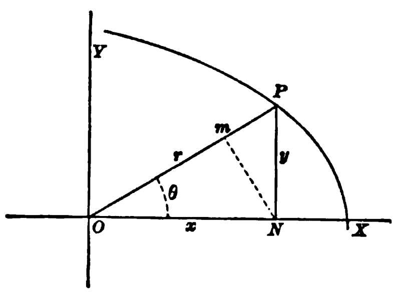

FIG. 244

If now, according to the law of gravitation, $F = \frac{\mu}{r^2}$, the equations become $\frac{d^2x}{dt^2} = -\frac{\mu x}{r^3}$ and $\frac{d^2y}{dt^2} = -\frac{\mu y}{r^3}$ (1). The integration of these equations will give the law of motion and the nature of the orbit.

2. *Equable description of areas.* Multiply the two equations (1) by y and x, respectively, and subtract the first product from the second: we get $x\frac{d^2y}{dt^2} - y\frac{d^2x}{dt^2} = 0$, or $\frac{d}{dt}\left(x\frac{dy}{dt} - y\frac{dx}{dt}\right) = 0$. Hence, by integrating, $x\frac{dy}{dt} - y\frac{dx}{dt} = h$, (2); h being the constant of integration and independent of the time.

If in (2) we substitute $r\cos\theta$ for x, and $r\sin\theta$ for y, and perform the necessary reductions, we get the corresponding polar differential equation $r^2\frac{d\theta}{dt}=h$ (3).

The left-hand members of both (2) and (3) are the well-known expressions for twice the increment of the area of the sector of the curve, corresponding to consecutive values of x and y, or of r and θ; so that the equations prove that this increment of area in a unit of time is constant, and constitute an analytical demonstration of the principle proved geometrically in Arts. 402–405.

From (3) we have also $\frac{1}{r^2}=\frac{1}{h}\frac{d\theta}{dt}$ (4).

3. *Nature of the orbit.* Substitute in equations (1) this value of $\frac{1}{r^2}$, from (4), and we get $\frac{d^2x}{dt^2}=-\frac{\mu}{h}\frac{x}{r}\frac{d\theta}{dt}=-\frac{\mu}{h}\cos\theta\frac{d\theta}{dt}$, and $\frac{d^2y}{dt^2}=-\frac{\mu}{h}\frac{y}{r}\frac{d\theta}{dt}=-\frac{\mu}{h}\sin\theta\frac{d\theta}{dt}$ (5).

Integrating, we have $\frac{dx}{dt}=-\frac{\mu}{h}\sin\theta+\alpha$, and $\frac{dy}{dt}=+\frac{\mu}{h}\cos\theta+\beta$ (6), α and β being the constants of integration, depending upon the initial conditions of motion at P.

4. Substitute these values of $\frac{dx}{dt}$ and $\frac{dy}{dt}$ in equation (2), and we have

$$x\left(\frac{\mu}{h}\cos\theta+\beta\right)+y\left(\frac{\mu}{h}\sin\theta+\alpha\right)=h,$$

or

$$\frac{\mu}{h}\left(x\cos\theta+y\sin\theta\right)+\beta x+\alpha y-h=0.$$

But $(x\cos\theta+y\sin\theta)=Om+mP$ (Fig. 244) $=r$, so that finally this equation of the gravitational orbit becomes

$$r+\frac{h}{\mu}\beta x+\frac{h}{\mu}\alpha y-\frac{h^2}{\mu}=0. \qquad (7)$$

This is a form of the general equation of a conic, with the origin at the focus. The coefficients of x and y depend upon the eccentricity of the curve, and the angle between its major axis and the axis of abscissas, while the absolute term is the semi-parameter, usually designated by p, or $a(1-e^2)$.

From (7) we see that in the gravitational orbit $p=\frac{h^2}{\mu}$, or $h=\sqrt{\mu p}$ (8); and since μ in the solar system is the mass of the *sun*, we have h *proportional to the*

***square root of the parameter of the orbit*, in accordance with the form of Kepler's third law given at the end of Art. 423.**

5. *Transformation of equation* (7) *to the normal polar form.* Put $\frac{h^2}{\mu} = p$, and make the coefficients of x and y, respectively, equal to $e \cos \psi$ and $e \sin \psi$: also for x write $r \cos \theta$, and for y, $r \sin \theta$, θ being the vectorial angle. The equation becomes

$$r + e \cos \psi \times r \cos \theta + e \sin \psi \times r \sin \theta - p = O,$$

or
$$r(1 + e \cos \theta \cos \psi + e \sin \theta \sin \psi) = p,$$

or
$$r(1 + e \cos [\theta - \psi]) = p,$$

or, finally,
$$r = \frac{p}{1 + e \cos(\theta - \psi)};$$

which is of the normal form, e being the eccentricity of the conic, while the major axis of the conic makes the angle ψ with the axis of abscissas, so that $(\theta - \psi)$ is the anomaly, v, in the equation as usually written, $r = \frac{p}{1 + e \cos v}$.

1007. Expression for the Velocity at any Point of the Orbit. (*Supplementary to Art. 428.*)—Suppose the orbit elliptical, and that the shaded sector in Fig. 245 has been described in a unit of time (say one second); then the area of this sector is the planet's *areal-velocity* $\frac{h}{2}$, and the short base of the sector (at P) is its linear velocity V. At P draw the tangent MPN, and from the two foci S and F draw the two perpendiculars to it, f and f'. Also join PF, or r'. The area of the sector equals $\frac{V \times f}{2} = \frac{h}{2}$, so that $V^2 = \frac{h^2}{f^2}$. But by equation (8) of the preceding article $h^2 = \mu p$; and, by the properties of the ellipse, $p = \frac{b^2}{a}$. We have, therefore, $V^2 = \frac{\mu b^2}{f^2 a}$.

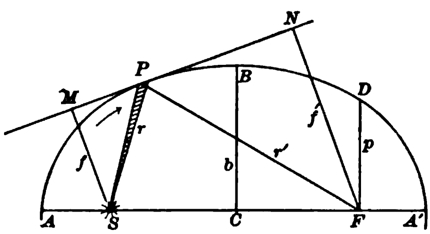

FIG. 245.

Again, from the properties of the ellipse, $ff' = b^2$; $\frac{f'}{f} = \frac{r'}{r}$ (similar triangles); and $(r + r') = 2a$.

Hence, $V^2 = \frac{\mu \times ff'}{f^2 a} = \frac{\mu f'}{af} = \frac{\mu}{a}\frac{r'}{r} = \frac{\mu}{a}\frac{(2a - r)}{r} = \frac{2\mu}{r} - \frac{\mu}{a}$, or, finally, $V^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)$, as given in Art. 428.

A corresponding demonstration holds for the hyperbola, bearing in mind that in that conic a and b^2 are both negative. The final equation is the same: in either case $\frac{\mu}{a} = \frac{2\mu}{r} - V^2$, and a is positive (ellipse) when $V^2 < \frac{2\mu}{r}$; becomes negative (hyperbola) when $V > \frac{2\mu}{r}$; and is infinite (parabola) when $V^2 = \frac{2\mu}{r}$.

1008. Proof that the "Parabolic Velocity," U, equals $\sqrt{\frac{2\mu}{r}}$.

(*Supplementary to Art. 429.*) — When the attracting force $= \frac{\mu}{r^2}$, we have for the acceleration of fall towards the attracting body $\frac{dv}{dt} = -\frac{\mu}{r^2}$. But $v = \frac{dr}{dt}$. Multiplying, $\frac{vdv}{dt} = -\left(\frac{\mu}{r^2}\right)\frac{dr}{dt}$. Integrating we get, $\frac{v^2}{2} = \frac{\mu}{r} + C$. If $v = 0$ when $r = s$, then $C = -\frac{\mu}{s}$; whence we have, $v^2 = 2\mu\left(\frac{1}{r} - \frac{1}{s}\right)$. If we make s infinite in this expression, v becomes the "velocity from infinity," or "parabolic velocity," denoted by U; and we have, therefore, $U^2 = \frac{2\mu}{r}$, or $U = \sqrt{\frac{2\mu}{r}}$.

1009. (*Supplementary to Art. 493.*) — Fig. 246 shows how the combination of the earth's motion with that of a planet produces an epicycloid as the relative (apparent) path of the planet. The earth's orbit is represented by the smaller circle, upon which are marked eight points, O–VII, occupied at eight equidistant times. AB is part of the circular orbit of a second planet with a period of twelve years (nearly the case of Jupiter), the points marked 0_1, 0_2, and 0_3 being those occupied by the planet when the earth is at O.

In the same way the points 1_1, 1_2, and 1_3 correspond to I in the earth's orbit, and similarly for the other points marked 2, 3, etc. At the start the earth is supposed to be at O, and the planet at 0_1. Around 0_1 as a centre draw a circle equal to the earth's orbit, and draw the radius $0_1O'$ parallel to SO. Then the line SO' will be parallel and equal to $O0_1$, so that 0_1 *has the same distance and direction from* S *as* 0_1 *has from* O: in other words, if the earth

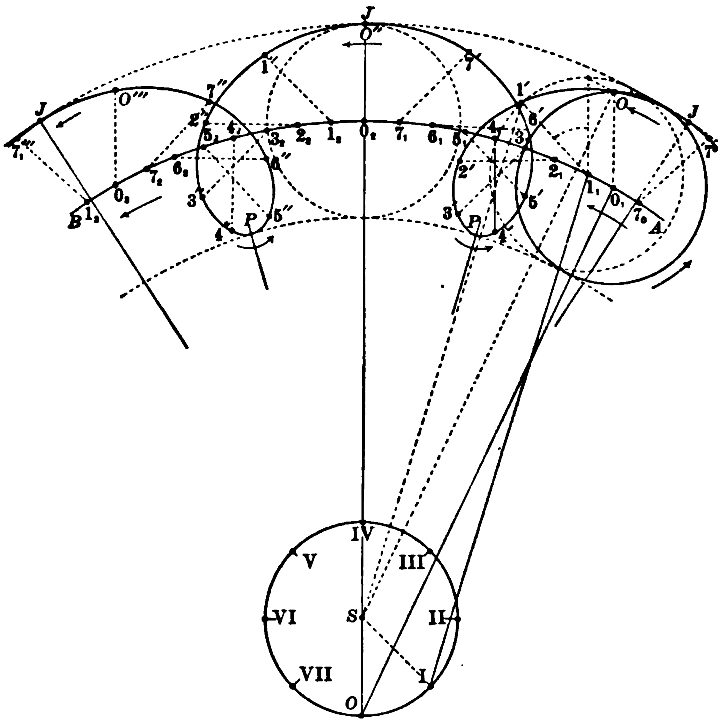

FIG. 246.

were transferred to S, O' would occupy precisely the same position in the celestial sphere that the planet actually does as seen from O. In an eighth of a year the earth will have moved to I, and the planet to 1_1: the line $\mathrm{I}1_1$ will represent the new direction and distance of the planet, and, repeating the same construction as before (*i.e.* drawing from 1_1 a line $1_11'$ parallel and equal to the radius $S\mathrm{I}$), we find $1'$ as the point which, seen from S, would hold the same relative position that 1_1 does with respect to I. Now a

glance at the figure shows that this point 1′ is on the circumference of a circle precisely like the one first drawn, but with its centre moved to 1_1, and is at the extremity of a radius inclined 45° to the original radius $0_1O'$, while the curved line $O'1'$ is the line along which the planet would have *appeared* to move, if the earth were regarded as stationary at *S*. Carrying out the construction for the successive positions of the earth and the planet, we find the points 2′, 3′, 4′, etc., for the apparent "geocentric" positions of the planet, and the looped curve is its apparent geocentric path. The points *P*, where the planet's distance from the earth is least, *and the apparent motion retrograde*, correspond to *opposition*, when planet and earth are on the same side of the sun. The points of maximum distance, marked J, are those of conjunction, when the earth and planet are on opposite sides.

THE GREEK ALPHABET.

Letters.	Name.	Letters.	Name.	Letters.	Name.
Α, α,	Alpha.	Ι, ι,	Iota.	Ρ, ρ ϱ,	Rho.
Β, β,	Beta.	Κ, κ,	Kappa.	Σ, σ ς,	Sigma.
Γ, γ,	Gamma.	Λ, λ,	Lambda.	Τ, τ,	Tau.
Δ, δ,	Delta.	Μ, μ,	Mu.	Υ, υ,	Upsilon
Ε, ε,	Epsilon.	Ν, ν,	Nu.	Φ, φ,	Phi.
Ζ, ζ,	Zeta.	Ξ, ξ,	Xi.	Χ, χ,	Chi.
Η, η,	Eta.	Ο, ο,	Omicron.	Ψ, ψ,	Psi.
Θ, θ ϑ,	Theta.	Π, π ϖ,	Pi.	Ω, ω,	Omega.

MISCELLANEOUS SYMBOLS.

☌, Conjunction.
□, Quadrature.
☍, Opposition.
☊, Ascending Node.
☋, Descending Node.

A.R., or α, Right Ascension.
Decl., or δ, Declination.
λ, Longitude (Celestial).
β, Latitude (Celestial).
ϕ, Latitude (Terrestrial).

ω, Angle between line of nodes and line of apsides. Also obliquity of the ecliptic.

DIMENSIONS OF THE TERRESTRIAL SPHEROID.

(According to Clarke's Spheroid of 1878. For the spheroid of 1866, see Art. 145.)

Equatorial semidiameter, —

20 926 202 feet = 3963.296 miles = 6 378 190 metres.

Polar semidiameter, —

20 854 895 feet = 3949.790 miles = 6 356 456 metres.

Oblateness (Clarke), $\frac{1}{293.46}$; (Harkness), $\frac{1}{300}$.

Length (in metres) of 1° of meridian in lat. $\phi = 111\,132.09 - 556.05 \cos 2\phi + 1.20 \cos 4\phi$.

Length (in metres) of 1° of parallel, in lat. $\phi = 111\,415.10 \cos\phi - 94.54 \cos 3\phi$.

1° of lat. at pole = 111 699.3 metres = 69.407 miles.
1° of lat. at equator = 110 567.2 metres = 68.704 miles.

These formulæ correspond to the Clarke Spheroid of 1866, used by the U.S. Coast and Geodetic Survey.

TIME CONSTANTS.

The sidereal day = $23^h\ 56^m\ 4^s.090$ of mean solar time.
The mean solar day = $24^h\ 3^m\ 56^s.556$ of sidereal time.

To reduce a time-interval expressed in units of *mean solar time* to *units of sidereal time*, multiply by 1.00273791; Log. of 0.00273791 = [7.4374191].

To reduce a time-interval expressed in units of *sidereal time* to units of *mean solar time*, multiply by 0.99726957 = (1 − 0.00273043); Log. 0.00273043 = [7.4362316].

Tropical year (Newcomb, reduced to 1900),	365^d	5^h	48^m	$45^s.98$.
Sidereal year " " "	365	6	9	8.97.
Anomalistic year " " "	365	6	13	48.09.

Mean synodical month (Neison),	29^d	12^h	44^m	$2^s.864$.
Sidereal month, ...	27	7	43	11.545.
Tropical month (equinox to equinox), .	27	7	43	4.68.
Anomalistic month (perigee to perigee), .	27	13	18	37.44.
Nodical month (node to node), . .	27	5	5	35.81.

Obliquity of the ecliptic (Newcomb),
$23° 27' 8''.26 - 0''.468 (t - 1900)$.
Constant of precession (Newcomb), $50''.248 + 0.000222 (t - 1900)$.
Constant of nutation (Paris Conference, 1896), $9''.21$.
Constant of aberration (Paris Conference, 1896), $20''.47$.
Solar parallax (Paris Conference, 1896), $8''.80$.
Velocity of light (Michelson and Newcomb),
186330 miles, 299860 km.

TABLE I.—PRINCIPAL ELEMENTS OF THE SOLAR SYSTEM.

	NAME.	SYMBOL.	Semi-Major Axis of Orbit.	Mean Dist. Millions of Miles.	Sidereal Period (mean solar days).	Period in Years.	Orbit-Velocity (miles per second).	Eccentricity.	Inclination to Ecliptic.	Longitude of Ascending Node.	Longitude of Perihelion.	Longitude at Epoch, Jan. 1, 1850.
Terrestrial Planets.	Mercury . . .	☿	0.387099	36.0	87.96926	0.24	23 to 35	.20560	7° 00′ 8″	46° 33′ 9″	75° 7′ 14″	327° 15′ 20″
	Venus	♀	0.723332	67.2	224.7008	0.62	21.9	.00684	3 23 35	75 19 52	129 27 15	245 33 15
	The Earth . .	⊕	1.000000	92.9	365.2564	1.00	18.5	.01677	0 00 00	172 37 40	100 21 22	100 46 44
	Mars	♂	1.523691	141.5	686.9505	1.88	15.0	.09326	1 51 2	48 23 53	333 17 54	83 40 31
	Ceres	(1)	2.767265	257.1	1681.414	4.60	11.1	.07631	10 37 10	80 46 39	149 37 49	103 25 3
	Eros	(433)	1.4581	135.5	643.1	1.76	15.3	.2228	10 49 30	303 29 50	121 8 12	
Major Planets.	Jupiter . . .	♃	5.202800	483.3	4332.580	11.86	8.1	.04825	1 18 41	98 56 17	11 54 58	160 1 10
	Saturn	♄	9.538861	886.0	10759.22	29.46	6.0	.05607	2 29 40	112 20 53	90 6 38	14 52 28
	Uranus . . .	♅	19.18329	1781.9	30686.82	84.02	4.2	.04634	0 46 20	73 13 54	170 50 7	29 17 51
	Neptune . .	♆	30.05508	2791.6	60181.11	164.78	3.4	.00896	1 47 2	130 6 25	45 59 43	334 33 29

	NAME.	SYMBOL.	Apparent Angular Diameter.	Mean Diameter. In Miles.	Mean Diameter. ⊕ = 1.	Mass. ☉ = 1.	Mass. ⊕ = 1.	Volume. ⊕ = 1.	Mean Density. ⊕ = 1.	Mean Density. Water = 1.	Axial Rotation.	Inclination of Equator to Orbit.	Oblateness.	Surface Gravity. ⊕ = 1.	Albedo (Müller).
	Sun . .	☉	32′ 04″ (mean)	866 400	109.4	1.000	332 000	1 300 000	0.25	1.39	$25^d\ 7^h\ 48^m \pm$	7°15′ (to ecliptic)	?	27.65	0.13
	Moon .	☾	31′ 07″ "	2 163	0.273	$\frac{1}{27\,058\,000}$	$\frac{1}{81.5}$	0.0204	0.61	3.39	$27^d\ 7^h\ 43^m$	6° 33′ (mean)	?	$\frac{1}{6}$	
Ter'strial Planets.	Mercury	☿	5″ to 13″	3 030	0.382	$\frac{1}{7\,000\,000}$?	$\frac{1}{21}$	0.056	0.85?	4.7?	88^d	?	?	0.43	0.14
	Venus .	♀	11″ to 67″	7 700	0.972	$\frac{1}{408\,000}$	0.82	0.92	0.89	4.94	225^d	small	?	0.82	0.76
	Earth .	⊕		7 917.6	1.000	$\frac{1}{332\,000}$	1.000	1.000	1.000	5.55	$23^h\ 56^m\ 4^s.09$	23° 27′ 08″	$\frac{1}{293}$	1.00	0.20?
	Mars . .	♂	3″.6 to 25″	4 230	0.534	$\frac{1}{3\,093\,500}$	$\frac{1}{9.33}$	0.152	0.71	3.92	$24^h\ 37^m\ 22^s.67$	24°	$\frac{1}{200}$	0.38	0.22
	Ceres .	(1)	0″.25 to 0″.5	488?	$\frac{1}{16.22}$?		$\frac{1}{7000}$?	$\frac{1}{4270}$?	Assumed same as moon?		?	?	?	$\frac{1}{35}$?	(low)
Major Planets.	Jupiter .	♃	32″ to 50″	86 500	10.92	$\frac{1}{1047.35}$	317.7	1309	0.24	1.32	$9^h\ 55^m \pm$	3° 05′	$\frac{1}{17}$	2.65	0.62
	Saturn .	♄	14″ to 20″	73 000	9.17	$\frac{1}{3501.6}$	94.8	760	0.13	0.72	$10^h\ 14^m \pm$	26° 49′	$\frac{1}{9.2}$	1.18	0.72
	Uranus .	♅	3″.8 to 4″.1	31 900	4.03	$\frac{1}{22\,756}$	14.6	65	0.22	1.22	?	?	$\frac{1}{11}$?	0.90	0.60
	Neptune	♆	2″.7 to 2″.9	34 800	4.39	$\frac{1}{19\,500}$	17.0	85	0.20	1.11	?	?		0.89	0.52

The masses given are substantially those adopted by Newcomb in his "Astronomical Constants" (Washington, 1895).

TABLE II.—THE SATELLITES

	Name.	Discovery.		Dist. in Equatorial Radii of Planet.	Mean Distance in Miles.	Sidereal Period.
	Moon	. . . .		60.27035	238 840	$27^d\ 7^h\ 43^m\ 11^s.5$
						SATELLITES OF
1	Phobos	Hall,	1877	2.771	5 850	$7^h\ 39^m\ 15^s.1$
2	Deimos	"	"	6.921	14 650	1^d 6 17 54.0
						SATELLITES OF
5	Nameless . . .	Barnard,	1892	2.551	112 500	$11^h\ 57^m\ 22^s.6$
1	Io	Galileo,	1610	5.933	261 000	1^d 18 27 33.5
2	Europa	"	"	9.439	415 000	3 13 13 42.1
3	Ganymede . . .	"	"	15.057	664 000	7 3 42 33.4
4	Callisto	"	"	26.486	1 167 000	16 16 32 11.2
6	Nameless . . .	Perrine,	1905	162.92	7 185 000	253.4
7	Nameless . . .	"	"	167.86	7 403 000	265.0
						SATELLITES OF
1	Mimas	W. Herschel,	1789	3.11	117 000	$22^h\ 37^m\ 5^s.7$
2	Enceladus . . .	" "	"	3.98	157 000	1^d 8 53 6.9
3	Tethys	J. D. Cassini,	1684	4.95	186 000	1 21 18 25.6
4	Dione	" "	"	6.34	238 000	2 17 41 9.3
5	Rhea	" "	1672	8.86	332 000	4 12 25 11.6
6	Titan	Huyghens,	1655	20.48	771 000	15 22 41 23.2
7	Hyperion . . .	G. P. Bond,	1848	25.07	934 000	21 6 39 27.0
8	Iapetus	J. D. Cassini,	1671	59.58	2 225 000	79 7 54 17.1
9	Phœbe	W. Pickering,	1898	213.5	8 000 000	546.5
10	Themis	" "	1905	24.3?	906 000?	20 20
						SATELLITES OF
1	Ariel	Lassell,	1851	7.52	120 000	$2^d\ 12^h\ 29^m\ 21^s.1$
2	Umbriel	"	"	10.46	167 000	4 3 27 37.2
3	Titania	W. Herschel,	1787	17.12	273 000	8 16 56 29.5
4	Oberon	" "	"	22.90	365 000	13 11 7 6.4
						SATELLITE OF
1	Nameless . . .	Lassell,	1846	12.93	221 500	$5^d\ 21^h\ 2^m\ 44^s.2$

OF THE SOLAR SYSTEM.

Synodic Period.	Inc. of Orbit to Ecliptic	Inc. to Plane of Planet's Orbit.	Eccen-tricity.	Diam'r in Miles	Mass in Terms of Primary.	Remarks.
29d 12h 44m 2s.7	5° 08′ 40″	– –	0.05491	2162	$\frac{1}{81.5}$	Specific gravity 3.44.

MARS.

– –	26° 17′.2	28° ±	0	35?	?	Orbits sensibly coincident with planet's equator.
– –	25 47.2	28° ±	0	10?	?	

JUPITER.

	2° 20′ 23″	– –	?	100?	?	
1d 18h 28m 35s.9	2 08 3	– –	0	2500	.00001688	The diameters are Engelmann's. The rest of the data are from Damoiseau.
3 13 17 53.7	1 38 57	– –	0	2100	.00002323	
7 3 59 35.9	1 59 53	– –	.0013	3550	.00008844	
16 18 5 6.9	1 57 00	– –	.0072	2960	.00004248	
		28°.4 } to plane of planet's		100?		
		31°.4 } equator		40?		

SATURN.

Long. of Ascend.	28° 10′ 10″	About 27°.	0	600?	?	The planes of the 5 inner orbits sensibly coincide with the plane of the ring.
Node of orbits	"	Inclination of the	0	800?	?	
on ecliptic for	"	5 inner satellites	0	1200?	?	
1900, 168° 10′ 35″.	"	to plane of celes-	0	1100?	?	
(5 inner satellites	"	tial equator	0	1500?	?	
and ring.)	27 38 49	= 6° 57′ 43″ (1900)	.0299	3500?	$\frac{1}{4600}$	
	27 4.8	– –	.1189	500?	?	Discovered independently by Lassell.
	18 31.5	– –	.0296	2000?	?	
?	5 6	– –	.22	50?	?	On photographs. Retrograde.
	39 00?	– –		30?	?	On photographs.

URANUS.

Long. of Ascend.	97° 51′ = – 82° 09′		0	500?	?	All Retrograde.
Node of orbits on	" "	Inc. to celestial	0	400?	?	
plane of ecliptic	" "	equator 75° 18′	0	1000?	?	
= 165° 32′ (1900).	" "	(1900).	0	800?	?	

NEPTUNE.

Long. Asc. Node, 184° 25′ (1900).	145° 12′ = – 34° 48′	120° 05′ (1900)	0	2000?	?	Retrograde.

TABLE III. — PERIODIC COMETS WHICH HAVE BEEN OBSERVED AT MORE THAN ONE PERIHELION PASSAGE. From the Annuaire du Bureau des Longitudes, 1904 (slightly abridged).

No.	Name.	Perihelion Passage.	Period. (Years.)	Perihelion Distance.	Aphelion Distance.	Eccentricity.	Longitude of Perihelion	Longitude of Asc. Node.	Inclination.
1	Encke	1901, Sept. 16	3.304	0.34163	4.09512	0.84600	158° 47′ 57″	334° 48′ 58″	12° 53′ 38″
2	Tempel	1899, July 29	5.281	1.38853	4.67638	0.54211	306 34 16	120 57 05	12 38 53
3	Brorsen . . .	1890, Feb. 25	5.456	0.58776	5.61038	0.81034	116 23 10	101 27 34	29 23 48
4	Tempel-L. Swift	1903, Jan. 25	5.678	1.15160	5.21390	0.63817	43 48 20	290 12 00	5 26 21
5	Winnecke . . .	1904, Jan. 22	5.828	0.92338	5.55377	0.71488	274 19 45	104 12 36	16 59 55
6	De Vico-E. Swift	1901, Feb. 14	6.400	1.66960	5.22477	0.51566	348 56 56	24 50 39	3 35 17
7	Tempel	1898, Oct. 4	6.538	2.09114	4.90196	0.40194	241 16 04	72 36 05	10 47 14
8	Finlay	1900, Feb. 17	6.556	0.96942	6.03616	0.72324	8 03 58	52 23 07	3 02 54
9	D'Arrest . . .	1897, June 3	6.686	1.32700	5.77140	0.62611	319 26 20	146 25 11	15 42 32
10	Biela (1) . . .	1866, Jan. 26	6.692	0.87915	6.22289	0.75242	109 40 18	245 46 11	12 21 58
11	Wolf	1898, July 5	6.845	1.60304	5.60708	0.55534	19 21 38	206 29 04	25 12 16
12	Holmes . . .	1899, April 29	6.874	2.12810	5.10230	0.41135	345 47 53	331 43 32	20 48 10
13	Brooks	1903, Dec. 7	7.101	1.95892	5.43019	0.46978	1 41 40	18 03 54	6 03 44
14	Faye	1903, June 4	7.390	1.64972	5.93798	0.56516	45 26 48	206 28 00	10 37 30
15	Tuttle	1899, May 5	13.667	1.01913	10.41330	0.82171	116 29 03	269 49 54	54 29 06
16	Pons-Brooks . .	1884, Jan. 26	71.560	0.77573	33.69805	0.95500	93 17 15	254 05 42	74 02 36
17	Olbers	1887, Oct. 9	72.650	1.19912	33.62339	0.93113	149 52 31	84 32 20	44 34 16
18	Halley due	1910, May 17	76.080	0.68710	35.22380	0.96173	168 42 52	57 10 33	162 13 09

TABLE IV. — STELLAR PARALLAXES AND PROPER MOTIONS.

From Oudemans' paper ("Astron. Nach.," Aug. 1889), with some modifications from later sources.

No.	NAME.	α (1900)	δ (1900)	Mag.	Parallax (p).	Weight.	Distance (Light-Years) $\frac{3.26}{p}$.	Proper Motion (μ).	Cross Motion (miles p. sec.) $2.94\frac{\mu}{p}$.
1	α Centauri. . . .	14h 32m.6	− 60° 25′	0.7	0″.75	5	4.4	3″.67	14.4
2	Ll. 21185	10 57.3	+ 36 56	6.9	0.50	3	6.5	4.75	28
3	61 Cygni	21 2.0	+ 38 12	5.1	0.40	6	8.1	5.16	37.9
4	η Herculis	17 9.6	+ 39 8	3.7	0.40 ?	1	8.1	0.08	0.6
5	Sirius	6 40.4	− 16 34	−1.4	0.39	4	8.4	1.39	10.5
6	Σ 2398	18 41.5	+ 59 28	8.2	0.35	2	9.3	2.40	20.3
7	η Cassiopeiæ . . .	0 42.9	+ 57 18	3.6	0.35	3	9.3	1.13	9.5
8	μ Cassiopeiæ . . .	1 01.0	+ 54 23	5.2	0.34	1	9.6	3.75	33.3
9	Groombridge 1618 .	10 4.8	+ 50 1	6.5	0.32	2	10	1.43	13.2
10	ν¹ and ν² Draconis .	17 30.3	+ 55 15	4.9 4.8	0.30	2	10.9	0.16	1.6
11	Groombridge 34. .	0 12.1	+ 42 24	7.9	0.29	2	11.2	2.80	28.4
12	Lac. 9352	22 58.8	− 36 29	7.5	0.28	3	11.6	6.96	73
13	Procyon	7 33.5	+ 5 30	0.5	0.27	4	12.1	1.25	13.6
14	Ll. 21258	11 0.0	+ 44 5	8.5	0.26	4	12.5	4.40	49.8
15	Arg.-Oeltzen 11677.	11 14.4	+ 66 26	9	0.26	2	12.5	3.04	34.4
16	70 Ophiuchi . . .	18 0.4	+ 2 33	4.1	0.25	2	13.2	1.13	13.3
17	σ Draconis . . .	19 32.6	+ 69 28	4.7	0.25	2	13.2	1.84	21.7
18	ε Indi	21 54.9	− 57 14	5.2	0.20	3	16.3	4.60	67.7
19	α Aquilæ . . .	19 45.4	+ 8 35	1	0.20	3	16.3	0.65	9.6
20	o² Eridani	4 10.2	− 7 49	4.5	0.19	2	17.2	4.05	62.7
21	Arg.-Oeltzen 17415-6	17 34.1	+ 68 27	9	0.18	2	18.1	1.27	20.8
22	Σ 1516	11 8.1	+ 74 4	7	0.17 ?	1	19.2	0.42	7.3
23	β Cassiopeiæ . . .	0 3.3	+ 58 33	2.4	0.16	2	20.4	0.55	10
24	Vega	18 33.5	+ 38 41	0.2	0.16	3	20.4	0.36	6.6
25	e Eridani	3 15.5	− 43 29	4.4	0.14	3	23.3	3.03	63.7
26	Arcturus	14 10.6	+ 19 45	0.3	0.13	1	25.1	2.28	51.6
27	α Tauri	4 29.6	+ 16 17	1	0.116	2	28.2	0.19	4.8
28	α Aurigæ	5 8.6	+ 45 53	0.2	0.107	2	30.4	0.43	11.8
29	α Leonis	10 2.5	+ 12 30	1.4	0.093	2	35.1	0.27	8.5
30	Groombridge 1830 .	11 46.6	+ 38 31	6.5	0.087 ?	1	37.5	7.05	239 ?
31	Polaris	1 18.5	+ 88 43	2.1	0.074	3	44	0.045	1.8
32	α Cassiopeiæ . . .	0 34.3	+ 55 56	2.2	0.071	1	46	0.05	2.1
33	β Geminorum . .	7 38.6	+ 28 17	1.1	0.068 ?	1	48	0.64	27.7
34	ζ Toucani	0 14.2	− 65 31	4.1	0.057	2	57	2.05	106
35	85 Pegasi	23 56.4	+ 26 30	5.8	0.054	1	60	1.29	70.3

Canopus, α Orionis, α Cygni, β Centauri, and γ Cassiopeiæ, all of them stars of the first or second magnitude, have also been carefully observed, and have yielded no parallax exceeding 0″.05.

In the table the column headed "weight" indicates roughly the probable reliability of the parallax given, — the estimate depending on the character, number, and accordance of the different determinations for the star in question. The average "probable error" for the parallaxes of the table may be taken as about 0″.04, *i.e. it is just as likely as not* that an average parallax, weighted 2 or 3, may be wrong by that amount.

The original paper of Oudemans contains all the data then available: in the cases of several of the stars they are very discordant and unsatisfactory, so that it is to be expected that ultimately some of the results tabulated above will prove seriously incorrect.

TABLE V.—ORBITS OF BINARY STARS.

(Mostly from Dr. See's List of Orbits, Astronomical Journal, Vol. XVI., 1896.)

No.	Name.	α (1900).	δ (1900).	Period.	a″.	e.	Periastron.	Magnitudes.	Authority.
1	Ll. 9091	4^h $45^m.7$	+ 10° 54′	5y.5 ± 0.1	0″.62	0.760	1896.40	8.0 : 8.0	See.
2	κ Pegasi	21 40.1	+ 25 11	11.42 ± 0.4	0.42	0.490	1896.03	4.3 : 5	"
3	δ Equulei, A.B. . .	21 9.6	+ 9 37	11.45 ± 0.2	0.45	0.165	1892.80	4.5 : 5	"
4	ζ Sagittarii. . . .	18 56.3	− 30 1	18.85 ± 1.0	0.69	0.279	1878.80	3.9 : 4.4	"
5	42 Comæ Ber. . .	13 5.1	+ 18 4	25.56 ± 0.1	0.64	0.461	1885.69	6 : 6	"
6	ζ Herculis	16 37.6	+ 31 47	35.00 ± 0.3	1.43	0.497	1864.80	3 : 6	"
7	η Coronæ Bor. . .	15 19.1	+ 30 39	41.60 ± 0.1	0.91	0.267	1892.50	5.5 : 6	"
8	Sirius	6 40.4	− 16 34	51.8 ± 0.2	7.62	0.600	1893.77	−1.4 : 9	Burnham.
9	γ Androm., B.C. .	1 57.8	+ 41 51	54.0 ± 1.0	0.37	0.857	1892.1	5.5 : 7	See.
10	ξ Ursæ Majoris . .	11 12.9	+ 32 6	60.00 ± 0.1	2.50	0.397	1875.20	4 : 5	"
11	ζ Cancri, A.B. . .	8 06.2	+ 17 58	60.0 ± 0.5	0.85	0.340	1870.40	5.5 : 6.2	"
12	γ Coronæ Bor. . .	15 38.5	+ 26 36	73.0 ± 2.0	0.73	0.482	1841.0	4 : 7	"
13	α Centauri	14 32.6	− 60 25	81.10 ± 0.3	17.70	0.528	1875.70	1 : 2	"
14	70 Ophiuchi . . .	18 0.4	+ 2 33	88.40 ± 1.0	4.54	0.500	1896.47	4.5 : 6	"
15	φ Ursæ Majoris . .	9 45.3	+ 54 33	97.0 ± 5.0	0.34	0.440	1884.0	5.5 : 5.5	"
16	ω Leonis	9 23.1	+ 9 30	116.2 ± 1.0	0.88	0.537	1842.1	6 : 7	"
17	ξ Boötis.	14 46.8	+ 19 31	128.0 ± 1.0	5.56	0.721	1903.9	4.5 : 6.5	"
18	γ Virginis	12 36.6	− 0 54	194.0 ± 4.0	3.99	0.897	1836.5	3 : 3.2	"
19	η Cassiopeiæ . . .	0 42.9	+ 57 18	195.8 ± 10.0	8.21	0.514	1907.8	4 : 7	"
20	σ Coronæ Bor. . .	16 11.0	+ 34 7	370.0 ± 25	3.82	0.540	1821.8	6 : 7	"
21	36 Andromedæ . .	0 49.6	+ 23 5	349.1 ?	1.54	0.634	1798.8	6 : 7	Doberck.
22	α Geminorum . . .	7 28.2	+ 32 8	997 ? ?	7.54	0.344	1750.3	2.5 : 3	Thiele.

TABLE VI. — VARIABLE STARS.

A selection from Dr. S. C. Chandler's third Catalogue (July, 1896) containing such as are visible to the naked eye, have a range of variation exceeding half a magnitude, and can be seen in the United States.

No.	NAME.	Place, 1900.		Range of Variation (Mag.).	Period (Days).	Remarks.
		α	δ			
1	T. Ceti	$0^h\,16^m.7$	$-20°\,37'$	5.1– 7.0	65 ±	Very irreg.
2	R. Andromedæ .	0 18.8	+ 38 1	5.6–12.8	410.7	
3	α Cassiopeiæ . .	0 34.5	+ 55 59	2.2– 2.8		Not periodic.
4	o Ceti (*Mira*) . .	2 14.3	− 3 26	1.7– 9.5	331.6	Large irregularities in date & brightness.
5	ρ Persei	2 58.8	+ 38 27	3.4– 4.2	33 ?	Very irreg.
6	β Persei (Algol) .	3 1.7	+ 40 34	2.3– 3.5	$2^d\,20^h\,48^m\,55^s.43$	Period now shortening.
7	λ Tauri	3 55.1	+ 12 12	3.4– 4.2	$3^d\,22^h\,52^m\,12^s$	Algol type. Irregular.
8	ϵ Aurigæ . . .	4 54.8	+ 43 41	3.0– 4.5		Not periodic.
9	α Orionis . . .	5 49.7	+ 7 23	0.7– 1.5		Not periodic.
10	η Geminorum .	6 8.8	+ 22 32	3.2– 4.2	231.4	
11	ζ Geminorum . .	6 58.2	+ 20 43	3.7– 4.5	$10^d\,3^h\,41^m\,30^s.6$	
12	R Canis Maj. . .	7 14.9	− 16 12	5.9– 6.7	$1^d\,3^h\,15^m\,46^s$	Algol type.
13	R Leonis Min. .	9 39.6	+ 34 58	6.0–13.0	370.5	
14	R Leonis . . .	9 42.2	+ 11 54	5.2–10.0	312.8	
15	U Hydræ . . .	10 32.6	− 12 52	4.5– 6.3	195 ± ?	Very irreg.
16	R Ursæ Maj. . .	10 37.6	+ 69 18	6.0–13.2	302.1	
17	R Hydræ . . .	13 24.2	− 22 46	3.5– 5.5	425.15	Period shortening.
18	S Virginis . . .	13 27.8	− 6 41	5.7–12.5	376.4	
19	R Boötis . . .	14 32.8	+ 27 10	5.9–12.2	223.4	
20	δ Libræ	14 55.6	− 8 7	5.0– 6.2	$2^d\,7^h\,51^m\,22^s.8$	Algol type.
21	R Coronæ . . .	15 44.4	+ 28 28	5.8–13.0		Not periodic.
22	R Serpentis . .	15 46.1	+ 15 26	5.6–13.0	357.0	
23	α Herculis . . .	17 10.1	+ 14 30	3.1– 3.9	60 to 90^d	Not periodic.
24	U Ophiuchi . .	17 11.5	+ 1 19	6.0– 6.7	$20^h\,7^m\,42^s.56$	
25	u Herculis . . .	17 13.6	+ 33 12	4.6– 5.4		Irreg.periodic.
26	X Sagittarii . .	17 41.3	− 27 48	4.0– 6.0	$7^d\,0^h\,17^m\,57^s$	
27	W Sagittarii . .	17 58.6	− 29 35	4.8– 5.8	$7^d\,14^h\,16^m\,13^s$	
28	Y Sagittarii . .	18 15.5	− 18 54	5.8– 6.6	$5^d\,18^h\,33^m\,24^s.5$	
29	R Scuti	18 42.1	− 5 49	4.7– 9.0	71.1	Very irreg.
30	β Lyræ	18 46.4	+ 33 15	3.4– 4.5	$12^d\,21^h\,47^m\,23^s.72$	
31	R Lyræ	18 52.3	+ 43 49	4.0– 4.7	46.4	
32	χ Cygni	19 46.7	+ 32 40	4.0–13.5	406.02	Period lengthening.
33	η Aquilæ . . .	19 47.4	+ 0 45	3.5– 4.7	$7^d\,4^h\,11^m\,59^s$	
34	S Sagittæ . . .	19 51.4	+ 16 22	5.6– 6.4	$8^d\,9^h\,11^m\,48^s.5$	
35	X Cygni	20 39.5	+ 35 14	6.4– 7.7	$16^d\,9^h\,15^m\,7^s$	
36	T Vulpeculæ . .	20 47.2	+ 27 53	5.5– 6.5	$4^d\,10^h\,27^m\,50^s.4$	
37	T Cephei . . .	21 8.2	+ 68 5	5.2–10.7	387	
38	μ Cephei . . .	21 40.4	+ 58 19	4.0– 5.5	430 ±	Irreg.periodic.
39	δ Cephei . . .	22 25.4	+ 57 54	3.7– 4.9	$5^d\,8^h\,47^m\,39^s.3$	
40	β Pegasi	22 58.9	+ 27 32	2.2– 2.7		Not periodic.
41	R Aquarii . . .	23 38.6	− 15 50	5.8–11?	387.16	
42	R Cassiopeiæ . .	23 53.3	+ 50 50	4.8–12	429.5	

TABLE VII. — VELOCITY OF STARS IN THE LINE OF SIGHT. — VOGEL.

The velocities are given in English miles per second. The sign, +, indicates recession.

NAME.	Mag.	Class.	Vel.	NAME.	Mag.	Class.	Vel.
α Andromedæ	2.0	Ia	+ 2.8 m.	γ Leonis	2.0	IIa	−24.1 m.
β Cassiopeiæ	2.1	Ia–IIa	+ 3.2	β Ursæ Majoris	2.3	Ia	−18.5
α Cassiopeiæ	var.	IIa	− 9.7	α Ursæ Majoris	2.0	IIa	−12.0
γ Cassiopeiæ	2.0	Ic	− 2.3	δ Leonis	2.3	Ia	− 8.8
β Andromedæ	2.3	IIa	+ 6.9	β Leonis	2.0	Ia	−12.0
α Ursæ Minoris	2.0	IIa	−16.1	γ Ursæ Majoris	2.3	Ia	−16.6
γ Andromedæ	2.4	IIa	− 7.8	ε Ursæ Majoris	2.0	Ia	−19.0
α Arietis	2.0	IIa	− 9.2	α Virginis	1	Ia	− 9.2
β Persei	var.	Ia	− 0.9	ζ Ursæ Majoris*	2.1	Ia	−19.5
α Persei	2.0	IIa	− 6.5	η Ursæ Majoris	2.0	Ia	−16.1
α Tauri	1	IIa	+30.1	α Bootis	1	IIa	− 4.6
α Aurigæ	1	IIa	+15.2	ε Bootis	2.0	IIa	− 9.7
β Orionis	1	Ib	+10.1	β Ursæ Minoris	2.0	IIa	+ 8.8
γ Orionis	2.0	Ia	+ 5.5	β Libræ	2.0	Ia	− 6.0
β Tauri	2.0	Ia	+ 5.1	α Coronæ Borealis	2.0	Ia	+19.9
δ Orionis	2.5	Ia	+ 0.5	α Serpentis	2.3	IIa	+13.8
ε Orionis	2.0	Ib	+16.6	β Herculis	2.3	IIIa	−22.2
ζ Orionis	2.0	Ia	+ 9.2	α Ophiuchi	2.0	Ia	−12.0
α Orionis	var.	IIIa	+10.6	α Lyræ	1	Ia	− 9.7
β Aurigæ	2.0	Ia	−17.5	α Aquilæ	1.3	Ia	−23.7
γ Geminorum	2.3	Ia	−10.1	γ Cygni	2.4	IIa	− 4.1
α Canis Majoris	1	Ia	− 9.7	α Cygni	1.6	Ib	− 5.1
α Geminorum*	2.8	Ia	−18.4	ε Pegasi	2.3	IIa	+ 5.1
α Canis Minoris	1	Ia–IIa	− 5.5	β Pegasi	var.	IIIa	+ 4.1
β Geminorum	1.3	IIa	+ 0.9	α Pegasi	2.0	Ia	+ 0.9
α Leonis	1.3	Ia	− 5.5	ζ Herculis* (Belopolsky)	3.1	IIa	−48.8

* The brighter component observed.

TABLE VIII. — MEAN REFRACTION.

Corresponding to temperature of 50° F., and to a barometric pressure of 29.6 inches.

Altitude.	Refraction.	Altitude.	Refraction.	Altitude.	Refraction.
0°	34′ 50″	11°	4′ 47″.7	30°	1′ 39″.5
1°	24 22	12°	4 24 .5	35°	1 22 .1
2°	18 06	13°	4 04 .4	40°	1 08 .6
3°	14 13	14°	3 47 .0	45°	57 .6
4°	11 37	16°	3 18 .2	50°	48 .3
5°	9 45	18°	2 55 .5	55°	40 .3
6°	8 23	20°	2 37 .0	60°	33 .2
7°	7 19	22°	2 21 .6	65°	26 .8
8°	6 29	24°	2 08 .6	70°	20 .9
9°	5 49	26°	1 57 .6	80°	10 .2
10°	5 16	28°	1 48 .0	90°	0 .0

For every 5° F. by which the temperature is *less* than 50° F., *add one per cent* to the tabular refraction, and decrease it in the same ratio for temperatures above 50° F.

Increase the tabular refraction by *three and a half per cent* for every inch of barometric pressure above 29.6 inches, and decrease it in the same ratio below that point. These corrections for temperature and pressure, though only approximate, will give a result correct within 2″ except in extreme cases.

INDEX.

See also the Supplementary Index.

[All references, unless expressly stated to the contrary, are to *articles* and not to *pages*.]

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SUPPLEMENTARY INDEX.

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